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Computer Generation of Random Variables and Vectors Used in Reliability

The aim of this paper is to present briefly some algorithms, each of them being used for generating one sampling value of a given random variable, with particular reference to those used in reliability theory. Some of the distributions mentioned are: negative exponential, Weibull, gamma, normal and log-normal, beta, geometric, negative binominal, hypergeometric and Poisson, [5]. *Computer generation of normal, multinominal and Dirichlet random vectors is also pointed out. The parameters of considered distributions are assumed to be known. Through this paper, a random number generator (called RND) is assumed to be available. (For example, in* [19], [20] *the* RND *is in the form* $N_{i+1} = 16807.\ N_i (\mathrm{mod}\ (2^{31} - 1))$, $0 < N_i < M = 2^{31} - 1$, *where* N_i *are integers. Uniform pseudo-random numbers over* (0, 1) *are* $U_{i+1} = (N_{i+1})/M$).

1. SOME GENERAL PROCEDURES

The following general theorems will be used to transform uniform random variates U over (0,1) into sampling values of non-uniform random variates X.

Theorem. 1 (Inverse method). Let X be a random variable whose cumulative distribution function (c.d.f) is $F(x)$, (known) and U an uniform random variate over (0,1) (assumed to be generated by RND). Then a sampling value of X is $X = F^{-1}(U)$ where F^{-1} is the inverse of F. (For proof see [8]).

The *inverse method* deriving from this theorem is applied to distributions for which F^{-1} can be easily calculated.

The following three theorems lead to so-called *rejection procedures*, [2], [13], [19].

Theorem 2. If $\{Z_n,\ n \geqslant 1\}$ is a sequence of independent random variables which have the same c.d.f. $F(x)$ then if Z_0 having the c.d.f. $F_0(x)$, is independent from Z_i and, if K is the integer random variable defined by the condition $Z_0 \geqslant Z_1 \geqslant \ldots \geqslant Z_{K-1} < Z_K$, then

$$H(x) = P[Z_0 \leqslant x;\ K = \text{odd}] = \frac{1}{p_a}\int_{-\infty}^{x} e^{-F(t)}\ dF_0(t) \tag{1}$$

$$p_a = \int_{-\infty}^{+\infty} e^{-F(t)}\ dF_0(t)$$

(For proof see [19]).

This theorem can be used to generate a random variable X corresponding to the c.d.f. $H(x)$, when the Z_0 and Z_i, $i \geqslant 1$ can be generated. The procedure is the following.

Procedure R1

1. Generate Z_0 with the c.d.f. $F_0(x)$ and a finite sequence of independent random variates $\{z_i,\ 1 \leqslant i \leqslant K\}$ where the random index K satisfies the condition $Z_0 > Z_1 > \ldots < Z_{K-1} < Z_K$.
2. If K is an even integer go to step 1.
3. (When K is odd). Take $X = Z_0$, where Z_0 belongs to the last (accepted) sequence $\{Z_i,\ 0 \leqslant i \leqslant K\}$.*

(In the following, "*", marks the end of the algorithm).

The efficiency of the algorithm is caracterized by the probability of acceptance p_a (see [2], [19]), or by the expected number of random variables $\{Z_i,\ i \geqslant 0\}$ needed for the generation of one Z_0 accepted, which is

$$N = \frac{1}{p_a}\left[1 + \int_{-\infty}^{+\infty}\left(\sum_{k=0}^{\infty}\frac{(F(t))^K}{k!}\right) dF_0(t)\right]. \tag{2}$$

Theorem 3. Assume that the probability density function (p.d.f), $f(x)$ of the random variable X may be written as

$$f(x) = q(1 - Q(x)r(x),\ x \in R,\ q = \text{const.} \tag{3}$$

where $Q(x)$ is the c.d.f. of a random variable Z and $r(x)$ is the p.d.f. of a random variable Y which is independent from Z. Then, the conditional p.d.f. of Y given that $Y \leqslant Z$ is $f(x)$. (For proof see [19]).

