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Mathematical Models in Demography

Romanian demography has greatly developed in the last few years; a big number of studies approach the application of the mathematical methods and models, the extension of the sphere of cybernetic and systemic interpretation of demographic phenomena. The most conclusive domains for applications are: mortality, nuptiality, fertility, urbanization and the territorial distribution of the population, the relationship between population and economy. Besides the classical mathematical models, such as the tables with simple and multiple decrement and the biometric functions, factorial models of demo-economic zoning have been elaborated and applied in the analysis, models of urban "attraction", demo-economic models based on social accounting etc.

Demography in Romania has just like the other sciences witnessed a great development, in the last few years, marked by a big number of studies with a theoretical, analytical and methodological character. An important contribution to this progress was made by the cybernetic and systemic interpretation of demographic phenomena, in the spirit of the conception of the school of economic cybernetics of academician, prof. Manea Mănescu [1], which opened up new prospects to this old science. The journal "Economic Computation and Economic Cybernetics Studies and Research" has been an active promoter of these preoccupations and it is a pleasant duty for a demographer to praise the journal, on its tenth anniversary, for this activity.

The priorities characterizing the demographic science are those determined by the characteristic aspects of the population development during the last few years and the needs of the demographic policy.

The first life table in the post-war period is that drawn up by academician G. Mihoc and contributors, on the basis of the 1956 census data [2] for the total population. Then followed the life tables for the population on sex (Lungu and Mada, 1956), the life table for the urban and rural population on sex (V. Gheţău, A. Bejan, L. Madaras, 1961) which outlined the supermortality in the rural population, measured by the main biometric functions, the life tables belonging to the Central Board of Statistics (1961, 1963, 1964—1967, 1970—1972 and 1972—1974), and a number of life tables for bigtowns (dr. P. Pruteanu, 1962, 1963 and 1967), for Bucharest municipality (G. N. Munteanu, 1971; Viorica Popescu and C. Tănase, 1972); those for the years 1956 and 1962, for Bucharest municipality (Th. Ilea and contributors), life tables by cause of death (dr. P. Pruteanu, 1966).

It should be pointed out that the life table of the Central Board of Statistics 1970—1972 [3] has calculated the probabilities of direct death based on the main collectivities of the deceased gr. I and the number of the population at each year of exact age.

The life table for the infant deaths which was drawn up for the first time in Romania by F. Muntean as part of his diploma paper for the Academy of Economic Studies (1975) represents a progress in the elaboration of the life tables.

The validity of many life tables has been checked also with the aid of methods, such as that of Reed-Merrell, of the UNO model tables and of the Coale-Demeny model tables. Life tables by cause of death and possibly on the cause of professional mortality, for this sector to be entirely covered, are to be drawn up in future.

The analysis of infant mortality formed the object of special studies. We mention the proposals on the improvement of the biometric analysis method of the French demographer J. Bourgeois-Pichat which are owed to dr. P. Pruteanu [4] and particularly to professor V. Sora [5] and [6].

The method elaborated by V. Sora, called the death probabilities method, starts from the finding that by establishing such probabilities on age groups in the first year of life, the trend can be noticed of a marked decrease of these probabilities in the first month of life, after which the decrease, though continuous, becomes almost uniform beginning with the age of 4—5 months. It can thus be estimated that in the second half of the age of 0, the endogeneous causes disappear almost entirely. The establishment of the exogenous mortality after Sora's method implies the following operations:

— the establishment of the death probabilities in the age interval 120—365 days and their graphic representation;

— the establishment of the abscissa point from where the decrease of the death probabilities marks a linear trend;

— the establishment of the linear function parameters expressing the dependence of the death probabilities on the age group of the deceased by the least square method:

$$q_{x_i} = a + bx_i$$

— the integration of the linear function into the definition domain 7—365 days, the result is the exogeneous death rate;

$$\int_{7}^{365} f(q_{x_i})\, d_{1xi}$$

— the exogeneous death rate

— the endogenous death rate is established by subtracting the exogeneous death rate from the infant mortality rate.

