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Management Model for a Production Section

The present work is the result of long preoccupations for the optimized use of resources existing in the productive systems, beginning with the work place and ending with the enterprise as a whole.

In the first part of the work, an analysis is made of the production section system with discrete technological processes made up of stages whose lengths of time are random. Further, the model is built for the management of the section.

The solution of the model is meant to establish a class of probable evolutions minimizing the queueing length of installations by taking into account the conditions imposed by the production plan and by the corresponding modelling level, thus optimizing the utilization of the productive capacities of the section.

1. PRESENTATION AND ANALYSIS OF THE PRODUCTION SECTION SYSTEM

The production section is considered as a system with technological processes which are working the raw materials, finished and semi-finished products being obtained in the end. The resources existing in the system are being grouped into two types. The resources which can be stocked for a certain period of time, their consumption taking place in quanta at different moments will be called resources of $\wedge$ type. The unstorable resources, in a permanent movement, with an instantaneous consumption, which are lost if not consumed, will be called resources of Ω type.

The section is a system with a double structure; on the one hand a material structure (equipment, stocks of raw and related materials, technology, energy, utilities, buildings, etc.), and on the other hand, an informational structure (data about the fulfilment of the plan targets, on supply, informational flows, etc.).

In the section, the high skilled manpower is playing a part in making certain operations, in supervising and maintaing the good functioning of the

system, in managing certain processes by guiding the values for certain characteristics.

The production section system may be decomposed according to some criteria (for example, the homogeneity of operations, the classification of installations, the phases in the making of a product, etc.) into a number of interdependent subsystems.

In each subsystem, there are elements performing the operations which make up each process (for example, lathes or autoclaves). Corresponding to an operation is at least a transition from a state to another of the element. Considering that these elements undergo a sufficiently big number of states for carrying out the operations, they will be further called active elements of the subsystems. They appear as complex finite automata. The other elements (for example, pipes, conveyor belts) which have at the most two states will be called passive elements.

The transition from one state to another of the active elements is not accidental, since there is a well established succession (for example, a container can be unloaded only after it was loaded). This succession forms a chain of states. An active element has at least one chain of states with a repetitive character; the complexity of the active element is given by the multitude of chains of states which may appear.

A chain of states — a cycle — allows for the making of the finished product in charges or batches, thus rendering the discrete character of the production process (for example, an active element passes from the initial state S_0 to state S_1, from S_1 to S_2 and afterwards from S_2 to S_0). The transition from a state to another is characterized by length, by consumptions of materials, of energy and by the obtaining of products and semifabs.

The active elements may be schematically represented by blocks.

To describe the active element it is necessary to specify the way in which its inputs and outputs are taking place. The relations between inputs and between outputs respectively can be of simultaneity (of "AND" type) or of alternation (of "OR" type).

There is also a relationship between the inputs and outputs taken as distinct entities, a relationship which renders a continuous (I "AND") or discontinuous (I "OR") character to the operations made (Fig. 1).

Inputs Outputs	Alternative	Simultaneous
Alternative	$I_{or} - E_{or}$	$I_{and} - E_{or}$
Simultaneous	$I_{or} - E_{and}$	$I_{and} - E_{and}$

Fig. 1

To describe the mechanism of the production processes within the subsystems, fictitious blocks may be introduced into the structure schemes associated with the subsystems (Fig. 2).

The digits existing on the oriented arces show the order in which the input and output respectively takes place in an active element.

