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Cybernetic Models of Economic Growth

Quantitative models are widely used in cosmology, physics and chemistry already for a long time. The use of these models has led to a thorough knowledge of the reality under study. During the last decades an important growth was recorded as regards the amount of quantitative models in social sciences, and particularly in economy. These researches are expected to allow a better description and control of economic phenomena, which are more complex than physical ones. We intend to present here some cybernetic models of economic growth.

At first, cybernetic systems are defined, then the possibility of studying certain types of equations with analogical methods is analized, and in the end several cybernetic models for economic growth are presented.

In the Report presented at the Eleventh Congress of the Romanian Communist Party, President Nicolae Ceauşescu said "The ample programm for socialist industry development forseen by the next Five-Year Plan calls for firm measures in view of organizing on a scientific basis production and work"...

"It is therefore essential to raise the whole management and organizational activity to the level of present requirements as regards development of national economy, to place it on a scientific basis" ...

In June 1968, President Nicolae Ceauşescu also pointed out economic cybernetics as one of the new branches of science called to play an important part in helping the politics of the Romanian Communist Party in view of a manysided development of the Romanian society. "Among the recently emerging fields in contemporary scientific research, an important place is taken by the management of economic processes, of production and work organization. The science of management has asserted itself to general attention in many countries throughout the world. High results are obtained

through modern technical computing and automation means, through analyse tools offered by the newest fields of mathematics, cybernetics and economic information science".

In view to develop productive forces and production relations, to improve patterns and methods of economic management, it is necessary to develop the scientific basis of political economy of socialism. A thorough knowledge of the most important aspects of economy, of its laws, is therefore essential. Models in economy are precisely useful in view of a better insight as regards economic facts. But economy is a complex phenomenon. It is therefore important to correlate the study of economic models with political economy.

The use of models in economy has become a necessity. "In economic cybernetics, just as in general cybernetics and in the science of management, the concept of model occupies a prevailing place" [1].

Models in the main, and particularly economic models are of two types: structural and dynamic. Structural models reflect the internal elements of an economy at a given moment. Dynamic models involve the body of parameters defining time-development of the system under study.

1. CYBERNETIC SYSTEMS

Cutting out a fraction of reality can be achieved following various criteria. This amounts to the possibility to consider a system as being composed of different subsystems according to the adopted criteria. Conversely, any system, exception made of Universe, may be considered as a subsystem belonging to another system which contains it. As regards substance, the existence of a lower limit for elementarity is admitted, the so-called quarcs. Systems can be oriented or non-oriented. Economics is interested mainly in oriented systems.

Let us consider the class of oriented systems. Such a system is described by input variables, output variables, operators linking together these values and the initial state of the system. Denoting by u the input variable, by x the state and by y the output variable of the system:

$$u \in U, \quad x \in X, \quad y \in Y$$

where U, X and Y are, respectively, the input, state and output spaces.

Between the characteristic variables are holding relations of the following type

$$y = y(x_0, u)$$

where x_0 is the initial state of the system.

In cybernetics we are interested by time varying systems. As a result relations of the type $y = y(t)$, $u = u(t)$ are also used, where t denotes time.

Under another form we may write as well

$$y = Au$$

where A is a transformation operator, which includes also the initial state of the system.

One may also use the approach of Prof. St. Bîrlea [2] which describes a system by

$$\mathcal{S} = [U,\ Y \mid A].$$

It can be considered that the above-mentioned operator and the initial state are proper to the system. It is assumed that the system is causal, in the contemporary meaning of the term (Nicolau, 1974) [3] and allows a single solution determined by the operator A — i.e. by the relation between x and y and by the initial condition.

Any oriented system can be represented by an arc oriented from the input towards the output. The arc is labelled by means of the operator corresponding to the element and of the initial state (Nicolau and Popovici, 1967) [4].

In this manner an oriented arc is associated to every subsystem of a given system. The system under consideration is put into correspondence with an oriented and labelled graph.

We define a cybernetic system as one which has an attached graph containing at least one loop.

It was admitted tacitly that the subsystems are such that the intersection between the output space of a subsystem and the input space of at least one subsystem with which the former is connected is not empty. In other words, among the subsystems connected at the output of a given subsystem, there is always at least one which accepts as input signals, the signals which are outputs for the first subsystem [5].