An alternative of this theorem is obtained when $f(x) = qQ(x)r(x)$. The multivariate case is considered in [8].

A rejection procedure for generating X when Y and Z can be generated is the following.

Procedure R2

1. Generate a random variable Y with the p.d.f. $r(y)$ and a random variable Z (independent from Y) with the c.d.f. $Q(z)$.
2. If $Y > Z$ go to step 1 (reject).
3. (When the pair (Y,Z) is accepted). Take $X = Y$. *

The efficiency of this generation procedure is characterized by the probability of acceptance (i.e. $p_a = 1/q$) or the expected number of pairs (Y, Z) to be generated for accepting an Y (i.e. $N = q$).

Theorem 4. Let X be a random variable with the p.d.f. $f(x)$ and Y a random variable with the p.d.f. $h(x)$. Assume that there exists a constant $\alpha > 0$, such as $f(x) \leqslant \alpha . h(x)$, $x \in R$. If U is an uniform random variable over (0,1) independent from Y, then the conditional p.d.f. of Y given that $U \leqslant f(Y)/(\alpha \cdot h(Y))$ is $f(x)$. (For proof see [8], [19]).

The (rejection) generation procedure for X, when Y can be generated is the following.

Procedure R3.

1. Generate an uniform random number U over (0, 1) and a random variate Y with the p.d.f. $h(x)$.
2. If $U > f(Y)/(\alpha \cdot h(Y))$ go to step 1. (i.e. reject the pair (U, Y)).
3. Take $X = Y$.*

The probability of accepting the pair (U, Y) is $p_a = \frac{1}{\alpha}$ and the expected number of pairs (U, Y) to be generated until accepting one pair is $N = \alpha$.

Composition procedures. Assume that the p.d.f. $f(x)$ of a random variable X is in the form

$$f(x) = \sum_{i=1}^{k} p_i f_i(x), \quad p_i > 0, \sum_{i=1}^{k} p_i = 1 \tag{4}$$

where $f_i(x)$ are densities corresponding to random variables X_i, $1 \leqslant i \leqslant k$. The p.d.f. f is the discrete mixture (or composition) of densities $\{f_i(x), 1 \leqslant \leqslant i \leqslant k\}$. If X_i, $1 \leqslant i \leqslant k$, can be generated, then the following composition procedure can be used to generate X, [8], [19].

Procedure CD

1. Generate the random index I, where $P(I = i) = p_i$, $1 \leqslant i \leqslant k$.
2. Generate X_I corresponding to the p.d.f. $f_I(x)$.
3. Take $X = X_I \cdot x$ *

Note that when some of p_i are big, and the corresponding X_i have a high speed of generation, then X is generated in an efficient way. If Δ is the set of values of X and

$$\Delta = \bigcup_{i=1}^{k} \Delta_i, \Delta_j \cap \Delta_l = \Phi, \; l \neq j, \; 1 \leqslant l, j \leqslant k$$

is a division of Δ, then denoting

$$p_i = P(X \in \Delta_i), \; f_i(x) = \begin{cases} \frac{1}{p_i} \cdot f(x), & x \in \Delta_i \\ 0, & x \overline{\in} \Delta_i \end{cases} \tag{5}$$

then the p.d.f. $f(x)$ can be represented in the form (4). Therefore, at least theoretically, every random variable X can be generated by the composition procedure CD, when Δ_i are chosen in a convenient way (for instance when $f_i(x)$ are "nearly linear", or "triangular", [11]).

Some distributions can also be represented in the form of a continuous mixture (the continuous analog of (4), see [19], etc.).

2. COMPUTER GENERATION OF RANDOM VARIABLES USED IN RELIABILITY

In the following, some selected algorithmus for generating particular random variables will be presented.

Exponential distribution. If the random variable X has the exponential distribution Exp (λ) (i.e. $f(x) = \lambda e^{-\lambda x}$, $x > 0$) of the parameter λ, then the generation of X can be done by the inverse method as follows.

