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Application of Statistical-Mathematical Methods for Determining the Money Reserves of the Population in a Given Territory

The elaboration of a method allowing to determine the volume of cash within a given territory, as a function of the data existing locally, is of particular importance at present.

The knowledge of the cash volume held by the population by regions and districts is necessary both on the occasion of the elaboration of the balance of the population's cash income and expenses and for the cash plan, as also on the occasion of the analyses made to establish the required correlation between the population's purchasing power on the one hand, and the stock of goods and services, on the other hand.

It is well known that the population's cash income, after deducting the compulsory payments for taxes, charges etc., and the sums constituting cash reserve in actual cash or in savings at the CEC (Savings Bank) represents the purchasing fund, i.e. the effective demand of the population. This effective demand must be met by a stock of goods and services sufficient in volume and of a structure corresponding to the requirements from the standpoint of the assortment of goods solicited.

Besides the cash income obtained in the current period, the population still disposes of a certain amount of money reserves which is to be found either in the form of cash or in the form of savings at the CEC. To the extent in which the quantity of cash in circulation corresponds to the objective requirements — which means that it should correspond to the demands of the payment and cashing-in turnover without containing sums representing a postponed consumption requirement — the cash available with the population does not manifest itself as an effective demand; on the contrary, its volume

increases from year to year concomitantly with the increase in the cash income and expense of the population. This, however, does not in any way modify the fact that the existing money reserves of the population represent a potential demand whose size should be known and whose evolution should be followed with care. Comparing the dynamics of cash with that of retail-sales, the ratio of the cash volume per inhabitant and the investigation of the changes in the speed of money circulation are a few highly efficient analyses which allow some predominant and characteristic trends to be singled out in the evolution of money circulation.

The upward and harmonious development of our national economy has ensured the conditions required for a healthy evolution of money circulation. Concomitantly with the increase of production and of goods circulation, the money turnover has also increased, setting complex tasks connected with the fulfilment of the cash requirements of economy and with the planning of money circulation, both to the central and to the local authorities.

The continuous increase of the population's cash income, which creates ever increasing saving possibilities, naturally leads to an increase of the population's financial possibilities, both in the form of cash and, particularly, in the form of saving deposits at the CEC. Under these conditions the manner in which the income is used by the population, the orientation of the consumption demand and the degree to which the money reserves of the population are attracted to the CEC play a particularly important part, the planning of money circulation and the operative management become more complex, thus requiring a higher level of planning and analysis in the sphere of the financial relations and of cash operations.

In the framework of the ensemble of problems on which should be focused the attention of the workers on whom tasks incumb in planning and following up cash operations, an important part falls on the most accurate knowledge, not only of the population cash income at a given moment, but also of the level of financial availabilities existing with the population, as well as the trends characterizing their forming and their evolution.

On the whole of the population's money reserves the most important part is to be found in the form of savings at the CEC. The volume of these cash means is known from the statistical and accounting records of the savings institutions. With regard to the volume of cash held by the population the situation is more complex since — as it is well known — in a given territory the bank records regarding the putting of cash into circulation do not offer sufficient indication to appreciate the cash volume held by the population of the respective territorial unit. To determine this, special computations and economic analyses are necessary, since the existing cash migration between regions and districts mean that, within a territory, the changes in the amount of cash put into circulation should only accidentally coincide with the changes of the existing cash within the respective territorial unit.

It is well known that for the whole country, the volume of existing cash in circulation at a given moment, from which the existing cash in the circulating cash accounts of the National Bank and in the cash accounts of the units in the socialist sector are to be subtracted, represents the cash held by the

population. The changes in the cash put into circulation within a certain period, respectively the balance between the payments and cashings-in of the cash-account plan in that period, represent the change, i.e. the increase or decrease of the existing cash in circulation.

For a given territory (region, district) the situation is not the same. The cash set in circulation by the local unit of the National Bank is not identical with the volume of cash existing in the respective territorial unit, and the balance between payments and cashings-in of the cash-account plan do not reflect the change in the volume of cash within that region or district. On account of cash migrations the balance of the cash-account plan reflects the putting into or withdrawing of the cash from circulation which occurs through the paying-desks of the National Bank in the respective territorial unit within a given time period, but it does not coincide with the change in the cash existing in circulation in the respective territorial unit. The cash-account plan reflects the cash variations occurring through the National Bank paying-desks without showing whether they refer to the population of the respective territorial unit or to that of other regions.

There results that for a given territory (region, district), neither the data regarding the putting of cash into circulation, not those referring to the fulfilment of the cash-account plan, give sufficient elements to allow to determine the volume of cash existing at a given instant in the framework of the respective territorial unit, but they represent — as it was shown — a potential consumption demand, whose knowledge is of particular importance.