2. TYPES OF CYBERNETIC SYSTEMS

To have a loop, a graph must contain at least two arcs which both link together points A and B, one being oriented from A to B, and the other from B to A. If the system has no input, and the output consists of signals from node B, we are faced with an autonomous system, in the actual meaning of the word.

Such a cybernetic system can be described by the following equations;

$$y_1 = L_1 u_1 \qquad u_2(t) = y_1(t) \qquad x_1(0) = x_{10}$$

$$y_2 = L_2 u_2 \qquad u_1(t) = y_2(t) \qquad x_2(0) = x_{20}$$

where y denotes outputs, u inputs (controls), and L an operator describing the two subsystems; are given, in addition, the initial states of the two subsystems, which determine the (unique) subsequent evolution of the system consisting of the two interconnected subsystems.

Of course, the relation between states and outputs is also specified for every subsystem.

In physico-mathematical and engineering problems operator L is often a deterministic linear operator. L may be a differential operator of the nth order — as in the case of functions for linear electrical networks, of linear systems in general, or a self -adjunct operator.

As a rule, L complies with a Green function, i.e. an integral kernel, so that equation

$$Lf(x) = s(x)$$

complies with the relation

$$f(x) = L^{-1}s(x) = \int_{-\infty}^{+\infty} k(x,\ x')\, s(x')\, \mathrm{d}x'.$$

Actually, however, not all phenomena are deterministic. In the case of stochastic models, we can write similarly

$$\mathcal{L}f(x) = s(x)$$

$$f(x) = \mathcal{H}s(x).$$

In the first case $\mathcal{L}$ is a differential operator, and $f(x)$ can be an ordinary function or a random function, with known statistical properties. In the latter case $f(x)$ is a stochastic process, for which the statistical properties must be determined.

The second type of equation which complies with stochastic methods brings in the operator $\mathcal{H}$, i.e. a random transformation which changes $s(x)$ into another random function, $f(x)$. From a certain point of view, $\mathcal{H}$ can be associated with a filter, described by several parametra, of which at least one is random (Adomian) [6].

For the stochastic case we have at our disposal various methods: symbolic formulation, perturbation theory, Green functions etc. We may resort to hierarchical equations — as in theoretical physics, or to step-by-step methods.

Fuzzy operators must be also mentioned (Zadeh, Negoiță and Ralescu) [7]. Lately, stochastic as well as fuzzy methods find ever increasing economic applications.

Remaining at the elementary level, we note that the most simple case of a cybernetic system is that in which the two arcs correspond to two algebraic operators, L_1 and L_2, of the type $y = \sum_{i=0}^{n} a_i\ u^i$ where operator L points out the transition from $u(t)$ to a polynomial in $u(t)$. Such opperators appear, for instance, in the case of some non-linear systems and of certain functional transformators. A simple case is that where the operators induce a delay

$$y_1(t) = au_1(t - \tau)$$

$$y_2(t) = bu_2(t - \tau).$$

Economic models often contain relations with delayed argument, a natural situation due to the time elapsed between an investment is started and the moment it becomes economically active.

It must be mentioned that in many cases econometric models have a deterministic character, a and b being constants.

3. ECONOMETRIC MODELS AND CYBERNETIC MODELS

L.F. Buchanan and F.E. Norton (1971), applying the techniques of automatic control to economic systems, have emphasized that it is possible to describe certain economic models as feedback systems, to which subsequently optimization techniques can be applied [8]. The idea was actually formulated earlier, in 1953 , by Tustin [9] and used later, in 1957, by Phillips [10].

In the study of L.F. Buchanan and F.E. Norton, the cybernetic model is presented as a series of interlinked standard blocks and contains following elements: adders with two inputs; devices subtracting the two inputs; delay circuits; multiplying devices with constant coefficients; an overall adder with n inputs.

These elements are linked together in view of modelling the system of relations which comply with the economic system under consideration.

4. ECONOMETRIC MODELS

Specific to econometric models is the fact that they establish a series of relations between the variables considered representative for a given economy. An econometric model is a set of algebraic relations expressing interactions between various variables which describe the evolution of an economy within the frame of the model under consideration.