According to this method, the endogeneous death rate is higher than that established through J. Bourgeois-Pichat's biometric analysis, but very close to that obtained when using the direct method of the death causes.

Potential demography whose concepts and indices are based on life tables is also present within the preoccupations of the Romanian demographers. Thus, Vasile Gheţău [7], [8] and [9] has calculated the total life potential of Romania's population for the years 1956, 1961 and 1963, the average life potential and "the life centre", according to formulas:

— the total life potential of a population (as the sum of the individual life potentials):

$$V(0, \omega; 0, \omega) = \sum_{x=0}^{x=M-1} P_x \frac{e_x + e_{x+1}}{2},$$

where: V — the total life potential;
M — the age until when the life span is extended;
e_x — the life expectancy at the age of x;

— the average life potential of the population aged $m - M$ years $(M > m)$

$$\bar{v}(m, M; 0, \omega) = \frac{\sum_{x=m}^{x=M-1} P_x \frac{e_x + e_{x+1}}{2}}{\sum_{x=m}^{x=M-1} P_x}$$

The method proposed by the Swiss demographer Liebmann Hersch already in 1944 can be successfully used in the study of mortality and of the decease causes, of the active population, of professional life. The population's reproduction as Louis Henry pointed out can be more correctly characterized with the aid of the indices of potential demography. In any case, economic demography, in putting itself the problem, of establishing the national income and consumption fund as age function, may sensibly benefit by potential demography. Favourable prospects are also created for the analysis of demographic ageing by using "the life centre".

POPULATION'S NUPTIALITY

Three nuptiality tables have been elaborated and published. Table 1956 [10] is based on the 1956 census and was constructed only for the decrease of the cohort of unmarried people following the marriage risk. The second [11],

for 1967 has taken into consideration the mortality effect too. The third one, for the 1971—1972 period [12], takes into consideration both nuptiality and mortality; the calculus of the marriage probabilities is made directly by relating the first marriages between two exact ages (the collectivity of marriages of I grade), x, x+1, to the number of the unmarried population of exact age x. The effect of mortality combined with that of nuptiality was calculated with the formula of interference between the two events.

A complete table of nuptiality for the years 1973—1974, which is under print, was constructed on the basis of an initial cohort at birth of 100,000 by taking into account the death probability for singles $\left(1 - \frac{q_x}{2}\right)$ and the marriage probability $\left((1 - \frac{ma_x}{2}\right)$ and, the establishment, in the end, of the stationary population and of the years of life as singles of the persons belonging to the initial cohort.

All these tables have provided important information about the mean, median and modal age at first marriage, "the model" of nuptiality — the distribution of marriages according to age — and the frequency of the final celibacy.

MIGRATION, URBANIZATION AND TERRITORIAL DEMOGRAPHY

Demography had to answer the many questions raised by the description and analysis of internal migration, of urbanization and of territorial distribution, a fact which put in the forefront the necessity of using some mathematical methods and models.

The mathematical model of internal migration in Romania drawn up by I. Measnicov [13] started from the idea that migration is determined by several factors and can be described by equation:

$$M_{ij} = f[(C_i - C_j),\ (I_i - I_j),\ A_i,\ D_{ji}],$$

where:

M_{ij} — the number of inhabitants attracted from locality j to locality i;
$C_i - C_j$ — the difference between the earning possibilities in locality i in comparison with locality j;
$I_i - I_j$ — the difference between the places of study in the two localities;
A_i — other factors of attraction;
D_{ji} — the distance between the two localities.

The first three factors act as a force of attraction whereas the distance represents the resistence to this force. The formula of a Pareto type describing this relation is:

$$M_{ij} = KP_iP_jD_{ji}^{-\alpha}$$

It is the author's merit to estimate the respective constant values, to this purpose, having processed a huge data obtained at the population census

in 1966. The theoretical curve calculated with the aid of the regression curve $Y = 55.820\ e^{-0,09406\ x}$ very close to the empiric curve has allowed the establishment of an important law: the bigger the town, the bigger the force of attraction is; the distance is inversely proportional to the force of attraction. This model may also become a previsional model, which is very important for the prognosis of internal migration in relation to urban growth.