Taking the above into consideration the determination of the quantity of cash held by the population in a given territory constitutes rather a difficult problem. As far as we know from special publications, this problem also constitutes the concern of the respective authorities in other socialist countries.

In our country the first endeavours to this end were already made in 1959, and since then the methods have gradually been improved as the necessary experience was acquired. The methods so far used have a common characteristic and consist, in the main, in distributing the existing volume of cash in the whole country by regions in accordance with the main factors bearing an influence on the cash level and on its speed of circulation.

It is evident that the methods of cash distribution can only be used by the central authorities for regions and by regional authorities for districts, which constitutes a great disadvantage since they cannot be directly used and applied by the territorial authorities which are the first to be interested in determining and knowing the quantity of cash within the respective territorial units.

The elaboration of a method allowing to determine the volume of cash within a given territory in keeping with the data existing locally, is therefore of particular importance at present. We shall expound hereunder, on the one hand, the main methods for distributing the cash by regions and, on the other hand, we shall give a new method, allowing to determine the quantity of cash held by the population in a given territory by using the statistical data existing locally.

I. CASH DISTRIBUTION FROM THE CENTRE, BY TERRITORIAL UNITS

The method of distribution may only be used centrally starting, on the one hand from the existing cash level at a given instant on the whole of the Socialist Republic of Romania and on the other hand from the elements which characterize the specific quality of the territorial units to which the cash is distributed.

The first problem which arises is to establish the main factors influencing the cash volume held by the population, and to find the practical means of including it in the computation. Generally, in the specialty publications it is considered that the determining factor in the distribution of cash by territory is constituted by the volume of the population's cash income, mention being made that under certain conditions the influence exerted by the level of the mean income should also be included in the computation.

As an explanation, we shall present a few methods which could be used to distribute the cash by regions, the peculiarities of each method being shown in detail. To this end, let us assume that there are only four regions in the country, for each of which we know the volume of the incomes, their distribution between the two main groups of population, both for the entire country and for each region in part. Likewise, we know the quantity of cash existing in the country, as well as its distribution between the two social categories. The objective we are seeking is to determine the volume of cash existing within each region. The conventional data which we shall use to illustrate the problem are the following:

Table 1

	Total per country (thousand)	of which			
		Region *A*	Region *B*	Region *C*	Region *D*
Incomes	4,000,000	1,000,000	1,250,000	1,000,000	750,000
of which:					
— wage earners	2,815,000	600,000	950,000	615,000	650,000
— peasants	1,185,000	400,000	300,000	385,000	100,000
Population	1,000	300	250	250	200
Cash	200,000	$N_1 = ?$	$N_2 = ?$	$N_3 = ?$	$N_4 = ?$
of which:					
— wage earners	90,000	—	—	—	—
— peasants	110,000	—	—	—	—

The first method that we shall use to determine the volume of cash existing within each region consists in distributing the cash in accordance with the volume of incomes achieved by the population of each region. The regions in which the population achieves higher incomes should have a higher volume of cash. Making abstraction of the influence exerted by other factors (structure of population, number of population, etc.), the volume

of cash existing within a region whatever must be directly proportional with the volume of incomes, i.e.:

$$N_i = N \frac{I_i}{\sum_{i=1}^{4} I_i},$$

where N_i is the cash which we are seeking to determine in a given territory; N represents the cash existing in the whole country; I_i — money income of the population in the respective territorial unit and $\sum I_i$ — total money income of the population for the whole country.

By applying the above formula we obtain the following results:

Table 2

Region	Method of computation	Cash
A	$200{,}000 \frac{1{,}000{,}000}{4{,}000{,}000}$	50,000
B	$200{,}000 \frac{1{,}250{,}000}{4{,}000{,}000}$	62,500
C	$200{,}000 \frac{1{,}000{,}000}{4{,}000{,}000}$	50,000
D	$200{,}000 \frac{750{,}000}{4{,}000{,}000}$	37,500
Total:	—	200,000

We notice that in regions *A* and *C*, in which the population achieved the same volume of income, we have obtained the same amount with respect to the volume of cash existing within each region; the cash in region *C* stands at a higher level, since the income of the population in this region is also higher; in region *D* the situation is reversed.

There results from the above that this method is very simple and easy to apply. It has, however, the disadvantage of only taking a single factor into account, even though this factor may be considered as decisive in the distribution of cash by regions. The application of this method — distribution of cash in accordance with the level of the income achieved — assumes that the rate of cashing-in and spending of income are the same in all the regions, a fact that, obviously, is not actual, if we take into account the fact that the structure of the population (wage earners, peasants) still differs considerably from one region to another.

The second method which we shall apply tries to turn round the shortcoming of the first method. To this end the distribution of cash by regions will be made in agreement with the population's income, taking into account the latter's structure.