Generally, mathematical models of the following type are involved

$$x_i = \sum_{j=0}^{n} a_{ij}x_j - b_i, \quad i = \overline{1, m} \quad m < n$$

or

$$X^* = AX - B$$

where X^* denotes the vector containing the n variables chosen to describe the given economy at moment t, X the same vector at moment $t-1$, A — a $n \times n$ matrix and B — a constant vector. Coefficients a_{ij} and b_i are inferred from econometric data. Usually these models hold for a projection horizon of a term, so that after each quarter A and B have to be updated. We specify that in this type of model matrix A and vector B have many zero coefficients.

The validity of econometric models is checked statistically, using certain indices, a current proceeding in statistics. Published data emphasize that under above-stated conditions (quarterly horizon) the models give values approaching those actually observed.

On this line are to be mentioned the model of the Belgian economy, the MIT model of the United States economy etc. The even brief description of some of these models will allow us to infer the main characteristics of econometric models.

5. SHORT-TERM FORECASTING MODEL FOR THE ECONOMY OF BELGIUM

The goal aimed at was the development of a model which, based on quarterly data, would allow to determine the national product in the next quarter of the year. The model is highly aggregated [11]. Thus, one can identify several groups of macroeconomic variables, prices, financial and exogenous variables, other variables — on the whole 30 variables. The model involves 23 relations between the above-mentioned variables or their equivalents, computed at constant prices.

At first, we will specify the groups of variables used: aggregated macroeconomic variables, prices, financial variables, exogenous variables, other variables.

The complete list of variables and symbols used is given below:

a) Macroeconomic aggregated variables (percentage change):

PNB — gross national product; DNB — gross national expenses; PIB — gross internal product; PID — industry output; C — private consumption; Cd — private consumption, deseasonalized and unlevelled; I — gross formation of fixed capital; X — exports; M — imports; G — public consumption; W — wages.

We specify that the same variables written with small characters are measured at constant prices.

b) Prices (percentage change)

$pPNB$ — price index for the gross national product; pM — price index for imports; pC — price index for private consumption; pdt — price index for retail trade; pG — price index for public consumption; pX — price index for exports.

c) Financial variables (percentage change)

Cr — credits granted to the private sector; L — available liquidities in the private sector; Sc — wages versus life cost; T — ratio between receipt of direct imports and PNG; Gh — hourly earnings in industry.

d) Exogenous variables (percentage change)

CEE — industrial output of the Common Market (European Economic Community) at constant prices; EU — industrial output of the United States at constant prices.

e) Other variables

com — estimation of the order protofolio — value obtained by quantization from conjucture studies; cm — orders for metallic production(percentage); Gh — unemployment (percentage); im — immigration (percentage); CH — absolute number of unemployed, thousands; ΔCH — variation of the number of unemployed, thousands.

In the following we will present the model's equations. We specify that the values calculated by means of the model's relations are denoted by an asterisk. Index t indicates the quarter of the year for which the forecast is made. All variables are previously deseasonalized and levelled.

The equations of the system are:

$$pnb_t^* = 0.183\, L_{t-2} + 0.085\, Cr_{t-1} + 0.041\, com_{t-1} + 0.035\, EU_{t-1} + \\ + 0.164\, CEE_t + 0.468$$

$$pPNB_t^* = 0.12\, pM_{t-2} + 0.19\, Sc_{t-1} + 0.80$$

$$pNB_t^* = pnb_t^* + pPNB_t^*$$

$$im_t^* = -0.0187\ CH_{t-1} + 4.50$$

$$\Delta CH_t^* = im_t^* - 7.59\ pnb_t^* + 4.30$$

$$CH_t^* = CH_{t-1} + CH_t^*$$

$$W_t^* = -0.014\ CH_t^* + 0.26\ pdt_{t-1} + 3.75$$

$$pdt_t^* = 0.29\ W_t^* + 0.027\ pM_{t-1} + 0.053\ T_{t-1} - 0.024$$

$$Ch_t^* = (CH_t^* - CH_{t-1})\ 100/CH_{t-1}$$

$$C_t^* = -0.11\ Ch_t^* + 1.32\ CH_{t-1} - 1.03\ pc_{t-1} + 1.40$$

$$pC_t^* = pdt_t^*$$

$$c_t^* = C_t^* - pC_t^*$$

$$i_t^* = 0.24\ cm_t + 1.14\ pnb_{t-1} + 0.17\ i_{t-1} - 0.12$$

$$I_t^* = 0.20\ cm_{t-1} + 0.94\ pnb_{t-1} + 0.30\ I_{t-1} + 0.14$$

$$X_t^* = 0.57\ CEE_t + 0.26\ EU_{t-1} + 0.59\ PNB_{t-1} + 0.42\ X_{t-1} - 1.29$$