The necessity of characterizing the territorial distribution of the socio-economic characteristics and of analysing the variation of the demographic characteristics has led to the application of a mathematical model on the basis of which they have come to a demographic and economic typology of the territorial-administrative units (counties) [14].

For the 39 counties, they have adopted 10 variables, five of which economic (the global product per inhabitant, fixed assets per inhabitant, the population employed in the non-agricultural sectors, the number of personell per 1000 inhabitants and the money incomes per inhabitant) and five cultural, medical and demographic variables (the urbanization rate, the proportion of persons with secondary and higher training, the number of radio per 1000 inhabitants, the number of inhabitants for one doctor and infant mortality)

The formula used is that of "relative distance" or "the discriminant" of the type:

$$D = \sum_{i=1}^{n} \frac{|d_i|}{\sigma_i}(1 - r_{ij}),$$

where:

d_i — the distance between the level of the characteristic of a county as compared to the level of the characteristic of the "fictitious" county;
r_{ij} — the correlation coefficient;
σ_i — the quadratic mean deviation.

The obtained values of distance D have allowed the hierarchization of the 39 counties; five groups of counties have been made up on this basis. This classification may serve as a basis for the factorial analysis of the demographic variables.

Other models of demo-economic zoning have also been tested during the last few years. Worth mentioning is an original model based on a theory of academician O. Onicescu [15] and on a scheduling algorithm [16]. An application is given in [17]. They start from two sets: C_j — the set of counties, where: $j = 1,2, ..., 40$ and C_i — the set of indices characterizing the level of development of the counties, where $1 = 1,2, ...,10$. Each county is characterized by a vector $a_j = (a_{ij})$, where $i = 1, 2, ... ,10$; $j = 1,2, ... ,40$, and whose elements are the values taken by the different characteristics C_i for the respective county. Matrix A is constructed on this basis:

$$A = (a_{ij}) = \begin{Vmatrix} a_{1,1}, & a_{1,2}, & a_{1,3}, & \cdots & a_{1,40}, \\ a_{2,1}, & a_{2,2}, & a_{2,3}, & \cdots & a_{2,40}, \\ c_{10,1}, & c_{10,2}, & c_{10,3}, & \cdots & c_{10,40}, \end{Vmatrix}$$

The town provides particular aspects in the study of the differential character of the demographic phenomena-mortality, fertility, nuptiality. Some of the demographic phenomena vary with the size of the town and its socio-economic type. To outline this differentiation, life tables of types of towns have been elaborated , for the first time, by the lecture Hristache [18].

His life tables, 11 in all, ware drawn up on three groups of size (over 100,000 inhabitants; 20,000—99,999 inhabitants and under 20,000 inhabitants) differentiated on towns in which the population employed in the primary, secondary and tertiary sectors is predominant.

DEMOGRAPHIC PROJECTIONS AND MODELS OF THE STABLE POPULATION

Thanks to the electronic computers and their wide-scale utilization in demography, the number of applications in the field of demographic projections has greatly extended. There are several demographic forecasts published for the whole of Romania and for various territorial-administrative units. Part of the first category is V. Gheţău's estimate [19] in which a normative type forecast was achieved, by using the morphological or components method. The models of stable population of a Coale-Demeny type as well as the United Nations model life tables have been used for check up. This forecast has outlined the demographic conditions necessary for the establishment of a number of the population established by the normative forecast.

Another project of the total population is that of I. Hristache [20]. The new element in this project is the utilization of the matricial method and of algorithms for electronic processing.

The matricial model was also used in the prognosis of the population of Bucharest municipality by Eugen Pecican and Viorica Popescu [21]. Interesting for this application is that the internal migration has also been taken into account which made the matricial model acquire a more complex character. As a matter of fact, E. Pecican has already used the matricial model in analysing the intercounty migration [22]. Generally, the matricial models are ever wider spread in Romanian demography.

The demographic forecasts of a territorial type have been multiplied during the last few years. We have mentioned that drawn up by M. Ţarcă for Jassy county, in several variants [23].