It is well known that the mode in which the peasants acquire and spend their income differs from that of the wage earners. The former acquire their income at longer periods, their expenses are directed, in the main, towards products other than food-stuffs, the use of savings institutions is more reduced and consequently the money speed of circulation is slower than for the wage earners, for whom the period of income cashing is shorter and the custom of using savings institutions is more developed. Consequently, for two regions having the same volume of income the mass of cash held by the population may be different, depending on the specific weight of the peasant income.

Maintaining the same numerical example in view of being able to compare the results obtained, we shall use the well-known relation existing between cash, income and money speed of circulation:

$$N = \frac{I}{S}\ ; \quad \text{and} \quad S = \frac{N}{I}$$

where N = cash in circulation; I = money income of population, and S = = coefficient of money speed of circulation. We shall compute the circulation speed for the total income:

$$S = \frac{N}{I} = \frac{4{,}000{,}000}{200{,}000} = 20 \text{ revolutions}$$

and then separately for wage earners and for peasants, using the data in table 1 to this end:

$$S_w = \frac{I_w}{N_w} = \frac{2{,}815{,}000}{91{,}000} = 31.3 \text{ revolutions}$$

$$S_p = \frac{I_p}{N_p} = \frac{1{,}185{,}000}{110{,}000} = 10.8 \text{ revolutions}$$

We notice that the money speed of circulation for wage earners is greater than for peasants, which means that for the same volume of income a higher quantity of cash will be held by the peasants than by the wage earners. As a function of the differentiated speed for the cash held by the wage earners and by the peasants, we determine the cash held by the wage earners and by the peasants in each region.

By summing up the amounts obtained for the wage earners and for the peasants we obtain the volume of cash held by the population within the region:

Table 3

Region	Income / Speed		Cash		
	Wage earners	Peasants	Wage earners	Peasants	Total
A	600,000 / 31.3	400,000 / 10.8	19,185	37,130	56,315
B	950,000 / 31.3	300,000 / 10.8	30,370	27,850	58,220
C	615,000 / 31.3	385,000 / 10.8	19,665	35,740	55,405
D	650,000 / 31.3	100,000 / 10.8	20,780	9,280	30,060
Total Romania			90,000	110,000	200,000

The results obtained by means of this method take into account both the action of the factor income and of the different mode in which the income is obtained and spent by the two main groups of population. Nevertheless, for the final interpretation of the results obtained, a supplementary analysis should be made, taking into account the specific economic features of each region in part and the structure of its agricultural production. It is well known that the structure of agricultural production and the seasonal acquirement of income also exert an influence on the cash held by the peasant population at the end of the year. The existence of a strong zootechnical and vegetable producing sector ensures a somewhat uniform realization of income in the course of the whole year, while a predominant vegetal (cereal) production leads to the forming of income after the production is achieved, towards the end of the year, a fact which will also determine the existence of larger money reserves in that period.

In the hypothesis that the volume of cash held by the population is mainly composed of sums representing money reserves necessary to effect current expenses, the results of this method could be considered as being satisfactory. In case, however, where the concrete analysis of the amount of cash held by the population would show that, on the whole, besides the current cash-account reserve there are also — in a noticeable proportion — cash availabilities destined to future expenses, the influence exerted by the level of the mean income should then also necessarily be included in the computation.

The third method starts from the results obtained by the second method. The reasoning from which we start is based on the fact that the level of the mean income is the one that determines the proportions and dynamics of that part of the population's money reserves which are not destined to current expenses, but represent an accumulation of resources for future purposes. The inclusion of the influence exerted by the level of the mean income into the computation is particularly required in cases where, between various

regions, there are noteworthy differences with regard to the income level achieved per inhabitant.

As an illustration we shall first compute the level of the mean income for the whole country and then for each region in part (conditional figures):

Table 4

	Income	Number of population	Mean income
Total country	4,000,000	1,000	4,000
Region *A*	1,000,000	300	3,333
Region *B*	1,250,000	250	5,000
Region *C*	1,000,000	250	4,000
Region *D*	750,000	200	3,750

From the above data there results that region *B*, having the largest volume of income, is also characterized by a high mean level. Region *D*, in which, although the population realizes the lowest level of income, has nevertheless a higher mean income level than region *A*, a fact which may influence the volume of cash existing in the respective districts.

The value to be added to or subtracted from the final results of the 2-nd method shown before, depending on a higher, respectively lower, mean income than the one at the level of the whole country, can be determined by means of the following formula:

$$\pm K = P_i(\bar{I}_i - \bar{I}) \frac{N}{\sum_{i=1}^{4} I_i}$$

where:

$\pm K$ — value of correction to be determined
P_i — number of population in a given region
I_i — mean income in the respective region;
$\bar{I}$ — mean income for whole country;
N — volume of cash existing in the whole country;
$\sum_{i=1}^{4} I_i$ — sum of income in all regions.

