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On Considering Imprecision in Dynamic Linear Programming

The study presents a new trend in the tackling of dynamic linear programming problems , namely the consideration of tolerances admitted by the coefficients occuring in the constraints expression. After underlining that a dynamic linear programming problem can be reduced to an ordinary linear programming one, the analysis concentrates on the various possibilities of taking imprecision into account. Thus, the study reviews the case of problems with set coefficients and also the case of problems with fuzzy set coefficients. The whole presentation is illustrated by examples especially from the garments industry where the authors have found current problems requiring an approach of this type.

1. INTRODUCTION

In the recent years, a tendency has been noticed to use dynamic linear programming techniques in planning and production scheduling [1], [2].

According to these techniques, the evolution of the economic process is described by a state equation to which if constraints and a performance functional are added, we come to the following general form of an optimzing problem:

$$\begin{aligned} & x(k+1) = A(k)\, x(k) + B(k)\, u(k) + S(k) \\ & P(k)\, x(k) + Q(k)\, u(k) \leqslant R(k) \\ & x(k), u(k) \geqslant 0 \ \ (lk = 0, ..., N-1) \\ & x(0) = x^{\circ} \\ & min \left\{ \alpha^T(N)\, x(N) + \sum_{k=0}^{N-1} [\alpha^T(k)\, x(k) + \beta^T(k)\, u(k)] \right\} \end{aligned} \tag{1}$$

where the symbols have the following economic significance:

Symbol	*Interpretation*	*Dimension*
k	Time period index	1
N	Programming horizon	1
x	Inventory vector	m
u	Vector of manufacturing orders	n
S	Supply and delivery vector	m
A	Matrix of connections between the inventories of two periods running.	$m \times m$
B	Matrix of production coefficients	mxn
P	Matrix with inventory utilization coefficients	pxm
Q	Specific consumption matrix	pxn
R	Resources vector	p
α	Inventory unitary costs	m
β	Order unitary costs	n

Characteristic of this approach is the grasping of the connections in time between the parameters of the examined economic process.

By solution to problem (1) we mean the series of values

$$\{x^*(k+1),\ u^*(k)\}_{k=0,\ldots,N-1}$$

satisfying the state equations and the constraints minimizing the performance functional.

There are several methods to solve this problem [3]; for instance, by linear programming, respectively by the utilization of the revized simplex algorithm with the inverse in a product form incorporated into several firm packages (OPALINE, SIMPMINI for the FELIX, C-256 LPS for IBM, etc.)

Indeed, problem (1) can be brought to form:

$$\begin{gathered} Hy = \lambda \\ y \geqslant 0 \\ min\ q^T y \end{gathered} \tag{2}$$

where:

$$H = \begin{bmatrix} -I & 0 & 0 & B(0) & 0 & 0 & 0 & 0 \\ A(1) & -I & 0 & 0 & B(1) & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -I & 0 & 0 & B(N-2) & 0 & 0 \\ 0 & 0 & A(N-1) & -I & 0 & 0 & B(N-1) & 0 \\ 0 & 0 & 0 & Q(0) & 0 & 0 & E & 0 \\ P(1) & 0 & 0 & 0 & Q(1) & 0 & 0 & E \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & P(N-1) & 0 & 0 & 0 & Q(N-1) & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} x(1) \\ \vdots \\ x(N) \\ u(0) \\ \vdots \\ u(N-1) \end{bmatrix}; \quad \lambda = \begin{bmatrix} -[Ax(0) + S(0] \\ -S(1) \\ \vdots \\ -S(N-1) \\ -P(0)\,x(0) + R(0) \\ \vdots \\ R(N-1) \end{bmatrix}; \quad q = \begin{bmatrix} \alpha(0) \\ \alpha(1) \\ \vdots \\ \alpha(N-1) \\ \beta(0) \\ \beta(1) \\ \vdots \\ \beta(N-1) \end{bmatrix}$$

Matrices I and E are unity matrices (mxm) and $(p \times p)$ respectively.

In problem (1) the coefficients, matrices A, B, P, Q, S are considered to be known with precision.