The stable population model is currently used in Romanian demography. It is instructively applied in the study of the demographic ageing of the population, of the causes and effects of this process [24].

In the same order of ideas, we should mention the attempts at interpreting the population as a system. The study [25] examines the active population as a cybernetic system; another study [26] extends the systemic interpretation over the entire population.

The necessity of describing and scientifically analysing the relationship between economy and population, in a systemic outlook, by using the concepts of modelling and of social and demographic accounting has led to the working out of a demo-economic model providing a large framework for the treatment of the demo-economic variables and of their interconnections. This model which is owed to academician O. Onicescu [27] and [28] has been developed in the studies of several mathematicians, cyberneticians and demograahers. These models aim at describing the relations between variables on territorial plane.

The elements of Academician O. Onicescu's demo-economic model as "actors" of the variant of representation of national life (social and economic) are in this model: persons P (and eventually the juristic persons as economic units) making up the population of the respective country; the state S — as a distinct personality; the exterior of the state E (or the socio-economic environment); the natural resources N (physical-material-and spiritual).

A diversity of accounts is attributed to each physical or institutional person, accounts which may be active or passive in direct or indirect correspondance with the accounts to be registered in the end, by a central Bank representing the State and at the same time with accounts of Nature and of the exterior World.

To describe the state of the component elements of the model, these accounts are considered certain in a first approximation and their variations will be deterministic. Thus, by associating with each person P of the population π ($P \in \pi$) at moment t, the account m_s the generally vectorial account will result

$$\overline{m}(P, t) = [m_0(P, t), ..., ms\ (P, t),\ \mu\ (P, t)$$

with

$$\mu(P, t) = \sum_{j=0}^{s} \alpha_j\, m_j\, (P, t)$$

where s is the fixed natural number and α_j — parameter ($j = 0, 1, ..., s$).

The component accounts $\{m_j\,(P, t), j = 0,1, ..., s\}$ have the significance of describing "the state" of person P at moment t, in relation to certain general categories of incomes or expenses (salary, consumption, taxes, savings, other receipts or services to the other factors, nature inclusively, etc.). The account $\mu(P, t)$ has the significance of a "sum account" of the component accounts.

The design of the demo-economic model meets with the conceptions aiming at covering, in the last analysis, the entire activity of social man in his relations with his fellowmen, with the state as the political form of organization of society, with the exterior environment and — on a vaster plane — with the biophysical universe.

MODELS OF DEMO-ECONOMIC ZONING ACCORDING TO MULTIFACTORIAL CRITERIA

To identify the zones with certain particularities towards which the statistical studies and the demo-economic analyses will be directed with priority, the territory of the Socialist Republic of Romania has been divided into demo-economic zones according to several economic, demographic and socio-cultural characteristics, common and variable, illustrating conspicuously the level of demo-economic development of each territorial micro-unit. The model drawn up by I. Marinescu and contributors [29] and [30] have several characteristics.

The country's territory was considered as a set Z made up of $N \approx 2700$ territorial microunits (communes, suburban communes, municipalities and towns) administrative units. 37 economic, demographic and socio-cultural characteristics significantly illustrating the level of demo-economic development of each unit has been associated with each unit $Z_i \in z$. In relation to characteristics $X_j \in X$ the subjacent elements of set Z were hierarchized by taking into account their level of development, on the basis of a synthetic characteristic which was conventionally called the unifactorial index and then they were reunited on homogeneous zones of bigger size, demo-economic zones which define a partition of set Z (18 in the rural zone and 8 in the urban zone).

A model of factorial analysis whose algorithm was programmed on the ICL 1900 electronic computer in the FORTRAN IV language (the INDT program) has been used for hierarchization and zoning. A model for removing the redundant variables, on the basis of the analysis of the main components has been incorporated into the program.

The applicative framework of the mathematical methods and models in demography will be extended in future especially by approaching problems of fertility of the female population. The obtaining of more detailed information on fertility, on the basis of nation-wide inquiries and of the data to be secured at the future population and dwelling census scheduled for 1977 represent favourable conditions for the working out and application of new mathematical methods and models.