If we therefore also take into account the influence of the mean income achieved in each region, the cash held by the population is distributed by regions as follows:

Table 5

	Cash volume depending on income	Influence of mean income ($\pm K$)	Cash volume corrected in agreement with mean income variations
Region *A*	56,315	—10,000	46,315
Region *B*	58,220	+12,500	70,720
Region *C*	55,405	—	55,405
Region *D*	30,060	— 2,500	27,560
Total country	200,000	—	200,000

The methods for the distribution of the cash volume held by the population in a given territory can be further improved. To this end it is, however, necessary to effect thorough investigations in connection with the action of savings deposits at the CEC on the cash volume by regions, as well as in connection with the structure of the cash held by the population, the knowledge of which (reserves for current expenses and reserves for future expenses) will permit a more concrete appreciation of the influence exerted by the mean income on the volume of cash.

II. DETERMINATION OF CASH VOLUME HELD BY THE POPULATION IN A GIVEN TERRITORY

We shall, hereunder, apply some statistical-mathematical methods in view of determining the cash volume held by the population in a given territory. To this end, we shall first determine some quantitative, functional relations, existing between the volume of cash held by the population and the factors that determine it. These relations will be determined on basis of the statistical data known of the national economy.

Determination of functional dependences between the volume of cash and the factors which determine it

The following notations will be used:

	Total	*Wage earners*	*Peasants*
Population	P	P_w	P_p
Money income of population (yearly)	I	I_w	I_p
Cash volume held by population	N	N_w	N_p
Money income per inhabitant (mean income)	M	M_w	M_p
Mean amount of cash per inhabitant (mean cash)	n	n_w	n_p

We have the following obvious relations:

$$P = P_w + P_p \qquad (1)$$

$$I = I_w + I_p \qquad (2)$$

$$N = N_w + N_p \qquad (3)$$

$$M = \frac{I}{P} \qquad M_w = \frac{I_w}{P_w} \qquad M_p = \frac{I_p}{P_p} \qquad (4)$$

$$M = \frac{P_w M_w + P_p M_p}{P_w + P_p} \qquad (5)$$

$$n = \frac{N}{P} \qquad n_w = \frac{N_w}{P_w} \qquad n_p = \frac{N_p}{P_p} \qquad (6)$$

$$n = \frac{P_w n_w + P_p n_p}{P_w + P_p} \qquad (7)$$

The economic financial analysis, as well as the statistical data, show that there is a close correlation between the money income of the population and the volume of cash (and therefore also between the money income per inhabitant and the amount of cash per inhabitant).

Thus, by computing the correlation coefficient according to formula:

$$c = \frac{n\Sigma IN - \Sigma I \times \Sigma N}{\sqrt{[n\Sigma I^2 - (\Sigma I)^2][n\Sigma N^2 - (\Sigma N)^2]}} \qquad (8)$$

on basis of the data in the income and expense balance of the period 1953—1955 for wage earners there results that the size of this coefficient is 0.99, i.e. nearly 1, which is the maximum value that the correlation coefficient can be. This points to a high degree of dependence between incomes and the amount of cash held by the population.

The analysis of this dependence shows, however, that the conclusion may not be drawn that between the amount of the money income of the population and the volume of cash held by the population there is a linear dependence of the form $N = \alpha I + \beta$.

This is because it is well known that the amount of cash held by the population changes from one period to another, not only in agreement with the level of the population money income, but also in accordance with its structure (wage earners, peasants), the predominance of mean income, either large or small, and the orientation of the consumption demand. The changes that occur in the manner in which income is formed and used do not allow the dependence between the level of income and the volume of cash to take the form of a linear dependence.

This has also been determined on basis of the figures in the balance of money expense and income of the population for the period 1953—1965.

Indeed, by establishing the relation:

$$N = \alpha I + \beta \qquad (9)$$

and determining the coefficients α and β by means of the least squares method between the actual amounts and those computed, deviations exceeding 40% were obtained. This invalidates the existence of a linear dependence between the amount of income and the amount of cash held by the population. Moreover, we consider that it is not even proper to seek a direct functional relation between the size of income and the amount of cash held by the population (within a region or in the whole country), but it is more proper to seek the functional relation linking the size of money income per inhabitant to the amount of cash per inhabitant. This requirement is also justified by the fact that the level of income per inhabitant is the one that determines, in the main, the evolution of the part of the income which is destined to current expenses and of the part destined to future expenses, the latter mostly finding its way to the savings institutions.