EXPI

1. Generate U, uniform over (0, 1).
2. Take $X = -\frac{1}{\lambda} \ln U$.*

Another method for generating an exponential Exp (1) random variate derives from Theorem 2. If (using notations of Theorem 2), $F_0(x) = F(x) = x$, $0 \leqslant x \leqslant 1$ and n is the random number of sequences $(U_0, \ldots, U_k)$ rejected until one sequence is accepted, then [1], [2], [13], the random variable $X = n + U_0$ is Exp (1). Therefore, the rejection algorithm is:

EXRJ

0. Initialize RND. Take $n = 0$.
1. Take $i = 0$. Generate U_0 uniform over (0,1). Take $U_a = U_0$.
2. Take $i \leftarrow i + 1$. Generate U_1, uniform over (0,1).
3. If $U_0 \geqslant U_1$, take $U_0 = U_1$ and go to 2.
4. (The sequence is completed). If i is even take $n \leftarrow n + 1$ and go to 1.
5. Take $X = n + U_a$.*

Note that if X is $Exp(\lambda)$ and Y is $Exp(1)$, then $X = \frac{1}{\lambda} Y$. In[1], [13], [20], other algorithms for generating exponential random variates are presented.

Weibull distribution. If X has a Weibull $W(\alpha, \beta, \nu)$ distribution, then $f(x) = \frac{\upsilon}{\beta} (x - \alpha)^{\nu - 1} \exp\left(-\frac{(x-\alpha)^{\nu}}{\beta}\right)$, $\beta, \nu > 0$, and X may be generated by $X = \alpha + \beta(-\ln U)^{1/\upsilon}$or in other terms $X = \alpha + \beta Z^{1/\upsilon}$ where Z is Exp (1).

Note, that Weibull distribution, together with the negative exponential, are the most used in reliability, as failure distributions.

Gamma distribution. The random variable Y has a gamma $G(\alpha, \lambda, \nu)$, $(\lambda, \nu > 0)$ distribution if its p.d.f. is in the form $f(x) = \lambda^{\nu}/(\Gamma(\nu))\ (x - \alpha)^{\nu-1} e^{-\lambda(x-\alpha)}$, where α, λ, ν are known parameters. The gamma distribution is a particular case of the generalized gamma distribution [15], [19]. The generation of Y reduces to generation of X which is $G(0,1, \nu)$ (see [19]), i.e. $Y = \alpha + X/\lambda$.

When ν is integer ($\nu = k$, *i.e.* Erlang distribution), the generation of X can be done by using the known property [3]: if $Z_1, \ldots, Z_k$ are independent $Exp(1)$ random variables, then $X = \sum_{i=1}^{k} Z_i$. The computing time in this case increases linearly with k and therefore this procedure is recommended for small k. Another procedure for generating X when $\nu > 1$ can be obtained by using theorem 4, when $h(x)$ is a Cauchy p.d.f. (see [3]) in the form $h(x) = k/\left(1 + \frac{(x-\nu+1)^2}{c}\right), c = 2\nu - 1$ which gives $\alpha = 1/(k\Gamma(\nu)) \cdot (\nu - 1)^{\nu-1} \cdot e^{-(\nu-1)}$. The algorithm is the following:

GAMCAU

0. Input ν, take $b \leftarrow \nu - 1$, $c \leftarrow \nu + b$, $s = \sqrt{c}$.
1. Generate U and take $T = s \cdot \mathrm{tg}(\pi(U - 0.5))$, $Y = b + T$. (Y has the Cauchy p.d.f. $h(x)$).
2. If $Y < 0$ go to 1. (Negative values of Y are eliminated).
3. Generate U_1. If $U_1 > exp\left(b \cdot \ln\left(\frac{Y}{b}\right) - T + \ln\left(1 + \frac{T^2}{c}\right)\right)$, go to step 1. (Reject).
4. $X = Y$. *

The expected number of pairs to be generated until one pair (U_1, Y) is accepted is [3]

$$N = \sqrt{\pi} \cdot c \cdot e^{-(\nu-1)} \cdot (\nu - 1)^{\nu-1} \cdot \Gamma(\nu). \tag{6}$$

Another procedure for generating $X \rightarrow G(0,1, \nu)$, $\nu > 1$ is based on the stability property of gamma distribution. If $k = [\nu]$ (integer part of ν) and $p = \nu - k$, $0 < p < 1$, then $X = E_k + G_p$ where G_p has a gamma distribution $G(0, 1, p)$, (see the case $0 < \nu < 1$), and E_k is an Erlang random variate.