But, it is well known that in economic systems one is always faced with the human subjectivity.

For this reason, we need an instrument able to operate also with inexact elements. Its elaboration is already in a very advanced stage. Soyster [4] and Tugan [5] have approached the so-called inexact programming problem, Bellman and Zadeh [6] have introduced the fuzzy dynamic programming Asai and Tanaka [7] have investigated fuzzy programming, Zimmermann [8] has examined an interesting case of optimization under fuzzy conditions by considering tolerances, Negoiță and Sularia [9] have reformulated the fuzzy programming problem and have applied the tolerance concept to the optimal modification of a linear programming problem without a solution, Negoiță, Flondor and Sularia [10] have given a rigorous foundation to the general problem of fuzzy programming.

In this study, we suggest a way of approaching the fuzzy dynamic linear programming.

In order to be able to present this problem, several stages have been covered, namely:

a) Linear programming with set coefficients,
b) Dynamic linear programming with set coefficients.
c) Fuzzy linear programming,
d) Fuzzy dynamic linear programming.

2. LINEAR PROGRAMMING WITH SET COEFFICIENTS

The essence of this problem consists in a new definition of the feasible region of a linear programming problem [4].

The ordinary linear inequalities $\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i$ are replaced by a finite sum of convex sets which has to be included into a given convex set.

Let $\{K_j\}_{j=1,\ldots,n}$ be n non-empty convex sets and K another non-empty convex set, all included into R^m. We have the following optimizing problem:

$$sup\ c \cdot x,$$

subject to

$$x_1K_1 + x_2K_2 + \ldots + x_nK_n \subseteq K \text{ and } x_j \geqslant 0 \tag{3}$$

where the binary operation + refers to the addition of sets. A set of n scalars $\{\bar{x}_j\}$ is admissible for (3) if and only if for each possible set of vectors $a_j \in K_j$, $j = 1, ..., n$ we have

$$\bar{x}_1 a_1 + \bar{x}_2 a_2 + ... + \bar{x}_n a_n \in K \quad (\bar{x}_j \geqslant 0).$$

If K contains the empty vector, then (3) has the solution $\bar{x}_j = 0$ for all j.

In the following, K has the form:

$$K = \{y \in R^m / y \leqslant b\}.$$

We notice that ordinary linear programming is a special case of problem (3) where the convex sets contain one point.

To solve problem (3) one constructs a matrix $\bar{A}$ with the components:

$$\bar{a}_{ij} = \sup_{a_j \in K_j} a_{ij},$$

a_{ij} being the i-th component of a_j.

It should be noticed that if there is $i \in \{1, ..., m\}$ so that $\bar{a}_{ij} = \infty$, then: $\sum_{j=1}^{n} \bar{x}_j K_j \subseteq K$ implies $\bar{x}_j = 0$. As variable $\bar{x}_j$ must be zero, generality is not lost if omitting the set K_j in problem (3).

Under these conditions we can suppose $\bar{a}_{ij} < \infty$ and the following linear programming problem is defined:

$$max\ c \cdot x, \quad \text{subject to } \bar{A}x \leqslant b \text{ and } x \geqslant 0. \tag{4}$$

It is proved in [4] that if x is an admissible solution for (4), then it is an admissible solution for (3) and vice versa.

It immediately follows that the set of admissible solutions of the two problems (3) and (4) are identical, which means that the optimal solutions of problem (3) can be obtained by solving the linear programming problem (4).

Sets K of the form

$$K = \{y \in R^m / y_i \leqslant b_i, \quad i = 1, 2, ..., m\}$$

can be extended by replacing the i-th constancy i by $y_i \geqslant b_i$. In this case, we replace $\bar{a}_{ij}$ for $j = 1, ..., n$ by $\inf_{a_j \in K_j} a_j$ and we use $\geqslant b_i$ in problem (4).

