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An Application of the Markov Chains and Simulation to the Study of Some Petrochemical Processes*)

The authors describe the possibility of applying the Markov chains theory and simulation to the study of petrochemical processes. By associating a finite Markov chain to a petrochemical process we can determine the quantities of raw materials and semifinished products required for the accomplishment of a given production plan.

The described problem is dwelt upon also through the numerical simulation method, which proves to be advantageous as the number of processing units increases.

1. INTRODUCTION

A number of practical problems can be considered in their all-round spread only through probabilistic methods. The complex scope of such problems is due to a huge volume of information, to the random character of variables and to the variation in time of the parameters characterizing the respective processes.

In this paper we are going to refer to the application of Markov chains to the analysis of production industrial processes of a continuous character. The described application concerns some processing units in the petrochemical industry. The earliest ideas as to determining resources for industrial processing by making use of Markov chains are contained in K. V. Richardson's work [1]. Here, we shall give some notions of the finite Markov chain theory maintaining the notation used in the work [2].

*) A paper submitted to the International Colloquium on the Constructive Theory of Functions, held in Cluj, Romania, from 6th to 12th September, 1973.

Consider a Markov chain having r states: $S_1, S_2, ..., S_r$. Its states can be divided into equivalence classes by introducing a partial order relationship: $S_i \frown\!\!\!\rightarrow S_j$ (state S_i is in connection with state S_j) if the process can pass from state S_i into state S_j. The maximal elements corresponding to the partial order relationship make up ergodic classes. The remaining classes make up the transition classes.

It is proper to transition classes that, as soon as they have been abandoned, the process does not go back to its states, whereas with ergodic classes the process no longer parts with the respective class as soon as its states have been attained.

Markov chains can be classified as follows:

1. Markov chains without transition classes;
2. Markov chains with transition classes.

If all ergodic classes are made up by one element, the chain is *absorbent.*

The transition matrix for an absorbent Markov chain having s transition states and $r - s$ absorbent states, to be noted first, is written under the form:

$$P = \begin{Vmatrix} I & 0 \\ R & Q \end{Vmatrix} \tag{1}$$

where I is a unit matrix of size $(r - s) \times (r - s)$; R is a matrix $s \times (r - s)$ whose elements are the probabilities of passing from a transition state to an absorbent one; Q is a matrix $s \times s$ whose elements are the probabilities of passing between the transition states; 0 is a matrix $(r - s) \times s$ with all the elements equal to zero. The fundamental matrix for an absorbent Markov chain is defined by $N = (I - Q)^{-1}$.

If we define n_j as the total number of times that the process is in a transition state, the elements of N have the following interpretation:

Theorem. $N = \{M_i[n_j]\}$ for any transition states S_i and S_j [2].

In other words, the element n_{ij} of matrix N yields the mean total number of times that the process is in transition state S_j on the assumption that the process started from state S_i.

For an initial probability vector π we have $\pi' N = \{M_\pi[n_j]$ where π' is made up of the former s components of π, corresponding to the transition states.

Theorem. If b_{ij} is the probability for the process to come into an absorbent state S_j starting from a transition state S_i, then $\{b_{ij}\} = B = NR$ [2].

2. A MARKOV CHAIN ASSOCIATED TO PRODUCTION DEVELOPMENT IN A PETROCHEMICAL PROCESS.

In industrial processes and particularly in petrochemical processes raw materials and semifinished products are worked out several times by passing through various processing units. Materials worked out in a unit I_i are directed to the next unit I_j in keeping with a determined technological succession or may be directed towards the delivery spots if processing is over.

Consider processing unit I_i (Fig. 1). receiving semifinished products from units $I_{\alpha_1}, I_{\alpha_2}, ..., I_{\alpha_p}$.

After processing, several semifinished products shall be directed to the units or the delivery spots $I_{\beta_1}, I_{\beta_2}, ... I_{\beta_q}$.

We shall note Q_{ij} the quantity of semifinished product to be directed from unit I_i to unit I_j $(j \in \{\beta_1, \beta_2, ..., \beta_q\})$.