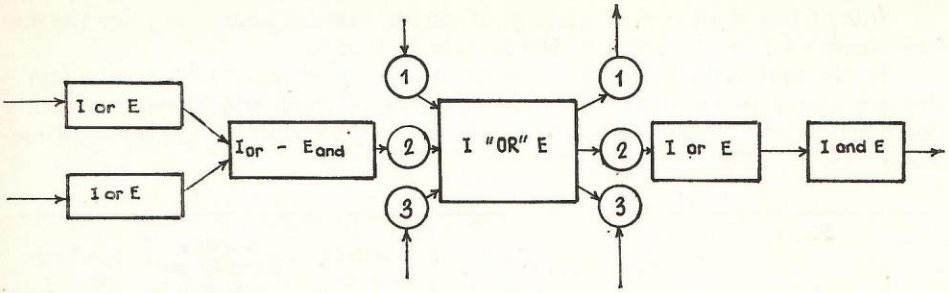

Fig. 2

Corresponding to each active element is a precedence matrix containing the states the element may cover. An element A_{ij} of the precedence matrix is equal to 1 if state i may be followed by state j and 0, in the opposite case.

In establishing the block scheme and the precedence matrices, the insignificant or constant elements have been eliminated. In the system, the active elements or groups of active elements are interconnected and the differences between the lengths of phases or their rate differences create queueing lines. The queueing lines influence the dynamics of the evolution of the stocks of resources of type $\wedge$ and the stocks of finished products.

The section has a group of active elements which play an essential part in the production process, in the fulfilment of the plan targets. This consideration determines only its inclusion into the model.

2. MODEL OF MANAGEMENT OF THE SECTION

The model of management of a section is a model with a time period at a day level (for example 1—5 days) concerning the evolution of a single group of active elements. It is subordinate to a global model of the section distributing the plan target for each assortment for a longer period (for example, decade, month) on groups of active elements. The global model is a deterministic static model meant to outline the best use of the section's resources. From the solution of the global model, certain elements represent input data for the management model.

The first variant of the model takes into consideration the production of a single assortment.

The established objective of the management model is the minimizing of the queueing lengths of the active elements in the section in the considered time interval $[0, \tau]$, where the time interval was denoted by τ. The implementation of this objective results in the improved internal rate of the system, having a positive impact on the fulfilment and overfulfilment of the plan targets and on the outlining of the materials and time reserves.

The main variables considered by the model are denoted X_k^h and represent the number of active elements which are found at moment k in phase h, $k = 0, 1, ..., \tau$ and $h = 0, 1, ..., H$, where H is the number of phases making up a chain of states for an active element; h is the class of rests modulo H, thus ensuring after phase H the active element to enter phase 0.

Out of the H phases of a chain of states, certain phases require the use of resources of type $\wedge$ given in the matrix in Fig. 3.

In the matrix in Fig. 3, $Q_{\xi h}$ represents the specific consumption in phase h from resource R_ξ on an active element, a consumption which may uniformly take place in the phase. In most cases, the matrix contains many null elements.

Phase / Resource	$0 \ldots h \ldots H$	Available resource	Loading quantity per time unit	Stocking capacity
R_1 $\vdots$ R_ξ $\vdots$ R_w	$Q_{\xi h}$	A_ξ	q_ξ	r_ξ

Fig. 3

The model also considers the use of a resource of type Ω, B_k representing its final flow at moment k. The resource is required in phases h_i, $i = 1, 2, \ldots, \Delta$, in quantities $\widetilde{Q}_i$ per time unit.

Besides the specific consumptions $Q_{\xi h}$ the followings are also considered as input data in the model:

N — the total number of active elements,

α^{inf} — the lower limit of the number of charges which should be produced in the time interval $[0, \tau]$,

α^{sup} — the higher limit of the number of charges which should be produced in the same time interval.

Worth mentioning are the variables $t_{l\nu}^h$ representing the final moment of phase h for the active element l in carrying out cycle ν, $l = 1, 2, \ldots, N$ and $t_{l\nu}^h \in [0, \tau]$.

These variables are closely linked with variables X_k and with the precedence matrices; the succession of the appearance of phases and, thus of moments $t_{l\nu}^h$, is given by the precedence matrix.

In interval $[0, \tau]$ the active element l may cover the chain of states multiplied by a certain number of I_e, where $I_e < I_\tau$ and I_τ represent the maximum possible number of chains covered by a certain active element in the considered time interval.