$$x_t^* = X_t^* - pX_t$$

$$M_t^* = 0.43\ PNB_t^* + 0.37\ X_t^* + 0.92\ pnb_{t-1} + 0.50\ pid_{t-2} -$$

$$-0.26\ X_{t-1} + 1.49$$

$$m_t^* = M_t^* - pM_t$$

$$G_t^* = 0.25\ G$$

$$g_t^* = g_t$$

$$dnd_t^* = 0.672\ c_t^* + 0.207\ i_t^* + 0.121\ g_t^* + 0.370\ x_t^* - 0.370\ m_t^*$$

$$pnb2_t = 0.85\ pnb_t^* + 0.15\ dnb_t^*$$

$$PNB2_t = pnb2_t + pPNB_t^*.$$

This model emphasizes a series of findings. Thus, though we have a system with 23 equations and 40 unknown quantities, out of the 690 possible coefficients only 79 are used in the equations, to which 11 free terms are to be added. This means that many bonds are unsignificant and therefore neglected. Matrix A is sparse, as well as matrix B, i.e. many coefficients are zero. It is also interesting to examine other aspects. Thus, it follows immediately that the economy of Belgium is strongly integrated into the European Economic Community — Common Market — and has strong links with the United States, considering that only these two economic structures appear in the model.

The analyse of the model also points out some political facts. Thus, the model uses immigration — alien manpower, these alien workers affecting the level of wages through the number of unemployed. One must note as

well an explanation of inflation by wage correlation with certain characteristic indices.

In view to estimate the quality of the model, statistical indices were used. These emphasized, that, on the whole, the model is satisfying.

6. THE "FEDERAL RESERVE — MIT" ECONOMETRIC MODEL

The model was developed by a team of economists working at the Massachusetts Institute of Technology, and by the Governors' Office of the Federal Reserve System, and covers a quarter of a year. Its goal is to foresee the consequences of monetary policy [12].

The model contains three blocks: finances, investments and consumption.

This econometric model tries mainly to emphasize the essence of monetary policy and its consequences upon economy. For this reason the authors of the model focused their attention upon financial markets, and upon the links between the financial markets and those of goods and services. Without entering into detail, we mention that the model has 12 equations in the financial block, 32 equations in the investment block and 21 equations in the consumption and inventory block. The number of characteristic values taken into consideration by the model varies, and even the values considered characteristic vary from one model to another.

In the following, a simple econometric model, mentioned in literature [13], will be presented:

$$C_t = \alpha_0 + \alpha_1 Y_t + \alpha_2 C_{t-1} + u_{1t}$$

$$I_t = \beta_0 + \beta_1 P_t + \beta_2 K_{t-1} + u_{2t}$$

$$W_t = \gamma_0 + \gamma_1 Y_t + \gamma_2 t + u_{3t}$$

$$Y_t = C_t + I_t + G_t$$

$$P_t = Y_t - W_t$$

$$K_t = K_{t-1} + I_t.$$

Here C denotes consumption, Y — net product, W — wages, P — other incomes, I — net investments, K — net capital stocks at the end of the period, t — time, G — federal expenses for goods and services. u_i denote disturbance factors. Index t indicates that the respective value is considered over a period t; index $t - 1$ indicates the preceeding period. In many models the period is a quarter.

7. THE "OBE — OFFICE OF BUSINESS ECONOMICS" MODEL

This model is an alternative of the model developed by the Whartan School of Finance and Commerce, University of Pennsylvania, and contains 36 equations which may be classified into six goups: components of the gross national product *(GNP)*, prices and wages, manpower, components

of the income, monetary variables, other variables. The model covers a quarter of an year [14].

Out of the 36 equations, 13 are identities; the relations involve 39 variables.