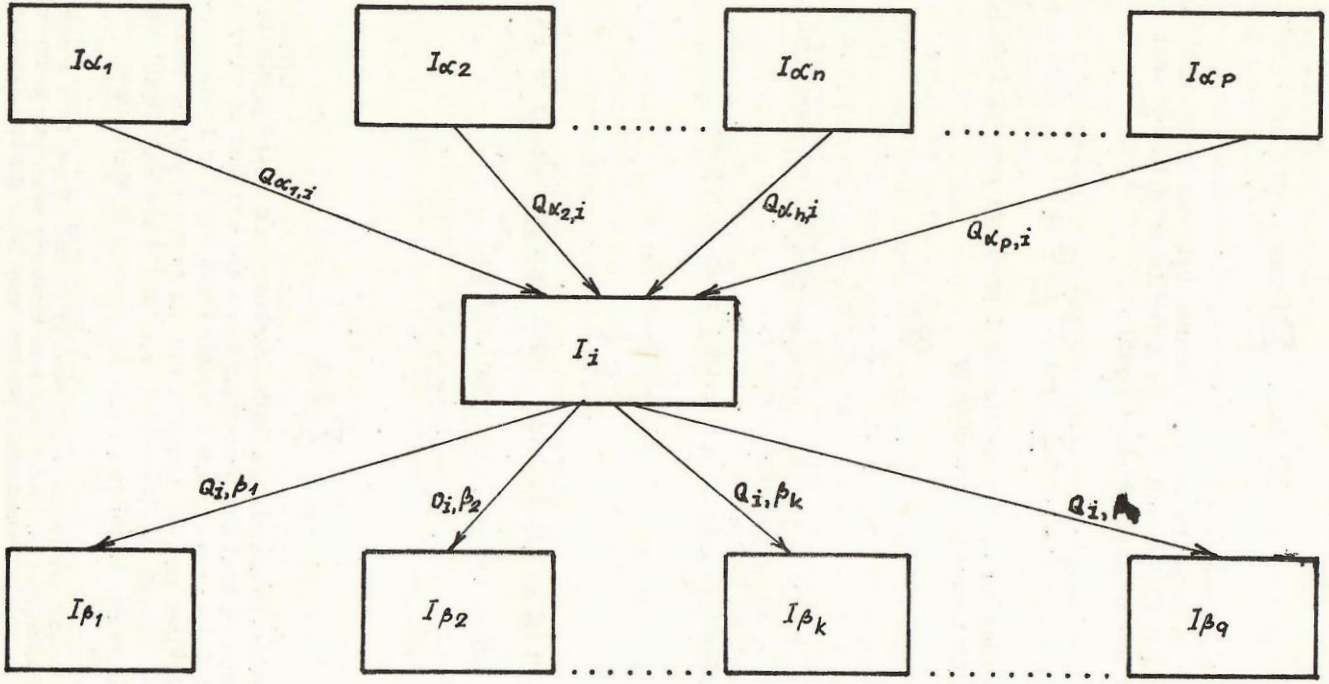
I_{α_1}
I_{α_2}
I_{α_n}
I_{α_p}
$Q_{\alpha_1,i}$
$Q_{\alpha_2,i}$
$Q_{\alpha_n,i}$
$Q_{\alpha_p,i}$
I_i
Q_{i,β_1}
Q_{i,β_2}
Q_{i,β_k}
I_{β_1}
I_{β_2}
I_{β_k}
I_{β_q}

Fig. 1

In general

$$\sum_{k=1}^{p} Q_{\alpha_k, i} > \sum_{k=1}^{q} Q_{i, \beta_k} \tag{2}$$

due to losses by processing.

However, in order to secure a balance between inputs and outputs we shall place in a fictitious unit $I_{\beta_{q+1}}$ or a fictitious delivery spot to receive a quantity of semifinished product equal to:

$$Q_{i, \beta_{q+1}} = \sum_{k=1}^{p} Q_{\alpha_k, i} - \sum_{k=1}^{q} Q_{i, \beta_k}. \text{ *)} \tag{3}$$

The distribution of the semifinished products coming from I_i can be characterized by means of relationships

$$p_{i, \beta_j} = \frac{Q_{i, \beta_j}}{\sum_{k=1}^{q} Q_{i, \beta_k}}. \tag{4}$$

In other words, $100 \times p_{i, \beta_j}$ is the percentage of a unit of semifinished product worked out in I_i and directed towards I_{β_j}.

In general, such reports, vary in time.

If we assume p_{i, β_j} to be a selection mean of reports

$$\frac{Q_{i, \beta_j}}{\sum_{k=1}^{q} Q_{i, \beta_k}},$$

then this mean (p_{i, β_j}) can be looked upon as a probability of passing from I_i to I_{β_j}.

Indeed, these values verify relationships

$$0 \leqslant p_{i, \beta_j} \leqslant 1$$

and

$$\sum_{k=1}^{q} p_{i, \beta_k} = 1. \tag{5}$$

This enables to associate a finite Markov chain to the production process.

It is assumed that the processing units are watched at various moments 1, 2, ..., T. If T does not surpass a certain limit, say 1 or 2 years, then the transition probabilities can be thought out as being constant (independent of time), because specific consumption does not have significant variations in such time intervals. It follows that the process is stationary.

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*) In order that sum totals in relationships (2) and (3) can have a meaning it is necessary that all quantities Q_{ij} should have been expressed previously in the same measure units. This, because certain semifinished products may be in gas state (and, consequently, expressed in volume units), or in liquid state (consequently expressed in weight units), or may be solid.