Example

We consider a linear programming problem with sets coefficients in the form of intervals:

$$\begin{aligned} &[0.02 - 0.03]x_1 + [0.05 - 0.06]x_2 \subseteq [10.5 - 15.6] \\ &[0.1 \ \ - 0.5]x_1 - [0.01 \ \ - 0.02]x_2 \supseteq [0.35 - 0.53] \\ &x_1, x_2 \geqslant 0 \\ &min\ 3x_1 + 2x_2 \end{aligned} \tag{5}$$

Coefficients K are the intervals including the length of fulfilment of the technological operations experimentally obtained (timing, holographs, etc.)

The average length of execution of operations represents an approximation which may lead to the situation in which a manufacturing program cannot be carried out. This is due to the fact that there are lengths of operations execution greater than the mean value which agglomerated into a short time interval may lead to disturbances in the production process.

Similarly, the specific consumptions of materials can be known only as value distributions in the form of K intervals. For example, in the ready-made clothes industry, the specific consumption (in square meter) of a material for making a suit of clothes with given characteristics is variable within the same type of texture because of the design or the different width of the material which implies another arrangement of patterns for cutting.

In this context, in the stages when the reservation of the production capacities is necessary and the design of the materials is not precisely known the interval in which the specific consumption of a material may assume values should be considered.

We similarly treat the case when the coefficients represent technological consumptions of the inter-branch flows, where it is more correct to consider intervals instead of fixed values, taking into account that these consumptions are the result of a multicriteria aggregation process of the branches of the national economy.

The free term in which are registered the supply-sales relationships or the enterprise resources, considering the high aggregation level of the economic information cannot be known with precision. Rather one knows the lower and higher limits within the free term.

According to those shown above, problem (5) can be written in the form of a linear programming problem:

$$\begin{aligned} 0.03x_1 + 0.06x_2 &\leqslant 15.6 \\ 0.5\ x_1 - 0.01x_2 &\leqslant 0.53 \\ 0.02x_1 + 0.05x_2 &\geqslant 10.5 \\ 0.01x_1 - 0.02x_2 &\geqslant 0.35 \\ x_1, x_2 &\geqslant 0 \\ min\ 3x_1 + 2x_2. & \end{aligned} \tag{6}$$

3. DYNAMIC LINEAR PROGRAMMING WITH SET COEFFICIENTS

If the production process is considered dynamically, the mode- has the form of state equations linking variables at different time inl tervals.

For the same reasons as described at point 2, a dynamic linear problem of type (1) appears, for example; in the following form:

$$x_1(k+1) - x_1(k) + [0.2 - 0.3]\, u_1(k) \subseteq [2.3 - 4.5] \tag{7.1}$$

$$x_2(k+1) - x_2(k) - [0.33 - 0.35)\, u_1(k) + [15.3 - 16.7]\, u_2(k) \subseteq$$

$$\subseteq [21.5 - 23.6] \tag{7.2}$$

$$x_3(k+1) - x_3(k) - [0.2 - 0.4]\, u_2(k) \subseteq [18.3 - 40.3] \tag{7.3}$$

$$x_1(k) - [0.2 - 0.3]\, u_1(k) \subseteq [10.3 - 13.2] \tag{7.4}$$ (7)

$$x_2(k) - [15.3 - 16.7]\, u_2(k) \subseteq [13.8 - 18.5] \tag{7.5}$$

$$x_3(k) \subseteq [18.3 - 40.3] \tag{7.6}$$

$$[0.3 - 0.5]\, u_1(k) + [0.8 - 1.7]\, u_2(k) \subseteq [15.3 - 35.8] \tag{7.7}$$

$$x_1(k),\ x_2(k),\ x_3(k),\ u_1(k),\ u_2(k) \geqslant 0 \quad k = 0, \ldots, 9$$

$$x_1(0) = x_2(0) = x_3(0) = 0$$

$$min \sum_{k=0}^{9} [2u_1(k) + 3u_2(k) + 0.3x_1(k) + 0.35x_2(k) + 0.4x_3(k)].$$

Constraint (7.1) describes the materials inventory dynamics in which the specific consumption of the material, as shown at point 2 is known in the form of an interval. The material inputs given also in the form of an interval take into account the suppliers' internal reserves. These are distributed within the limits of the production capacities' utilization.

Constraint (7.2.) shows the evolution of the stocks which from the point of view of the execution stage are unfinished production situated between the initial stage (raw material) and the final stage / finite product.