Let, therefore,

$$n = f(M) \qquad (10)$$

be the functional link between the size of mean cash (n) and the size of mean income (M). In order to determine the analytical form of function f, we shall

start from the analysis of the functions played by the cash held by the population. This analysis shows that for a reduced level of income per inhabitant, the rate of cash increase is small. This situation is maintained up to a certain value M_1 of the mean income. Beginning from this level the rate of cash increase in relation with the mean income increase becomes much more pronounced. This new situation continues until a certain value M_2 of the mean income, after which the rate of cash increase shows a marked tendency to slow up. Likewise, analysis shows that starting from a certain level of the mean income its increase no longer implies a noticeable increase in the mean cash.

The analysis of the data in family budgets also leads to the same conclusion. Taking the above into account we estimate that the diagram of the function $n = f(M)$ is of the following shape:

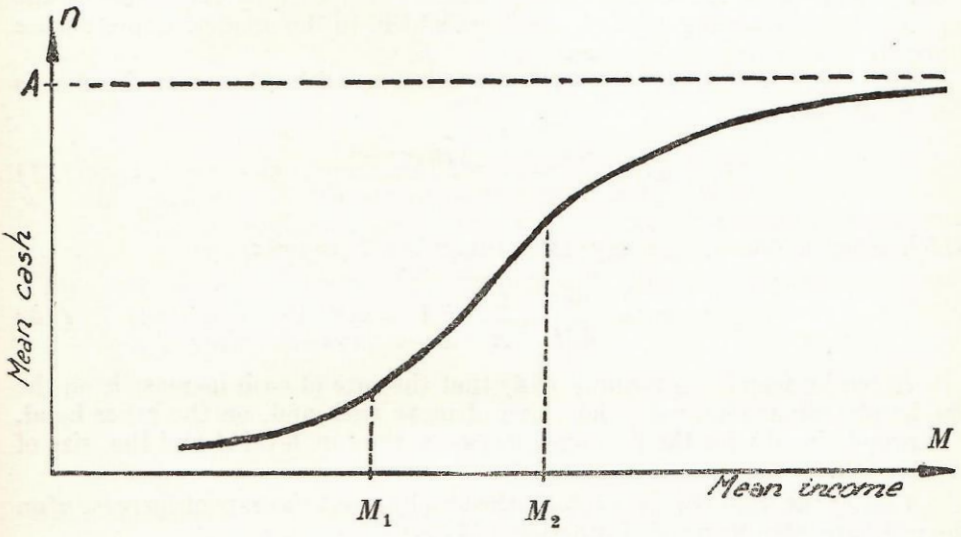

Diagram 1

We notice in the first place that the diagram has a horizontal asymptote. This corresponds to the fact that starting from a certain level of the mean income its increase no longer implies a noticeable increase of the mean cash.

It can be seen from diagram 1 that the functional dependence between the amount of mean cash and the size of the mean income can be approximated by the so-called "logistic function". Hence, the function $n = f(M)$ is of the form:

$$n = \frac{A}{1 + Be^{-CM}} \qquad (11)$$

where A, B and C are constants to be determined and

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = 2.71828 \ldots$$

It can be seen from formula *(11)* that the straight line

$$y = A \tag{12}$$

is asymptote to the function diagram.

Indeed:

$$\lim_{m \to \infty} \frac{A}{1 + Be^{-CM}} = A$$

In other words, the magnitude A may be considered as the "saturation level". This means that under normal conditions, taking into account the economic role played by the cash held by the population, its mean size will not exced the top value A, whatever be the increase of the mean income level. This, inasmuch as the whole income surplus will be directed towards the purchase of long-lasting objects, a process which, to the greatest extent, takes place by the agency of savings.

The increase rate mean cash in relation with the mean income is the following:

$$n' = \frac{dn}{dM} = \frac{ABCe^{-CM}}{(1 + Be^{-CM})^2} \tag{13}$$

which after a few elementary transformations becomes:

$$n' = \frac{dn}{dM} = \frac{C}{A} n(A - n) \tag{14}$$

It can be seen from formula *(14)* that the rate of cash increase is on the one hand proportional with the value of mean cash and, on the other hand, it is proportional with the difference between the top level A and the size of mean cash.

Taking the value of the cash on the abscissa and the rate of increase n' on the ordinate, the diagram of function *(14)* is:

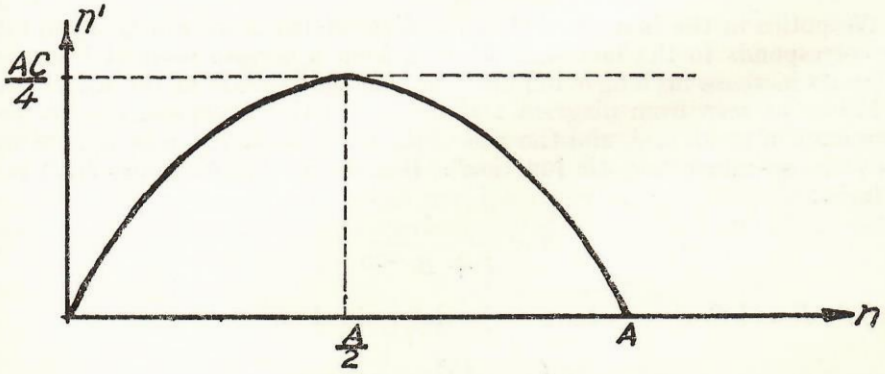

Diagram 2

As it can be seen from diagram 2, the mean cash increases up to the value $\frac{AC}{4}$, where it reaches the maximum value $\left(\text{the size of the cash then being } \frac{A}{2}\right)$, after which it begins to decrease.