The relation between variables $t_{l\nu}^h$ and X_k^h is given by equation (1) which appears as a double recurrent equation: a noncyclic recurrence of moments and a cyclic recurrence of phases:

$$X_{k+1}^h = X_k^h + \sum_{l=1}^{N} \delta(t_{l\nu}^{h-1}; k) - \gamma_k^{h+1}$$
$$k = 0, 1, \ldots, \tau - 1; \quad h = 0, 1, \ldots, H; \quad \nu < I_\tau \tag{1}$$

where:

$$\gamma_k^{h+1} = \begin{cases} \sum_{l=1}^{N} \delta(t_{lv}^0;\ k) & \text{if } h = 0 \text{ in condition Y} \\ 0 & \text{if } h = 0 \text{ and noncondition Y} \\ \sum_{l=1}^{N} \delta(t_{lv}^h;\ k) & \text{if } h \neq 0 \end{cases} \tag{2}$$

and where condition Y is:

$$\text{for } (\forall)\ k \in [t_{lv}^{h_i - 1} + 1,\ t_{lv}^{h_i}],\ (\exists)\ B_k \geqslant \widetilde{Q}_i \sum_{l=1}^{N} \delta(t_{lv}^0;\ k) \tag{3}$$

$$(\forall)\ i = 1, 2, ..., \Delta;\ (\forall)\ l = 1, 2, ..., N.$$

In relations (1), (2), (3) Kronecker's symbol was used:

$$\delta(u, v) = \begin{cases} 1 & , \text{ if } u = v \\ 0 & , \text{ if } u \neq v. \end{cases} \tag{4}$$

The constraints regarding the fulfilment of the plan targets are:

$$\alpha^{inf} \leqslant \sum_{k=0}^{\tau} X_k^H \leqslant \alpha^{sup}. \tag{5}$$

The constraint of comprising all active elements into the production process, for the analysed period, at a given moment, is:

$$\sum_{h=0}^{H} X_k^h = N, \qquad k = 0, 1, ..., \tau. \tag{6}$$

The constraints regarding resources of type Λ are:

$$\sum_{k=0}^{\tau} \sum_{h=0}^{H} Q_{\xi h}\ X_k^h \leqslant A_\xi, \qquad \xi = 1, 2, ..., W. \tag{7}$$

The constraints concerning resource of type Ω, are:

$$\sum_{i=1}^{\Delta} \widetilde{Q}_i X_k^h \leqslant \overline{B} \qquad , k = 0, 1, ..., \tau \tag{8}$$

$$B_{k+1} = B_k + \bar{B} - \sum_{i=1}^{\Delta} \widetilde{Q}_i \sum_{l=1}^{N} \theta(k; t_{lv}^{h_i - 1} + 1,\ t_{lv}^{h_i}) \tag{9}$$

$$k = 0, 1, ..., \tau - 1.$$

where $\bar{B}$ represents the input flow of the resource and θ is defined as follows:

$$\theta(x;\ u, v) = \begin{cases} 1 & ,\text{if } x \in [u, v] \\ 0 & ,\text{if } x \notin [u, v]. \end{cases} \tag{10}$$

The objective function of the model which must be minimized is:

$$\sum_{k=0}^{\tau} X_k^0, \tag{11}$$

representing the total queueing length corresponding to state S_0 of the active elements in the analysed period.

Obviously, the conditions of membership of variables X_k^h to the set of natural numbers are supposed.

Variables $t_{l\nu}^h \in [0, \tau]$, $l = 1, 2, ..., N$; $h = 0, 1, ..., H$ are essential variables for the model, through which the guidance of the processes in the system takes place.

Practice has imposed the building of the model by assuming that $t_{l\nu}^h$ are random variables.