The dynamics of the unfinished production stock implies the consideration of tolerances for consumptions and inputs, respectively the outputs of subunits within inter-plant cooperation, particularly when they are given in conventional units.

Constraint (7.3) reflects the dynamics of the finished product stock.

Constraints (7.4), (7.5), (7.6) and (7.7) are due to material succession delivery and capacity, respectively.

Thus, the material constraint makes it necessary that the material stock should cover the material consumption implied by the launching of the order and it should remain between the confidence (safety) stock and of the standard stock.

Constraint (7.5) expresses the fact that the stocks must cover the consumption implied by the launching of the order for the finished product provided that an unfinished production stock remains which should allow the process continuity.

Similarly (7.6) makes it necessary that the finished product stock be comprised between the confidence stock and the maximum limit of demand.

Constraint (7.7) refers to the production capacity limits.

Problem (7) is reduced to a dynamic linear programming problem of the following form:

$$
\begin{aligned}
& x_1(k+1) - x_1(k) + 0.3\,u_1(k) \leqslant 4.5 \\
& x_1(k+1) - x_1(k) + 0.2\,u_1(k) \geqslant 2.3 \\
& x_2(k+1) - x_2(k) - 0.33\,u_1(k) + 16.7\,u_2(k) \leqslant 23.6 \\
& x_2(k+1) - x_2(k) - 0.35\,u_1(k) + 15.3\,u_2(k) \geqslant 21.5 \\
& x_3(k+1) - x_3(k) - 0.2\,u_2(k) \leqslant 40.3 \\
& x_3(k+1) - x_3(k) - 0.4\,u_2(k) \geqslant 18.3 \\
& x_1(k) - 0.2\,u_1(k) \leqslant 13.2 \\
& x_1(k) - 0.3\,u_1(k) \geqslant 10.3 \\
& x_2(k) - 15.3\,u_2(k) \leqslant 18.5 \qquad (8) \\
& x_2(k) - 16.7\,u_2(k) \geqslant 13.8 \\
& x_3(k) \leqslant 40.3 \\
& x_3(k) \geqslant 18.3 \\
& 0.5u_1(k) + 1.7\,u_2(k) \leqslant 35.8 \\
& 0.3u_1(k) + 0.8\,u_2(k) \geqslant 15.3 \\
& x_1(k),\ x_2(k),\ x_3(k),\ u_1(k),\ u_2(k) \geqslant 0 \quad k = 0, \ldots, 9 \\
& x_1(0) = x_2(0) = x_3(0) = 0 \\
& \min \sum_{k=0}^{9} [2u_1(k) + 3u_2(k) + 0.3x_1(k) + 0.35x_2(k) + 0.4x_3(k)].
\end{aligned}
$$

4. FUZZY LINEAR PROGRAMMING

We shall now consider the situation when the coefficients are fuzzy sets [10]. This corresponds to the cases when a preference for certain values is expressed.

The problem to be solved is similar to (3); the only difference is that sets K_1, K_2, ..., K_n and K are fuzzy sets, that it is application:

$$\mu_j : K_j \to [0,1] \quad (j = 1, \ldots, n)$$

and $\mu : K \to [0, 1]$ called membership functions are given in addition.

At this point, it is necessary to define addition and multiplication of the fuzzy sets [10]. Considering a fuzzy set (A, μ_A), where $\mu_A : A \to [0,1]$ and $A \subset R^n$ for a scalar $\lambda \in R$ we can form the fuzzy set $(\lambda A, \mu)$ where:

$$\lambda A = \{\lambda a / a \in A\}$$

and with each element of λA let us say λa. We associate the membership grade $\mu_A(a_0)$; we have thus defined the characteristic function:

$$\mu_{\lambda A} : \lambda A \to [0, 1].$$

Let (A, μ_A), (B, μ_B) be two fuzzy sets, where $A, B \subset R^n$ and $\mu_A : A \to [0, 1]$ and $\mu_B : B \to [0,1]$.