Some other efficient algorithms for generating * $X \rightarrow G(0, 1, \nu)$, $\nu > 1$ *can be found in* [3], [19], [20].

The case $0 < \nu < 1$, [19]. A first procedure in this case is derived from theorem 4 when $h(x)$ is Weibull $w(0, 1, \nu)$ which gives

$$\alpha = \frac{1}{\Gamma(\nu+1)}\ e^{\zeta(\nu-1)},\ \zeta = \nu^{\frac{1}{1-\nu}}. \tag{7}$$

*) The notation $X \rightarrow$ A means that X follows the probability distribution A.

The algorithm is the following:

GRJNL1

0. Input ν, take $c = \frac{1}{\nu}$, $\zeta = \nu^{\frac{1}{1-\nu}}$, $a = e^{\zeta(\nu-1)}$

1. Generate a Weibull random variate Y. (This is done by:
   1.1. Generate U;
   1.2. Take $Z = \ln U$;
   1.3. Take $Y = Z^c$).
2. Generate $U \cdot$ If $U > a \cdot e^{Z-Y}$ go to 1. (Z is from step 1).
3. Take $X = Y$. *

Another procedure is based on combining a composition procedure with some results of theorems 2 and 3, [19]. Assume that the p.d.f. $f(x)$ of $X \to G(0, 1, \nu)$ is written in the form $f(x) = p_1 f_1(x) + p_2 f_2(x)$, $f_1(x) = 0$, $x \geqslant 1$, $f_2(x) = 0$, $x < 1$; $p_1 = \frac{\Gamma(1;\nu)}{\Gamma(\nu)} p_2 = 1 - p_1$, where $\Gamma(b;\ \nu)$ denotes incomplete gamma function. Therefore, generation of X reduces to generation of X_i, $i = 1, 2$ corresponding to $f_i(x)$. The generation of X_1 can be done by using theorem 2 with $Z_0 = U_0^{1/\nu}$, and $Z_i = U_i$ (uniform over (0,1), $i \geqslant 1$), and the generation of X_2 is done by using theorem 3 with $Q(x) = 1 - x^{\nu-1}$, $x \geqslant 1$, $r(x) = e^{-x+1}$, $x \geqslant 1$. The obtained algorithm is [19].

GAMNL1

0. Input ν, $0 < \nu < 1$, take $a = \frac{1}{\nu}$, $b = -1/(1-\nu)$.

Calculate $\Gamma(1;\nu)$, $\Gamma(\nu)$ and take $p = \Gamma(1, \nu)/\Gamma(\nu)$.
1. Generate U_s with RND .If $U_s > p$ go to 7.
2. Generate U and take $U_0 = U^a$. Set $k = 0$, $U_a = U_0$.
3. Generate U_1 and set $k \leftarrow k + 1$.
4. If $U_0 \geqslant U_1$ take $U_0 = U_1$ and go to 3.
5. If k is even go to step 2.
6. Take $X = U_a$ and go to step 11.
7. Generate U and take $Z = U^b$.
8. Generate U and take $Y = -\ln U + 1$.
9. If $Y > Z$ go to step 7.
10. Take $X = Y$.
11. Deliver X. *

Steps 1—6 generate X_1 and steps 7—10 generate X_2. The algorithm is very fast.

Normal and log-normal distributions. There are various algorithms for generating a normal random variate $X \to N(0, 1)$. (the generation of a $N(\mu, \sigma)$

random variate Y may be obtained as $Y = \mu + \sigma \cdot X$). Here we present a simple algorithm for generating X based on theorem 4, when $Y \to Exp(1)$. This algorithm is [19].