As it is well known a widely used indicator in economic analysis is the elasticity of the function. In our case the elasticity of the mean cash in relation with the level of the mean income is:

$$E = \frac{dn}{dM} : \frac{n}{M} = \frac{d\ln n}{d\ln M}$$
$$F = \frac{C}{A} n\,(A - n) \times \frac{M}{n} = \frac{C}{A} M\,(A - n) \qquad (15)$$

One notices from formula *(15)* that the elasticity of mean cash in relation with the mean income also has a contradictory trend: on the one hand, it has the tendency to increase together with the increase of income, but concomitantly, it has the tendency to decrease with the increase of the mean cash. This can be seen still more clearly if we express the elasticity only as a function of the mean cash level.

From formula *(11)* there results:

$$M = \frac{1}{C} \ln \frac{Bn}{A - n} \qquad (16)$$

By introducing this in formula *(15)* we have:

$$E = \frac{A - n}{A} \ln \frac{Bn}{A - n} \qquad (15')$$

from which the fact results still more pregnantly that the elasticity of mean cash decreases together with the increase of the cash amount.

We also have:

$$\lim_{n \to A} E = 0$$

in other words, the closer the amount of cash comes to the top value A, the closer the elasticity comes to the value 0.

All these properties of the logistic function allow us to consider that it can approximate to a sufficiently satisfactory extent the functional dependence existing between the level of mean income and the size of mean cash.

III. DETERMINATION OF THE CASH VOLUME WITHIN A GIVEN REGION

We shall assume that, on basis of the existing statistical data, we have reached the following formulae which determine the relation between the level of the mean income and the cash amount, separately for wage earners and for peasants.

$$n_w = \frac{A_w}{1 + B_w e^{-C_w M_w}} \tag{17}$$

$$n_p = \frac{A_p}{1 + B_p e^{-C_p M_p}} \tag{18}$$

In this situation we want to determine the level of cash held by the population in every region or, which amounts to the same thing, to find out the mean cash in every region.

Let n_{wi}, respectively M_{wi} be the level of the mean cash, respectively the level of the mean income, for the wage earners in region i $(i = 1, 2,...q)$ and n_{pi} respectively M_{pi}, the level of the mean cash, respectively the level of the mean income, for the peasants in region i $(i = 1, 2,..., q)$.

We shall consider that:

$$n_{wi} = f_{wi}(M_{wi}, \omega) \tag{19}$$

$$n_{pi} = f_{pi}(M_{pi}, \omega) \tag{20}$$

where we have noted by f_{wi}, f_{pi} the functional relation between the level of mean cash and the level of mean income in region i for wage earners and peasants, and ω is an aleatory variable.

Since, due to lack of statistical data, we do not have the possibility to determine the functional relationship between the level of mean income and the level of mean cash for every region, we shall consider that this functional relationship is unique, viz. that it is the one determined on basis of the statistical data for the whole country. Likewise, we shall, for the time being, make abstraction of the aleatory factor.

In this case, the level of mean cash n_{wi} and n_{pi} will be determined on basis of the formulae *(17)* and *(18)*.

That is

$$n_{wi} = \frac{A_w}{1 + B_w e^{-C_w M_{wi}}} \tag{17'}$$

$$n_{pi} = \frac{A_p}{1 + B_p e^{-C_p M_{pi}}} \tag{18'}$$

In this case the cash level in region i will be:

$$N_i = P_{wi} \times n_{wi} + P_{pi} \times n_{pi} \, (i = 1, 2, ..., q). \tag{21}$$

where we have noted by N_i the cash level in region i, and by P_{wi} and P_{pi} the number of wage earners, and, respectively, of peasants in region i.

Hence:

$$N_i = P_{wi} \frac{A_w}{1 + B_w e^{-C_w M_{wi}}} + P_{pi} \frac{A_p}{1 + B_p e^{-C_p M_{pi}}} \qquad (21')$$

A certain correction should still be made to formula *(21')*. It arises from the following considerations:

The logistic function

$$f(x) = \frac{A}{1 + Be^{-Cx}} \qquad (22)$$

has an inflexion point which causes it to be convex up to a certain value of the independent variable x, after which it becomes concave.