Associated with these two fuzzy sets is the fuzzy set $(A + B, \mu_{A+B})$, defined as follows:

a) $A + B = \{a + b / a \in A,\ b \in B\}$

b) $\mu_{A+B} : A + B \to [0,1]$,

$$\mu_{A+B}(x) = \bigvee_{a+b=x} (\mu_A(a) \wedge \mu_B(b)).$$

Consider the fuzzy set (A, μ_A) denoted by $A = \{1, 2, 3\}$

A	1	2	3
μ_A	0.5	0.6	0.4

.

We can form the fuzzy set $(5A, \mu_{5A})$ denoted by:

$$5A = \{5,\ 10,\ 15\}$$

$5A$	5	10	15
μ_{5A}	0.5	0.6	0.4

.

Also consider the fuzzy sets (A, μ_A) and (B, μ_B) in the form:

$$A = \{2,\ 3,\ 4\}, \qquad B = \{4,\ 5,\ 6\}$$

A	2	3	4
μ_A	0.8	0.2	0.4

B	4	5	6
μ_B	0.7	0.5	0.6

.

Then, the sum of these two fuzzy sets will be: $(A + B, \mu_{A+B})$ denoted by:

$$A + B = \{6,\ 7,\ 8,\ 9,\ 10\}$$

$A + B$	6	7	8	9	10
μ_{A+B}	0.7	0.5	0.6	0.4	0.4

.

Thus:

$$\mu_{A+B}(8) = max\{min\{\mu_A(2),\ \mu_B(6)\},\ min\{\mu_A(3),\ \mu_B(5),$$

$$min\{\mu_A(4),\ \mu_B(4)\} = 0.6.$$

The inclusion of the fuzzy sets should also be defined.

Let (A, μ_A) and (B, μ_B) be two fuzzy sets where $A, B, \subset R^r$, and $\mu_A : A \to [0,1]$ and $\mu_B : B \to [0, 1]$. We say that $(A, \mu_A) \subset (B, \mu_B)$ if $\forall x \in A \cup B$, the applications $\mu'_A : A \cup B \to [0, 1]$ and $\mu'_B : A \cup B \to [0, 1]$ denoted by:

$$\mu'_A(y) = \begin{cases} \mu_A(y) & y \in A \\ 0 & y \in B-A \end{cases}$$

$$\mu'_B(y) = \begin{cases} \mu_B(y) & y \in B \\ 0 & y \in A-B \end{cases}$$

have the property: $\mu'_A(x) \leqslant \mu'_B(x)$.

The problem we shall further deal with is:

$$sup \ c \cdot x$$

$$x_1 \underset{\sim}{K}_1 + x_2 \underset{\sim}{K}_2 + \ldots + x_n \underset{\sim}{K}_n \subseteq \underset{\sim}{K}, \quad x_j \geqslant 0 \tag{9}$$

where $\underset{\sim}{K}_j$ and $\underset{\sim}{K}$ are fuzzy sets (K_j, μ_{kj}), (K, μ_k)

To solve problem (9) by known methods the representation theorem for fuzzy sets [11], gives us the possibility of acting on level sets.

Let $\underset{\sim}{A} = (A, \mu_A)$ be a fuzzy set. We denote the level set:

$$\sigma_r(\underset{\sim}{A}) = \{x \in A / \mu_A(x) > r\}.$$

The level sets have the properties:

a) $r_1 \leqslant r_2 \Rightarrow \sigma_{r_2}(\underset{\sim}{A}) \subseteq \sigma_{r_1}(\underset{\sim}{A})$

b) $\underset{\sim}{A} \subseteq \underset{\sim}{B} \Leftrightarrow \sigma_r(\underset{\sim}{A}) \subseteq \sigma_r(\underset{\sim}{B}), \ \forall r$

c) $\sigma_r(\underset{\sim}{A} + \underset{\sim}{B}) = \sigma_r(\underset{\sim}{A}) + \sigma_r(\underset{\sim}{B})$

d) $\sigma_r(\lambda \underset{\sim}{A}) = \lambda \sigma_r(\underset{\sim}{A})$

Since the level sets are ordinary sets, by using properties b, c, a, problem (9) has the form (3):