RJNORM

1. Generate U and take $Y = -\ln U$.
2. Generate U_0. If $U_0 < exp(Y - \frac{Y^2}{2} + 0.5)$ go to step 1.
3. Generate U_1. If $U_1 < 0.5$ take $X = -Y$, otherwise take $X = Y$. *

The algorithm generates first a positive normal deviate (steps 1—2) and then assigns a random sign to this (step 3). Other good algorithms for generating $X \to N(0, 1)$ can be found in references [1], [2], [3], [9], [11].

In order to generate a log-normal random variate X which has the expectation μ_x and variance σ_x^2, the following algorithm can be applied [18]:

LNORM

0. Input μ_x, σ_x. Calculate $\sigma_y^2 = \ln[(\sigma_x/\mu_x)^2 + 1]$, $\mu_y = ln\mu_x - \frac{\sigma_y^2}{2}$
1. Generate $Z \to N(0, 1)$ and take $Y = \mu_y + Z \cdot \sigma_y$
2. $X = e^Y$. *

Beta distribution. If X is a beta $B(p, q)$ distribution then [18] it can be generated by the formula $X = Y/(Y + Z)$ where $Y \to G(0, 1, p)$, $Z \to G(0, 1, q)$. Other efficient algorithms for generating X can be found in [3], [10], [20]. In reliability, the non-standard beta random variable V is sometimes used (extended over an interval (a,b)), which is $V = a + (b - a)X$. (see [18]).

Negative binomial distribution. This discrete distribution (denoted $NB(k, p)$) has the frequency function

$$f(x) = \binom{x-k+1}{x} p^k (1-p)^x, \quad x = 0, 1, 2, \ldots, k \in N, 0 < p < 1. \qquad (8)$$

It can be generated either by counting "Bernoulli trials" or by different procedures (see [3], [13], [20]). A Bernoulli trial is generated as follows:

0. Input p, $Z = 0$.
1. Generate U with RND.
2. If $U < p$, take $Z = 1$.
3. Deliver Z. *

Therefore, the event defining the Bernoulli trial occurs (*i.e.* $Z = 1$) with probability p, and does not occur (*i. e.* $Z = 0$) with probability $1 - p$. The negative binomial random variable X is interpreted as the number of Bernoulli trials needed until k times the event produces. The case $k = 1$, reduces to *geometric* distribution.

Hypergeometric distribution. If A, n, N ($A < N$) are parameters of a hypergeometric $H(A, n, N)$ random variable X, the generation of X is done by the following algorithm [20]:

HYRV 1

0. Input N, p, n (where $p = A/N$). Take $i = 0$, $X = 0$.
1. Generate U, take $i \leftarrow i + 1$, set $S = 0$.
2. If $U \leqslant p$ take $S = 1$.
3. Set $X \leftarrow X + S$.
4. Calculate $p \leftarrow \frac{N \cdot p - S}{N-1}$, $N \leftarrow 1$, $N - 1$.
5. If $i < n$ go to step 1.
6. Deliver X. *

The algorithm simulates in fact n random extractions from an urn "without replacement" and counts "successes" (which have the variable probability p).

Poisson distribution. Good algorithms for generating this discrete random variable can be found in [3]. A simple algorithm for generating a Poisson random variable X of the parameter λ is based on the property that if Z_i, $i \geqslant 1$, are independent random variables distributed as $Exp(1)$ then X is defined by the condition

$$\sum_{i=1}^{X} Z_i \leqslant \lambda < \sum_{i=1}^{X+1} Z_i \tag{9}$$

(If $Z_1 > \lambda$), then $X = 0$). This algorithm works only for small values of λ, [20].

Some randon vectors. If Z_i, $1 \leqslant i \leqslant k + 1$ are independent $G(0, 1, \nu_i)$ random variables, then [21] the random vector $(X_1, \ldots, X_k)$, $X_i = Z_i/(Z_1 + \ldots + Z_{k+1})$ has a *Dirichlet* distribution; this property can be used for generating such a vector. The Dirichlet distribution is a multivariate extension of beta distribution.