In order to dermine the point of inflexion we compute the second derivative of the function $f(x)$, which is

$$f''(x) = \frac{ABC^2 e^{-Cx} (B^2 e^{-2Cx} - 1)}{(1 + Be^{-Cx})^4} \qquad (23)$$

and which becomes null for the value

$$x = \frac{\ln B}{C} \qquad (24)$$

Hence, the logistic function is convex in the interval $\left(0, \frac{\ln B}{C}\right)$ and concave in the interval $\left(\frac{\ln B}{C}, \infty\right)$.

On the other hand, it is well known that convex functions have the property that:

$$\sum_{i=1}^{q} \alpha_i f(x_i) \geqslant f(x) \qquad (25)$$

where:

$$\sum_{i=1}^{q} \alpha_i = 1 \quad \alpha_i \geqslant 0 \qquad (26)$$

$$x \in (x', x'') \text{ where}$$

$$x' = \min x_i ; \; x'' = \max x_i$$

and concave functions have the property that

$$\sum_{i=1}^{q} \alpha_i f(x_i) \leqslant f(x) \qquad (27)$$

where the notations are the ones shown before.

Hence, there results that in our case

$$\sum_{i=1}^{q} N_i \leqslant N, \quad M_i < \frac{\ln B}{C} \qquad (28)$$

$$\sum_{i=1}^{q} N_i \geqslant N, \quad \boldsymbol{M} > \frac{\ln b}{C} \qquad (29)$$

In other words, by computing the existing cash in every region, using the formulae *(17')*, *(18')*, and *(21')*, the sum of the existing cash in all the regions will yield a larger or a smaller amount than the existing cash at the level of national economy, which is a known amount. This depends on the fact that we are either in the convex or in the concave part of the logistic curve.

In view of investigating this deviation we suggest that the difference

$$\Delta = N - \sum_{i=1}^{q} N_i \qquad (30)$$

between the actual and computed values of existing cash at the level of national economy be distributed directly proportional to the values N_i.

Numerical example

Let us assume that our country is divided into only 3 regions, for which we know the volume of income, their distribution in the two main groups (wage earners, peasants), as well as the cash and structure of the population.

Table 6 shows the income of the population (total, wage earners and peasants), the exisitng cash, as well as the number and structure of the population, for the whole country, for the period 1961—1965.

Table 6

Year	Income (million lei)			Cash (million lei)			Number of inhabitants (thousand)		
	Total	Wage earners	Peasants	Total	Wage earners	Peasants	Total	Wage earners	Peasants
1961	144,572	109,239	35,113	11,055	5,006	6,049	18,567	7,810	10,757
1962	160,189	121,514	38,675	12,407	5,750	6,657	18,681	8,114	10,567
1963	175,703	132,712	42,991	13,250	6,750	6,500	18,813	8,314	10,468
1964	188,787	143,823	44,964	14,250	7,500	6,750	18,927	8,750	10,177
1965	209,079	159,562	49,517	15,277	8,427	6,850	19,050	9,350	9,700

Table 7 shows the mean income and the mean cash (total, wage earners and peasants).

Table 7

Year	Mean income (lei)			Mean cash (lei)		
	Total	Wage earners	Peasants	Total	Wage earners	Peasants
1961	7,787	13,987	3,265		690	535
1962	8,575	14,975	3,660		773	575
1963	9,340	15,962	4,107		842	618
1964	9,975	16,437	4,417		870	645
1965	10,975	17,065	5,105		902	703

On basis of the data in table 7 the mathematical relationship existing between the level of the mean income and the level of the mean cash has been determined for wage earners and peasants.

They are:

$$n_w = \frac{1{,}040}{1 + 120\, e^{-M_w/2560}} \qquad (31)$$

$$n_p = \frac{930}{1 + 3.2\, e^{-M_p/2014}} \qquad (32)$$

Let us assume that we want to compute the cash level in the regions *A*, *B* and *C* for 1965.

The data relating to 1965 are shown in table 8.

Table 8

Year 1965	Income (million lei)			Number of inhabitants (thousands)			Mean income (lei)		
	Total	Wage earners	Peasants	Total	Wage earners	Peasants	Total	Wage earners	Peasants
Total Romania	209,079	159,562	49,517	19,050	9,350	9,700	10,975	17,065	5,105
Region A	80,039	65,039	15,000	8,050	4,500	3,550	9,943	14,453	4,225
Region B	60,040	40,523	19,517	5,000	2,000	3,000	12,008	20,261	6,500
Region C	69,000	54,000	15,000	6,000	2,850	3,150	11,500	18,947	4,761

By applying formulae *(31)* and *(32)* there results that the mean cash in the 3 regions is (in lei):

	Wage earners	*Peasants*
Region *A*	730	629
Region *B*	996	793
Region *C*	969	675

Multiplying the mean cash by the number of inhabitants in each region, and distributing the difference $N - (N_1 + N_2 + N_3)$ directly proportional to the values N_1, N_2, N_3 the following amounts are obtained for the existing cash in each region (in million lei):