$$sup \ c \cdot x$$

$$x_1 \sigma_r(\underset{\sim}{K}_1) + x_2 \sigma_r(\underset{\sim}{K}_2) + \ldots + x_n \sigma_r(\underset{\sim}{K}_n) \subseteq \sigma_r(\underset{\sim}{K}) \tag{10}$$

$$x_j \geqslant 0, \quad \forall r \in [0,1].$$

Following [10] we shall use the proposition: If the fuzzy set K in problem (9) has the property:

$$\{\mu_K(a)/a \in K\} = \{r_1, r_2, \ldots, r_p\}$$

$$0 \leqslant r_1 < r_2 < \ldots < r_p \leqslant 1$$

then problem (9) is equivalent to:

$$sup \ c \cdot x$$

$$x_1 \sigma_{r_i}(K_1) + x_2 \sigma_{r_i}(K_2) + \ldots + x_n \sigma_{r_i}(K_n) \subseteq \sigma_{r_i}(K)$$

$$x_j \geqslant 0, \quad j = 1, 2, \ldots, \quad n, \ i = 1, \ldots, p.$$

Thus, the fuzzy linear programming problem is reduced to a problem of type (3).

The economic intepretation of the fuzzy linear programming model is the following:

The coefficients of the variables are fuzzy sets. This goes back to the fact that the specific consumptions of materials and of capacity are not deterministic but may assume values in an interval with certain membership grades.

Thus, a certain agglomeration of specific consumptions of materials corresponding to geographical zones to which the garments production is meant, which can be described by means of the membership function as in figure 1, has been noticed in practice.

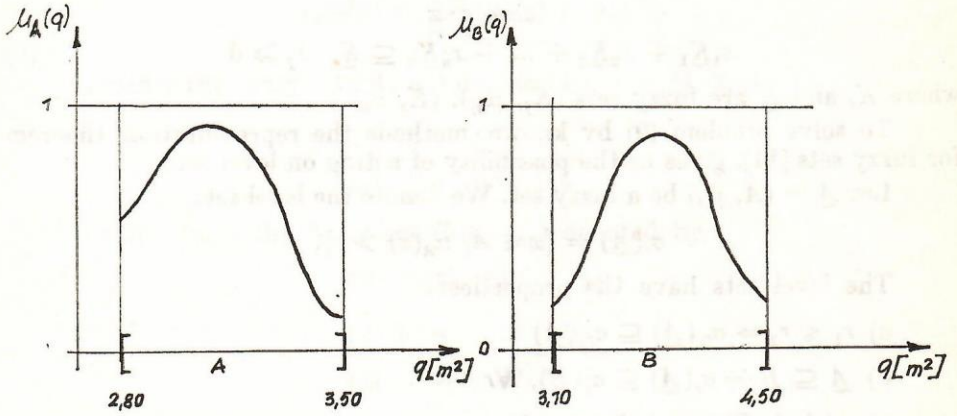


Fig. 1 Distribution of material consumption described by means of the membership function for two different sale zones

5. FUZZY DYNAMIC LINEAR PROGRAMMING

A natural generalization of the cases described at points 1, 2, 3 and 4 leads to the fuzzy dynamic linear problem. We shall further consider a simple example:

$$\begin{gathered} x(k+1) - x(k) + A_{11}u(k) \subseteq B_1 \\ A_{21}x(k) \subseteq B_2 \\ x(k),\ u(k) \geqslant 0, \quad k = 0, \ldots, 9 \\ x(0) = 0 \\ min \sum_{k=0}^{9} [0.5x(k) + 0.8u(k)] + 0.5x(10) \end{gathered} \tag{11}$$

where A_{11}, A_{21}, B_1 and B_2 have the significance given in Fig. 2.