If we denote by $d_{l\nu}^h$ the length of phase h for the chain of state ν for the active element l, also a random variable, we have relation:

$$d_{l\nu}^h = t_{l\nu}^h - t_{l\nu}^{h-1}. \tag{12}$$

The length of the chain of states ν for the active element l is calculated as follows:

$$d_{l\nu} = t_{l\nu}^H - t_{l\nu}^0 + 1. \tag{13}$$

Between indices h and ν of variables $t_{l\nu}^h$ there is a link given by relation:

$$t_{l\nu}^H + 1 = t_{l,\nu+1}^0. \tag{14}$$

Appearing for an active element l is a number of random variables, made up of moments $t_{l\nu}^h$, $h = 0, 1, ..., H$; $l = 1, 2, ..., N$, between the components of which there is a Markovian form dependence.

Also appearing is a dependence between the lines associated with the active elements, imposed, in the first place, by the allocation of resource of type Ω.

The statistical analysis for a previous period of variables $d_{l\nu}^h$ has enabled the knowing of certain characteristics for variables $t_{l\nu}^h$, characteristics reflected in the obtaining of the solution.

The model presented so far contains the hypothesis for the production of a single assortment and the hypothesis for the evolution of the stocks of resources of type Λ, an evolution given by the global model.

The model which will be further analysed eliminates these two hypotheses, ensuring a more faithful reflection of the realities in the section.

We shall consider that in the section, M assortments can be produced on the N active elements providing that the structure of the chain of states remains the same for all assortments, the phases having a different length from an assortment to another.

The storing of the raw materials of type Λ is achieved in containers of a given capacity; it is required that the loading and unloading of the storing containers should not cause supplementary queueings of the active elements.

Further, we shall denote by:

$S_{\xi k}$ — the stock of resource ξ at moment k;

X_{kj}^h — the number of active elements which at moment k are in phase h, for the production of assortment j;

t_{lvj}^h — the final moment of phase h, for the active element l, in achieving the chain of state ν, for the production of assortment j;

α_j^{sup}, α_j^{inf} — the number of higher limit charges, lower limit respectively, which should be produced in interval $[0, \tau]$ of assortment j;

Under these conditions, the structure of the model is the following:

$$[min] \sum_{k=0}^{\tau} \sum_{j=1}^{M} X_{kj}^0 \tag{15}$$

$$X_{k+1,j}^h = X_{kj}^h + \sum_{l=1}^{N} \delta(t_{lvj}^{h-1}; k) - \gamma_{kj}^{h+1} \tag{16}$$

$$h = 0, 1, ..., H; \quad j = 1, 2, .., M; \quad k = 0, 1, ..., \tau - 1; \quad \nu < I_\tau$$

where:

$$\gamma_{kj}^{h+1} = \begin{cases} \sum_{l=1}^{N} \delta(t_{lvj}^0; k) & \text{,if } h = 0 \text{ in condition } Y' \\ 0 & \text{,if } h = 0 \text{ and noncondition } Y' \\ \sum_{l=1}^{N} \delta(t_{lvj}^h; k) & \text{,if } h \neq 0 \end{cases} \tag{17}$$

and where Y' is given by relations (18), (19):

$$\text{for} \quad (\forall)\ k \in [t_{lvj}^{h_i - 1} + 1, t_{lvj}^{h_i}], (\exists)\ B_k \geqslant \widetilde{Q}_{ij} \sum_{l=1}^{N} \delta(t_{lvj}^0; k) \tag{18}$$

$$(\forall)\ i = 1, 2, ..., \Delta; \quad (\forall)\ l = 1, 2, ..., N;$$

$$(\forall)\ j = 1, 2, ..., M.$$

$$S_{\xi k} \geqslant Q_{\xi hj} \sum_{l=1}^{N} \delta(t_{lvj}^0; k), \quad (\forall)\ \xi = 1, 2, ..., W. \tag{19}$$

$$S_{\xi, k+1} = S_{\xi k} - Q_{\xi hj} \sum_{l=1}^{N} \delta(t_{lvj}^0; k) + \omega_\xi \tag{20}$$

$$\xi = 1, 2, ..., W; \quad k = 0, 1, ..., \tau - 1,$$

where:

$$\omega_\xi = \begin{cases} q_\xi & \text{,if } S_{\xi k} \leqslant r_\xi \\ 0 & \text{,otherwise.} \end{cases} \tag{21}$$