	Total	*Wage earners*	*Peasants*
Region *A*	5,652	3,392	2,260
Region *B*	4,545	2,136	2,407
Region *C*	5,056	2,904	2,152

For comparison, we also give hereunder the volume of the cash distributed, in the case of the chosen example, proportionally to the income achieved in each region (in million lei):

	Total	*Wage earners*	*Peasants*
Region *A*	5,504	3,458	2,046
Region *B*	4,835	2,108	2,727
Region *C*	4,914	2,868	2,046

As it may be seen, if, with respect to the cash held by wage earners there are no noticeable differences, the situation in the case of peasants is altogether different.

From the above the conclusion may be drawn that the method of determining the volume of cash existing in a given territory, by using the logistic function, constitutes a forward step in comparison with the computations so far made. The method expounded in this paper should, however, only be considered as the first stage of a path still to be followed, in the sense that there are still a number of elements which have not been included in the computation and whose determination and inclusion still requires complex analyses and studies.

The starting point in carrying out the computations was the hypothesis that the manner in which the income is used by the population and, implicitly, the level of the cash held by it depends on the mean income level and of the population structure (wage earners, peasants), which differs noticeably from one region to the other. Thus, a few of the essential elements which determine the cash level in each region have been included in the computation.

Concomitantly, however, it was considered that the wage earner group and the peasant group are homogeneous in the whole country, since at the present stage of investigation it has not been possible to take into account the differences existing between the manner in which the income is used by the population of some regions, in comparison with that in other regions. It is, however, well known that by virtue of tradition formed in the historical, relief, climatic and specific means of communication conditions, there are differences not to be neglected from one region to another. Besides, the nature

of agricultural crops and the periodicity of their turning to account, the custom of using savings institutions etc., bear an influence on the level of existing cash in circulation, by regions.

In order to include in the computation the specific features outlined above, complex monographic studies are required concerning each region in part, thus requiring particular conditions and time. The extent to which it will be possible to accurately determine the peculiarities mentioned will create the necessary prerequisites in view of applying the methods of computation suggested and of passing on to a better form of the computations regarding the volume of cash held by the population within a given territory.

APPLICATION DES METHODES STATISTIQUES-MATHEMATIQUES POUR LA DETERMINATION DES DISPONIBILITES FINANCIERES DE LA POPULATION DANS UN TERRITOIRE DONNE

Partant de la nécessité de connaître le numéraire possédé par la population de certaines régions, les auteurs présentent quelques méthodes de calcul, qui diffèrent par les éléments pris en considération. Ainsi, outre le volume des revenus, on propose d'introduire dans le calcul le niveau du revenu moyen, ainsi que la structure de la population (salariés, paysans), considérant que le volume du nombre sera plus élévé et les revenus des paysans ont un plus grand poids (la vitesse de circulation du numéraire des paysans est plus lente que chez les salariés).

Prenant pour base l'analyse, on propose une formule de calcul ayant la forme de la fonction logistique afin de déterminer le volume du numéraire existant dans la region donnée pour la population toute entière et, séparément, pour les salariés et les paysans. Partant de ces formules, connaissant le revenu moyen dans chaque région, tant pour les salariés que pour les paysans, on donne des exemples regardant le calcul du niveau moyen et le volume du numéraire dans chaque région.

ПРИМЕНЕНИЕ СТАТИСТИКО-МАТЕМАТИЧЕСКИХ МЕТОДОВ ДЛЯ ОПРЕДЕЛЕНИЯ НАЛИЧИЯ ДЕНЕЖНЫХ СРЕДСТВ У НАСЕЛЕНИЯ ПО ОБЛАСТЯМ И РАЙОНАМ

Исходя из необходимости знания количества денежных средств имеющихся в наличии у населения, в зависимости от экономического профиля данной территории, авторы, с этой целью, излагают несколько методов исчисления, отличающихся один от другого по принимаемым в расчет элементам. Так, помимо объема доходов, предлагается ввести в исчисление средний уровень доходов и структуру населения (служащие, крестьяне), исходя из соображения, что количество находящихся в наличии денежных средств будет большим в более доходных территориальных единицах, в которых доходы крестьян имеют больший удельный вес (у крестьян, денежные средства обращаются с меньшей быстротой, чем у служащих).

На основе анализа, предлагается формула исчисления в форме логической функции для определения количества денежного наличия на территории, имеющей данный профиль, у всего ее населения или отдельно, — у служащих и крестьян. Исходя из этих формул и зная средний доход, осуществляемый в каждой области служащими и крестьянами, приводятся примеры исчисления среднего уровня денежного наличия и величина денежного наличия по каждой области.