The generation of a *multinominal* random vector which has parameters p_i, $1 \leqslant i \leqslant k$ and n, $(\sum_i p_i = 1)$ can be performed by the following algorithm:

MULNUM

0. Input p_α, $1 \leqslant \alpha \leqslant k$ and n. Store the array $F[i]$, $1 \leqslant i \leqslant k$ and calculate $F[i] = \sum_{\alpha=1}^{i} p_\alpha$. Initialize the array $X[i] = 0, 1 \leqslant i \leqslant k$.

Take $j = 0$.
1. Generate U, take $m = 0$, set $j \leftarrow j + 1$.
2. Set $m \rightarrow m + 1$.
3. If $U \geqslant F[m]$ go to step 2, otherwise $X[m] \leftarrow X[m] + 1$.
4. If $j < n$ go to step 1, otherwise deliver the array X. *

In reliability, normal random vectors can also be used, or vectors for which some of the components are normal and the other are long-normal (components are correlated). The generation of such vectors is studied in [17]. The random vector X has a multivariate normal distribution $N(\mu, \Sigma)$, $X \in R^k$, if its p.d.f. is

$$f(x) = 1/(2\pi)^{k/2} \cdot 1/(\det \Sigma) \exp\left(-\frac{1}{2}(x-\mu)' \Sigma^{-1}(x-\mu)\right) \qquad (10)$$

where μ is the mean vector and Σ is the covariance matrix of X. If Z is $N(0, I)$, with I the unit matrix of order k x k, then the computer generation of X, [13] can be done by the formula $X = \mu + CZ$ where C is an upper triangular matrix such as $CC' = \Sigma$, $\det \Sigma > 0$.

In order to generate the random vector $X = (Y_1, \ldots, Y_s, X_{s+1}, \ldots, X_k)'$ where Y_i are lognormally distributed, $1 \leqslant i \leqslant s$ and X_j are normally distributed $s+1 \leqslant j \leqslant k$, when the mean vector $(m_1, \ldots, m_s, \mu_{s+1}, \ldots, \mu_k)'$ and the corresponding covariance matrix

$$S = \begin{pmatrix} S_{11}^* & S_{12} \\ S_{21} & \Sigma_{22} \end{pmatrix} = \left(\begin{array}{c|c} s^*_{ij} & s_{ij} \\ \hline s_{ji} & \sigma_{ij} \end{array}\right)$$

are known, the following procedure [17] must be performed.

0. Given $m_i, s_{ij}, 1 \leqslant i \leqslant s, s+1 \leqslant j \leqslant k, s^*_{ij}, 1 \leqslant i, j \leqslant s, \mu_j, s+1 \leqslant j \leqslant k, \sigma_{ij}, s+1 \leqslant i, j \leqslant k$, calculate:

$$\mu_i = \ln m_i - \ln\left[\frac{s^*_{ij}}{m_i^2} + 1\right], \quad 1 \leqslant i \leqslant s$$

$$\sigma_{ij} = \frac{s_{ij}}{m_i}, \quad 1 \leqslant i \leqslant s, \quad s+1 \leqslant j \leqslant k$$

$$\sigma_{ij} = \ln\left[\frac{s^*_{ij}}{m_i m_j} + 1\right], \quad 1 \leqslant j, i \leqslant s.$$

The random vector $X = (X_1, \ldots, X_s, X_{s+1}, \ldots, X_k)'$ with $X_i = \ln Y_i, 1 \leqslant i \leqslant s$ has normal $N(\mu, \Sigma)$ distribution with $\mu = (\mu_1, \ldots, \mu_k)'$, $\Sigma = (\sigma_{ij})$.

1. Generate a random vector $X \to N(\mu, \Sigma)$ with μ and Σ defined at step 0.
2. Take $Y_i = e^{X_i}, 1 \leqslant i \leqslant s$, where $X_i, 1 \leqslant i \leqslant k$ are the components of X.
3. Deliver $X^* = (Y_1, \ldots, Y_s, X_{s+1}, \ldots, X_k)'$.