Writing all constraints, we obtain a fuzzy linear problem which was treated at point 4. In the present example, this problem will have 20 constraints with fuzzy set coefficients, each constraint turning into 4 linear con-

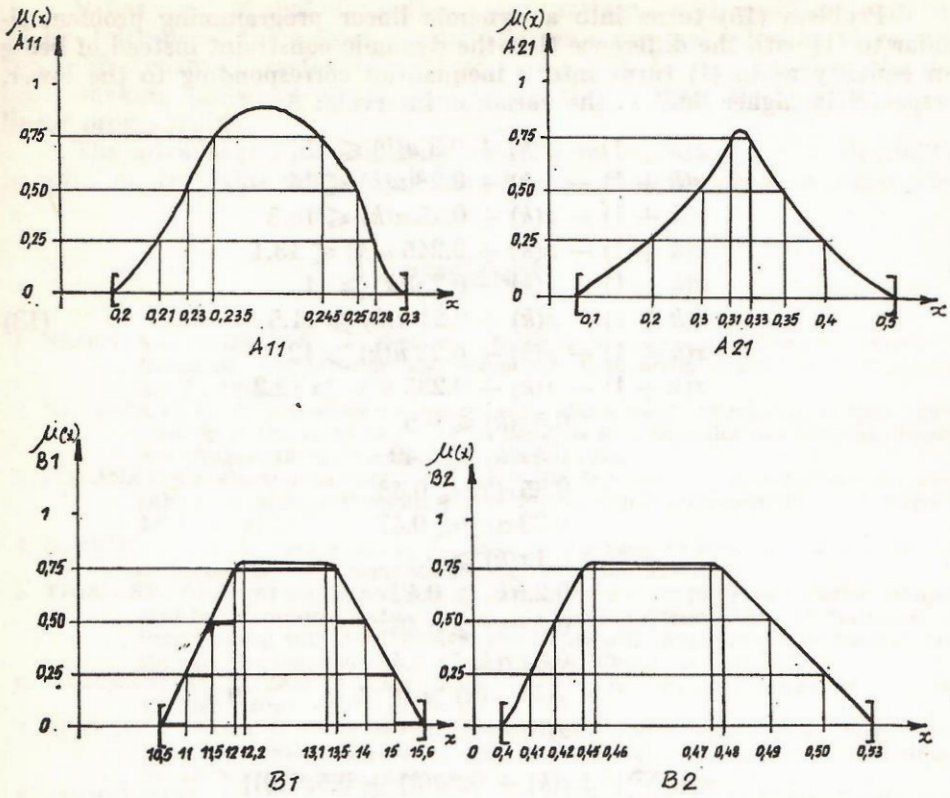


Fig. 2

straints with deterministic set coefficients. We shall thus obtain a total of 80 constraints with deterministic set coefficients corresponding to which are 160 linear constraints.

Thus, problem (14) is equivalent to the following set coefficient programming problem:

$$
\begin{aligned}
&x(k+1) - x(k) + (0.2 - 0.3)\,u(k) \subseteq [11 - 15]\\
&x(k+1) - x(k) + (0.21 - 0.28)\,u(k) \subseteq [11.5 - 14]\\
&x(k+1) - x(k) + (0.23 - 0.25)\,u(k) \subseteq [12 - 13.5]\\
&x(k+1) - x(k) + (0.235 - 0.245)\,u(k) \subseteq [12.2 - 13.1]\\
&(0.1 - 0.5)\,x(k) \subseteq [0.41 - 0.5]\\
&(0.2 - 0.4)\,x(k) \subseteq [0.42 - 0.49] \qquad (12)\\
&(0.3 - 0.35)\,x(k) \subseteq [0.45 - 0.48]\\
&(0.31 - 0.33)\,x(k) \subseteq [0.46 - 0.47)\\
&x(0) = 0, \quad x(k),\ u(k),\ u(k) \geqslant 0, \quad k = 0, \ldots, 0\\
&min \sum_{k=0}^{9} [0.5x(k) + 0.8u(k)] + 0.5x(10).
\end{aligned}
$$