REFERENCES

1. MANEA MĂNESCU: *Preocupări cu privire la tehnica de elaborare a modelelor cibernetice în economie* (Preoccupations on the Technique of Elaborating Cybernetic Models in Economy). Studii şi cercetări de calcul economic şi cibernetică economică, no 2, 1967.
2. * * * *For the life tables in the 1956—1975 period see a complete bibliography.* In: VI. Trebici "Mică enciclopedie de demografie". Editura ştiinţifică şi enciclopedică. Bucharest, 1975. pp. 221—222.
3. * * * *Anuarul demografic al Republicii Socialiste România 1974.* (The Demographic Year Book of the Socialist Republic of Romania, 1974). The Central Board of Statistics and the National Demography Commission, Bucharest, 1974, pp. 328—336.
4. PRUTEANU, P: *Mortalitatea infantilă exogenă şi endogenă.* (Exogeneous and Endogeneous Infant Mortality). Revista de statistică, no. 12, 1966.
5. SORA V.: *O nouă metodă de estimare a mortalităţii infantile endogene şi exogene.* (A New Method of Estimating Endogeneous and Exogeneous Infant Morality). Revista de statistică no. 2, 1971.

6. SORA V.: *Considerații pe marginea evoluției mortalității infantile în țara noastră. Lucrările celei de a IV-a Consfătuiri științifice de statistică.* (Considerations on the Evolution of Infant Mortality in Our Country). Debates of the Fourth Scientific Conference of Statistics, Bucharest, 1966, pp. 1331—1361.
7. GHETAU V.: *Aspecte din demografia potențială* (Aspects of Potential Demography). Revista de statistică, no. 9, 1966.
8. GHETAU V.: *Înaintarea în vîrstă și decesele în demografia potențială.* (Ageing and Deceases in Potential Demography). Revista de statistică no. 1, 1967.
9. GHETAU V., TREBICI VL.: *Longitudinal Analysis, Potential Demography and Demographic Accounts* (II). Economic Computation and Economic Cybernetics Studies and Research, no.4, 1973, Bucharest.
10. GÂNDAC I. D., GHETAU V.: *Tabela de nupțialitate a populației din țara noastră* (Table of Nuptiality of the Dopulation of Our Country). Revista de statistică, no. 1 1966.
11. GHETAU V.: *Interferența dintre nupțialitate și mortalitate* (The Interference between Nuptiality and Mortality). Debates of the Seventh Scientific Conference of Statistics, June 11—14, 1969, Vol. II. The Central Board of Statistics, 1972, pp. 1702—1708.
12. ȘERBAN G.: *Tabelele de nupțialitate a populației României 1971—1972,* (Tables of Nuptiality of Romania's Population 1971—1972). The Central Board of Statistics, 1975.
13. MEASNICOV I.: *Un model matematic al migrației interne în România* (A Mathematical Model of Internal Migration in Romania). Viitorul social, a review of sociology and politology, no. 1, 1976, Bucharest, pp. 50—59.
14. TREBICI VL.: *Populația României și creșterea economică* (The Population of Romania and the Economic Growth). Studii de demografie economică, Editura Politică, Bucharest, pp. 274—297.
15. ONICESCU O.: *Procedee de estimare comparativă a unor obiecte purtătoare de mai multe caracteristici,* (Methods of Comparative Estimation of Objects Bearing Several Characteristics). Revista de statistică, no. 4, 1970.
16. SAPHIER I. I., RUSU A. V.: *Algoritm de ordonare a unei mulțimi finite de obiecte pe baza caracteristicilor comune* (Algorithm for Scheduling a Finite Set of Objects on the Basis of their Common Characteristics). Revista de statistică, no. 8, 1971.
17. SOCOLESCU VALENTINA: *Classification des districts de Roumanie d'après des critères socio-économiques.* (A paper presented at the symposium of demography, Buzău, December 1975.
18. HRISTACHE I., TREBICI VL.: *Urbanizare și demografie diferențială în România.* (Urbanization and Differential Demography in Romania). Viitorul social, no. 3, 1975, pp. 489—499.
19. GHETAU V.: *O perspectivă condițională a populației României în următoarele decenii. Metodă, Rezultate.* (A Conditional Prospect of Romania's Population in the Next Decades, Method, Results-Revista de statistică, no. 3, 1973.
20. HRISTACHE I.: *Prognoza populației R. S. România pînă în anul 1990.* (A Forecast of the Population of the Socialist Republic of Romania until the Year 1990). Studii și cercetări de calcul economic și cibernetică economică, no. 2, 1975.
21. PECICAN E., POPESCU V.: *Un model matematic de prognoză a populației municipiului București.* (A Mathematical Model of Forecasting the Population of Bucharest Municipa lity). Revista de statistică, no. 11, 1973.
22. PECICAN E.: *Probleme ale migrației interjudețene.* (Problems of Inter-County Migration). Revista de statistică, no. 5, 1973.
23. TARCA M.: *Introducere în prognoza demografică* (Introduction to Demographic Forecast). Editura Junimea, Jassy, 1974.
24. TREBICI VL.: *Îmbătrînirea demografică a populației României.* (Demographic Ageing of the Population of Romania) Viitorul social, no.1, 1972.
25. TREBICI VL.: *The Cybernetic model of Manpower.* Economic Computation and Economic Cybernetics Studies and Research, no. 3, 1974, Bucharest.
26. TREBICI VL.: *Populația ca sistem.* (The Population as a System). Studii și cercetări de calcul economic și cibernetică economică, no. 3, 1975.
27. ONICESCU O., BOTEZ M.: *Un Model demoeconomic. Preliminarii.* (A Demo-Eonomic Model. Preliminaries), Revista de statistică, no. 8, 1973.
28. ONICESCU O., BULGARU V., BOTEZ M.: *Considerații privind modelul demoeconomic în contextul creșterii populației.* (Considerations on the Demoeconomic Model in the Context of the Population's Growth). Revista de statistică, no. 8, 1974.