$$\alpha_j^{inf} \leqslant \sum_{k=0}^{\tau} X_{kj}^H \leqslant \alpha_j^{sup}, \quad j = 1, 2, ..., M. \tag{22}$$

$$\sum_{h=0}^{H}\sum_{j=1}^{M} X_{kj}^{h} = N,\ k = 0, 1, ..., \tau. \tag{23}$$

$$\sum_{j=1}^{M}\sum_{k=0}^{\tau}\sum_{h=0}^{H} Q_{\xi hj}\, X_{kj}^{h} \leqslant A_{\xi}, \qquad \xi = 1, 2, ..., W. \tag{24}$$

$$\sum_{j=1}^{M}\sum_{i=1}^{\Delta} \widetilde{Q}_{ij}\, X_{kj}^{hi} \leqslant \bar{B}, \qquad k = 0, 1, ..., \tau. \tag{25}$$

$$B_{k+1} = B_k + \bar{B} - \sum_{j=1}^{M}\sum_{i=1}^{\Delta} \widetilde{Q}_{ij} \sum_{l=1}^{N} \theta(k;\ t_{lvj}^{hi-1} + 1,\ t_{lv}^{hi}) \tag{26}$$

$$k = 0, 1, ..., \tau - 1.$$

X_{kj}^{h} being natural numbers.

3. SOLUTION OF THE MODEL

We shall call the solution of the analysed management model Production Operational Program, effective at the modelling level existing at present. The solution of the model is obtained, as a rule, by covering the stages resulting from the processing scheme in Fig. 4. Considering the random character of the lengths for some phases, the solution is obtained by successive simulations. The testing of the existence of resources and their allocation is made in the order imposed by the type of active elements the interaction of which is presented in schemes of the type of those in Fig. 2.

The production operational programme gives possible trajectories for the unfolding of the production processes in the section.

By guiding the generations of the lengths, trajectories are obtained which give information about the evolution of the production processes under the conditions given. Thus, optimistic, medium and pessimistic variants can be obtained by establishing certain variation intervals for the parameters of the length of the phases. The guidance of simulations is based on the observations of the decision-making factor in the section, the solution providing him a comprehensive image of the evolution of the section.

The operational program can be obtained by knowing the state of the system at a given moment, considered as the initial moment $k = 0$. Constraints regarding the human factor can also be associated and the program can be updated by introducing the changes concerning the moments of starting the phases in relation to a given reference moment. All updatings aim at following the implementation of the plan targets.

The model implies a rythmic exchange of information between the production section and the computation system on which the solution is obtained, based on methods which will be presented in a future study.

In conclusion, the Operational Program, closely linked with the solution of the global model provides information for the management of the processes in the section for the fulfilment of the plan targets.