Problem (15) turns into a dynamic linear programming problem similar to (1) with the difference that the dynamic constraint instead of being an equality as in (1) turns into 2 inequalities corresponding to the lower, respectively higher limit of the variation intervals:

$$
\begin{aligned}
& x(k+1) - x(k) + 0.3\,u(k) \leqslant 15. \\
& x(k+1) - x(k) + 0.28\,u(k) \leqslant 14. \\
& x(k+1) - x(k) + 0.25\,u(k) \leqslant 13.5 \\
& x(k+1) - x(k) + 0.245\,u(k) \leqslant 13.1 \\
& x(k+1) - x(k) + 0.2\,u(k) \geqslant 11. \\
& x(k+1) - x(k) + 0.21\,u(k) \geqslant 11.5 \\
& x(k+1) - x(k) + 0.23\,u(k) \geqslant 12 \\
& x(k+1) - x(k) + 0.235\,u(k) \geqslant 12.2 \\
& 0.5x(k) \leqslant 0.5 \\
& 0.4x(k) \leqslant 0.49 \\
& 0.35x(k) \leqslant 0.48 \\
& 0.33x(k) \leqslant 0.47 \\
& 0.1x(k) \geqslant 0.41 \\
& 0.2x(k) \geqslant 0.42 \\
& 0.3x(k) \geqslant 0.45 \\
& 0.31x(k) \geqslant 0.46 \\
& x(k),\ u(k) \geqslant 0 \quad k = 0, \ldots, 9 \\
& x(0) = 0.
\end{aligned}
\tag{13}
$$

$$
min \sum_{k=0}^{9} [0.5x(k) + 0.8u(k) + 0.5x(10)]
$$

It should be noticed that a problem of type (1) with fuzzy set coefficients where $\underset{\sim}{S}(k)$ and $\underset{\sim}{R}(k)$ are characterized by $\sum_{j=1}^{m} r_i^j(k)$ and respectively $\sum_{i=1}^{p} r_K^i(k)$ samples is equivalent to a linear programming problem with at most

$$
N\left(\sum_{j=1}^{m} r_s^j(k) + \sum_{i=1}^{p} r_R^i(k)\right)
$$

constraints.

Not all the constraints, however, are independent. That is why it is necessary to apply a procedure of eliminating the redundant constraints before solving the linear programming problem.

6. CONCLUSIONS

In this paper we have tried to present some generalizations of the linear programming problems in order to extend their applicability also in the case of taking into account imprecision.

It has resulted that also in the case of problems with coefficients affected by tolerances, there exist possibilities for a rational solution. Moreover, the subjectivism characteristic of preferences may be considered in the decision-making processes which recourse to the mathematical apparatus of linear programming.

The advantage of considering imprecision is marked by a flexibility in solution. But, this flexibility is paid by enlarging the problem dimension.

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POINTS DE VUE SUR L'IMPRECISION DANS LA PROGRAMMATION LINEAIRE DYNAMIQUE

Les auteurs présentent dans cet article une nouvelle tendance qui se manifeste dans la façon d'aborder les problèmes de programmation linéaire dynamique, c'est- à-dire la tendance d'envisager certaines tolérances admises par les coefficients qui apparaissent dans l'expression des restrictions. Après avoir montré qu'un problème de programmation linéaire dynamique peut être réduit à un problème linéaire courant, l'analyse est dirigée vers les diverses possibilités de tenir compte de l'imprécision. Ainsi, on passe en revue le cas des problèmes à coefficients — ensembles et ensuite le cas des problèmes ä coefficients ensembles fuzzy. L'exposé est illustré d'exemples, surtout de l'industrie des confections, où les auteurs ont identifier des problémes courants qui réclament un abord de ce genre.

ОБ ОТНОШЕНИИ К НЕТОЧНОСТИ В ДИНАМИЧЕСКОМ ЛИНЕЙНОМ ПРОГРАММИРОВАНИИ

Работа представляет новую тенденцию в трактовке вопросов динамического линейного программирования, а именно, учёт допущений коэффициентов, возникающих в выражении рестрикций. После того, как указывается, что проблема динамического линейного программирования может быть доведена до проблемы обычного линейного программирования, анализ сосредатачивается на различных возможностях учёта неточности. Так делается обзор случая вопросов с множественными коэффициентами и затем случай вопросов с множественными коэффициентами фуззи. Вся работа сопровождается примерами, в особенности промышленности готового платья, где автори нашли текущие вопросы, требующие трактовки такого типа.