29. MARINESCU I.: *Zonări după criterii multifactoriale.* (Zoning According to Multifactorial Criteria) Studii de statistică informațională. Reports and papers presented at the seminar on the application of statistical-mathematical models in economy. The Central Board of Statistics, Bucharest, 1975, pp. 19—35.
30. MARINESCU I.: *Demo-régionalisation et analyse territoriale de la mortalité infantile.* Seminaire international sur l'analyse de la mortalité. Roumanie, Mamaia, 20 septembre-3 octobre, 1972.

MODELES MATHEMATIQUES EN DEMOGRAPHIE

La démographie roumaine a connu ces dernières années un développement remarquable. Un grand nombre de travaux s'occupent de l'application des méthodes et des modèles mathématiques, de l'élargissement de l'interprétation cybernétique et systémique des phénomènes démographiques. Les domaines les plus caractéristiques pour les applications sont: la mortalité, la nuptialité, la fécondité, l'urbanisation et la répartition territoriale de la population, le rapport entre la population et l'économie. Parallélement aux modèles mathématiques classiques, tels que les tables à décroissance simple et multiple et les fonctions biométriques, les modèles de l'attraction urbaine, démo-économiques, à base de comptabilité sociale et autres ont été élaborés et appliqués à l'analyse des modèles factoriels de répartition par zone démo-économique.

МАТЕМАТИЧЕСКИЕ МОДЕЛИИ В ДЕМОГРАФИИ

Румынская демография познала в последние годы крупное развитие. Большое число работ занимается применением математических методов и моделей, расширением рамок кибернетической и системной интерпретации демографических явлений. Самыми характерными областями применения являются: смертность, брачность, плодовитость, урбанизация и размещение населения, соотношение между населением и экономикой. Наряду с классическо-математическими моделями, как таблицы простого и множественного уменьшения и биометрические функции, были разработаны и применены в анализе факториальные модели демо-экономического зонирования, модели городского «тяготения», демо-экономические модели, основавыющиеся на социальном счетовостве и т.д.

30. VI. 1976.