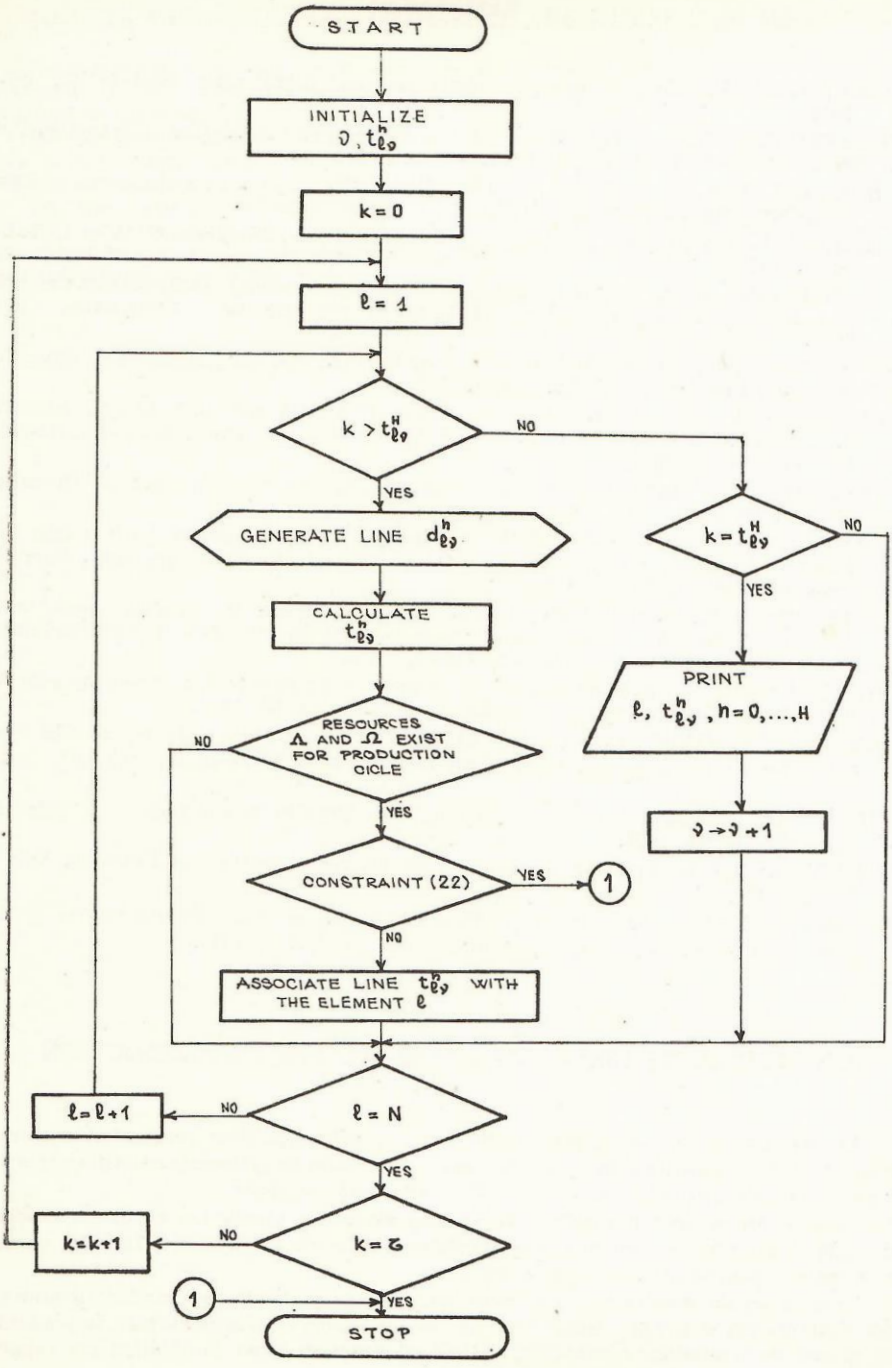
START
INITIALIZE
ϑ, $t^h_{\ell\vartheta}$
k = 0
ℓ = 1
$k > t^H_{\ell\vartheta}$
NO
YES
GENERATE LINE $d^h_{\ell\vartheta}$
CALCULATE
$t^h_{\ell\vartheta}$
$k = t^H_{\ell\vartheta}$
NO
YES
PRINT
ℓ, $t^h_{\ell\vartheta}$, h = 0,...,H
$\vartheta \to \vartheta + 1$
RESOURCES
Δ AND Ω EXIST
FOR PRODUCTION
CICLE
NO
YES
CONSTRAINT (22)
YES
1
NO
ASSOCIATE LINE $t^h_{\ell\vartheta}$ WITH
THE ELEMENT ℓ
$\ell = N$
NO
$\ell = \ell + 1$
YES
k = σ
NO
k = k+1
1
YES
STOP