8. A GENERAL THEOREM

A different class of models tries to grasp the economic facts in their dynamics. These are dynamic models, which take into account a number of characteristic time-changing variables, as well as their interrelations. The models belonging to this class are often described by linear algebraic relations and by linear differential equations. In certain cases, relations with delayed argument appear, corresponding to the delayed transformation of investments into productive units. An important property may be stated for the models described by linear integral-differential equation with constant coefficients.

Theorem. In any model described by linear integral-differential equations with constant coefficients, the characteristic variables are linear combinations of time exponential functions, if the characteristic equation has only simple roots.

The demonstration of this statement is given below. An aggregated model is considered, in which economy is reduced to a single synthetic index denoted x. It satisfies the equation:

$$\dot{x} = ax + b\int_0^t f(t-z)\, x(z)\, dz + c, \quad x(0) = d$$

where a, b, c and d are constants. Applying the onesided Laplace transformation, we obtain:

$$sX(s) - d = aX(s) + bF(s)\, X(s) + c$$

from which

$$X(s) = (c + D)/(s - a - bF(s)).$$

Function $F(s)$ is the transformation of function $f(t - z)$ which measures the effect — at moment t — of investments made at a previous moment — z. We assume that function $F(s)$ is such that equation

$$s - a - bF(s) = 0 \tag{1}$$

has only N distinct roots. In this case we write

$$X(s) = \sum_{i=1}^{N} m_i/(s - s_i)$$

therefore

$$x(t) = \sum_{i=1}^{N} m_i\ exp\ s_i t$$

s_i being the roots of equation (1) which relates s to $F(s)$.

The demonstration can be extended to a system of linear relations, under the same assumption of a characteristic equation with distinct roots.

9. CONCLUSIONS

As could be observed, growth models proposed by literature — either continuous, or with discrete time — may be considered cybernetic models, due to the loops appearing in the graph.

In the case of systems with discontinuous time, equations have the following form

$$x_i^{\bullet} = \sum a_{ij} x_j + b_j$$

which can be simulated resorting only to delay lines denoted τ (which introduce the gap between x and x), "coefficient" devices (which multiply with generally positive and always real scalars) and adders. If b are random, it is necessary to include in the model generators of random numbers. In the case of continuous time, the equations have the form

$$\dot{x}_i = a_{ij} x_j + b_j.$$

We are again faced with a feedback system, which can be modelled giving up delay devices and using integrators.

Concluding, economic growth models can mostly be expressed as feedback systems, i.e. cybernetic systems. Another result is the possibility of using cybernetic techniques to ensure stability and optimality of the modelled systems.

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MODELES CYBERNETIQUES DEVELOPEMENT ECONOMIQUE

Il y a longtemps que les modèles quantitatifs sont utilisés amplement en cosmologie, physique et chimie. L'utilisation de ces modèles a mené à la connaissance approfondie de la réalité étudiée. Ces dernières décennies ont connu une augmentation importante du nombre des modèles quantitatifs utilisés, en sciences sociales et particulièrement en économie. Il est à prévoir que ces recherches permettent une meilleure description et la maitrise des phénomènes économiques, qui s'avèrent être de beaucoup plus complèxes que les phénomènes physiques. Dans cette étude, l'auteur se propose de présenter quelques modèles cybernétiques de développement économique.

On définit d'abord les systèmes cybernétiques, aprés quoi l'on montre la possibilité d'étudier certains types d'equations à l'aide des techniques analogiques et l'on présente quelques modéles de développement économique, du type cybernétique.

КИБЕРНЕТИЧЕСКИЕ МОДЕЛИ ЭКОНОМИЧЕСКОГО РАЗВИТИЯ

Каличественные модели уже давно используются довольно широко в космологии, физики и химии. Использование этих моделей привело к глубокому познанию изучаемой действительности. В течение последних десятилетий констатируется значительный рост количественных моделей, используемых в общественных науках и особенно в экономике. Весьма возможно, что эти исследования дадут возможность лучше описать и познать экономические явления, которые значительно сложнее физических. В настоящей работе представляются некоторые кибернетические модели экономического развития.

Вначале определяются кибернетические системы, в которых показывается возможность изучения некоторых типов уравнений, используя аналогическюу технику и представляются некоторые модели экономического развития кибернетического типа.