The random vector X^* arrises in the situation when $Y_i, 1 \leqslant i \leqslant s$ represent failure times and $X_j, s+1 \leqslant j \leqslant k$ are other characteristics defining the quality of an item or of a device, and all these quantities are intercorrelated.

3. FINAL REMARKS

a) The exponential, gamma, beta, Weibull, normal and lognormal distributions in reliability theory are usully failure distributions. Discrete distributions are distributions of the number of failures per unit time (e.g. Poisson, negative binomial) or the distributions of defects per lot (e.g. hypergeometric).

b) In reliability simulation models, the random failures of the components are generated and the reliability characteristics of the system are derived by the simulation program, according to interrelations between components. Discrete distributions can also be used in order to simulate external random events which influence the system's reliability.

c) The multinomial distribution can be used to simulate frequences corresponding to various states of a component (non dichotomuous-component).

d) Composition procedures are used sometimes to simulate failures which might have different distributions, each distribution occuring with a given probability. (For instance, the running time of a device has a given distribution when it is under charged, another distribution when it is fully charged, etc., and these situations have different probabilities to occur).

Finally, we mention the fact that [20] contains generation FORTRAN subroutines for all classical distributions, for discrete Markov Chains and stationary gaussian processes. These subroutines have successfully passed statistical tests.

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ENGENDREMENT A L'AIDE DE L'ORDINATEUR DES VARIABLES ET DES VECTEURES ALEATOIRES UTILISES DANS LA THEORIE DE LA SURETE

Le but de cet ouvrage est celui de présenter brièvement quelques algorithmes, chacun étant utilisé pour engendrer une valeur de sélection d'une variable aléatoire donnée se referrant tout spécialement aux variables aléatoires utilisées dans la théorie de la sûreté. En particulier, on envisage les répartitions: exponentielle négative, Weibull, gamma normale et lognormale, beta, géométrique, binomiale à exponent négatif, hypergéométrique et Poisson 5.

De même, on considère la possibilité d'engendrer à l'aide de l'ordinateur des vecteurs aléatoires normaux, multinomiaux et Dirichlet. Les paramètres des répartitions considérées sont supposés connus. Dans l'ouvrage on suppose donné un générateur de nombres aléatoires (nommé RND). (Par exemple dans [19], [20] RND a la forme $N_i + 1 = 16807.\ N_i \pmod{2^{31} - 1}$), $0 < N_i < M = 2^{31} - 1$, où N_i sont entiers. Les nombres pseudo-aléatoires, (uniformes sur (0, 1) sont $U_i + 1 = (N_i + 1)/M$).

ПРОИЗВОДИМОСТЬ ВЫЧИСЛИТЕЛЕМ АЛЕАТОРИЧЕСКИХ ПРОИЗВОЛЬНЫХ И АЛЕАТОРИЧЕСКИХ ВЕКТОРОВ, ИСПОЛЬЗУЕМЫХ В ТЕОРИИ СОХРАННОСТИ

Цель этой работы представить вкратце несколько алгорифмов каждый из них будучи использован для создания стоимости селекции данной алеаторической произвольной со специальным адресом к тем алеаторическим произвольным, используемым в теории сохранности. В частности учитываются распределения: негативный экспоненциал, Вейбулл, гамма, нормальную и логонормальную бету, геометрическую, двухноминальную с негативным экспонентом, гипергеометрическую и Пойссон 5.

Освещается и производимость вычислителем нормальных алеаторических, многономинальных векторов и Диришле. Параметры распределений предполагается, что они известны. В работе предполагается генератор с алеаторическими числами /названный *RND*/. (Например 19, 20, *RND* формы $N_{i+1} = 16807.\ N_i \pmod{2^{31} - 1}$), $0 < N_i < M = 2^{31} - 1$, где N_i являются целыми. Псевдоалеаторическими, однородными номерами на (0,1) являются $U_i + 1 = (H_i + 1)/M$).