Fig. 4

REFERENCES

1. AGAPOVA L. V.: *Zadacea o raspredelenii resursa.* Tehnicescaia Kibernetica, No. 4, 1976.
2. BESTWICK P. F., HASTINGS N. A. J.: *A New Bound for Machine Schedulling.* Operational Research Quarterly, Vol. 27, No. 2, 1976.
3. BOGOMOLOV A. M., TVERDOHLEBOV V. A.: *Tzelenapravlennoe povedenie avtomatov.* Izdatelstvo Nauka Dumka, Kiev, 1975.
4. BRANDWAJN A.: *Control Schemes in Queueing Networks.* Management Science, Vol. 22, no. 7, 1976.
5. CIHIKVADZE G. M.: *Imitatzionaia modeli functzionirovania snabjenceskosbitovoi bazi.* Matematiceskaia i Tehniceskaia Kibernetica, Izdaltestvo Metzniereba, Tbilisi, 1975.
6. CROOKES J. G., SPINELLI DE CARVALHO R.: *Cellular Simulation.* Operational Research Quarterly, Vol. 27, No. 1, 1976.
7. EDWARDS W. J., CARLTON A. J., HARVEY G. F., EVANS R. F. K., MC KERROW P. J.: *Coating Mass Control System Design for Continuous Galvanizing Line.* Automatica, Vol. 12, No. 3, 1976.
8. FISHMAN G. S.: *Statistical Analysis of Multiserver Queueing Simulations.* Operational Research Quarterly, Vol. 27, No. 4, 1976.
9. GEOFFRION A. M., GRAVES G. W.: *Schedulling Parallel Production Lines with Changeover Cost. Practical Application of a Quadratic Assignment LP Approach.* Operations Research, Vol. 24, No. 4, 1976.
10. GOLENKO D. I., DIMOV E. M., OSOKIN V. V.: *Voprosi imitatzionogo modelirovania discretno-neprerivnih proizvodstvennih protzessov.* Matematiceskaia i Tehniceskaia Kibernetica, Izdaltestvo Metzniereba, Tbilisi, 1975.
11. HENDERSON B. P., STEIGLITZ K.: *A Problem in Single-machine Sequencing with Nonlinear Delay Costs.* Management Science, Vol. 22, No. 12, 1976.
12. MASANORI KODAMA: *Probabilistic Analysis of a Multicomponent Series, Parallel System under Preemptive Repeat Repear Discipline.* Operations Research, Vol. 24, No. 3, 1976.
13. MIHAIL R.: *Modelarea reactoarelor chimice* (Modelling Chemical Reactors). Editura tehnică, Bucharest, 1976.
14. RAO M. R.: *Optimal Capacity Expansion with Inventory.* Operations Research, Vol. 24, No. 2, 1976.
15. TABAK D., ROY T. S., CARON W. J.: *Identification of Unknown Parameters in Cash Flow Systems of Insurance.* Automatica, Vol. 12, No. 1, 1976.

UN MODELE DE DIRECTION D'UNE SECTION DE PRODUCTION

Le présent ouvrage est le résultat de certaines préoccupations constantes concernant l'optimisation de l'utilisation des ressources, qui existent dans les sytèmes productifs en commençant par le lieu de travail et finissant par l'ensemble de l'entreprise.

La première partie fait l'analyse du système-section de production ayant des processus technologiques discrets, comprenant des phases dont la durée est aléatoire. Ensuite, on construit le modèle pour la direction de la section.

La solution du modèle fixe une classe d'évolutions probables qui rendent minimes les durées d'attente des outillages, tenant compte des conditions exigées par le plan de production et du niveau de simulation convenable, optimisant de cette façon l'utilisation des capacités de production de la section.

МОДЕЛЬ УПРАВЛЕНИЯ ПРОИЗВОДСТВЕННЫМ ЦЕХОМ

Настоящая работа является результатом исследования оптимизации использования наличных ресурсов в производственных системах, начиная с рабочего места работы и кончая предприятием в целом.

В первой части работы проводится анализ системы призводственного цеха с дискретными технологическими процессами, состоящими из фаз случайной продолжительности. Далее даётся модель для управления цехом.

Решение модели состоит в определении вероятной эволюции, которая сводит до минимума срок ожидания оборудования, учитывая условия производственного плана и соответствующий уровень моделирования, оптимизируя таким образом использование производственных мощностей цеха.

1.II.1977