In order to state that the process is a Markov chain we must prove that the probability for the process to be in a certain state only depends on the preceding state. If we consider a particle going successively through various processing units, then the probability of its passing from I_i into I_j depends upon having been in unit I_i or not.

States of the chain are the processing units (real or fictitious) and the delivery spots.

If I_i is a delivery spot, that means that the products are to leave the production process and I_i will consequently be an absorbent state of the Markov chain.

If I_j is a fictitious delivery spot (irrecoverable losses or recoverable for another processing unit which does not pertain to the system), our previous statement is still true.

Fictitious units $I_{\beta_{q+1}}$ can be assumed to be absorbent states if losses are not recoverable or their recovery is carried out in order to be directed to an unit not pertaining to the system.

The processing units shall be the transition states of the Markov chain.

The petrochemical process can be characterized by means of a matrix of passings from processing unit to another of the form (1) where Q will be a matrix of transition probabilities between the processing units, R will be the matrix of probability of passing from processing unit to the delivery spots and I will be the unit matrix expressing the fact that a delivered product never comes back to the system.

The matrix of the total processing flows is yielded by $N = (I - Q)^{-1} = = ((n_{ij}))$.

In other words, n_{ij} is the total quantity of semifinished product reaching spot I_j generated by a unity of material at spot I_i.

Likewise, the matrix elements

$B = NR = (I - Q)^{-1}R = ((b_{ij}))$ are the total quantity to be delivered. That is to say b_{ij} is the quantity to be sold at the delivery spot I_j of semifinished products coming from unit I_i.

Consider X to be a vector whose elements are the input quantities in units E_i. This is the initial probability vector for the associated Markov chain.

By writing Y for the sales (the quantities reaching the selling spots), we get relationship:

$$Y = X(I - Q)^{-1}R. \tag{6}$$

Hence, we can say that, given the input quantities in the unit, we can infer the quantities that are to reach the selling spots.

In a reciprocal way, it follows from relationship (6) that:

$$X = YR^{-1}(I - Q) \tag{7}$$

which means we have to determine the input quantities corresponding to a given delivery plan.

In relationship (7) matrix R in most cases is not square and, consequently, the reverse R^{-1} is meaningless. On this account, in practical applications it is assumed that an infinitesimal share ε of the semifinished product coming from each unit which does not precede a delivery spot will reach a fictitious delivery spot.

In this way, matrix R shall become square with elements different from zero only on the diagonal and only if its reverse is trivial.

3. AN EXAMPLE OF A PETROCHEMICAL PROCESS MODELLED BY MEANS OF MARKOV CHAINS.

This has been applied to the Borzeşti Petrochemical Combine Works,*) more exactly to the vinyl polychloride producing units. To achive this product, 34 raw materials (methyl alcohol, mercuric chloride, natrium sulphite, sulphuric acid, a.s.o.) and 11 semifinished products (acetylene, vinyl chloride, trichlorine ethylene, hydrochloric acid, etc.) are wanted; they are achieved in eleven units.

By using the Markov chains theory the problem of the necessary materials for a given plan was solved up, in the hypothesis of some perfect correlations between the plan and the proportions in which products are obtained in keeping with chemical reactions. For instance, if the proportions governing the products are $r_1, r_2, \ldots, r_p$, then the components of the plan vector $y = (y_1, y_2, \ldots, y_p)$ shall satisfy relationship:

$$\frac{y_1}{r_1} = \frac{y_2}{r_2} = \ldots = \frac{y_p}{r_p} = \lambda. \tag{8}$$

In general, such perfect correlation is not possible. On this account some products will be yielded in unwished quantities, which may lead to exaggerated storage expenses. To avoid this, some original orders are diminished; a number of penalities may ensue from the non-delivery of the respective product. Hence the task of determining a production plan rendering minimal the sum of these expenses (storage and penalties).

Consider y_h^* to be the quantities required by users which do not observe relationship (8), and y_h the quantities to be effectively produced and which observe relationship (8). We consider the difference:

$$\delta_h = y_h^* - y_h = y_h^* - \lambda r_h.$$

If $\delta_h > 0$, a finite stock of products will follow, and if $\delta_h < 0$ a shortage of finished products will follow.

Cost c_h is attached, having this expression:

$$c_h = D_h \quad \text{if} \quad \delta_h > 0$$

$$c_h = P_h \quad \text{if} \quad \delta_h < 0$$

where:

D_h is the storage cost for a unit of product h;

P_h is the penalty cost for a missing unit of product h.

Thus, the optimum function becomes

$$F = \sum_h \delta_h c_h$$

and it is to be minimized.

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*) The numerical example cannot be inserted in this paper because of the considerable amount of computation.

PRINCIPLE DIAGRAM OF THE ELECTROLYSIS PROCESS OF THE NaCl SOLUTION

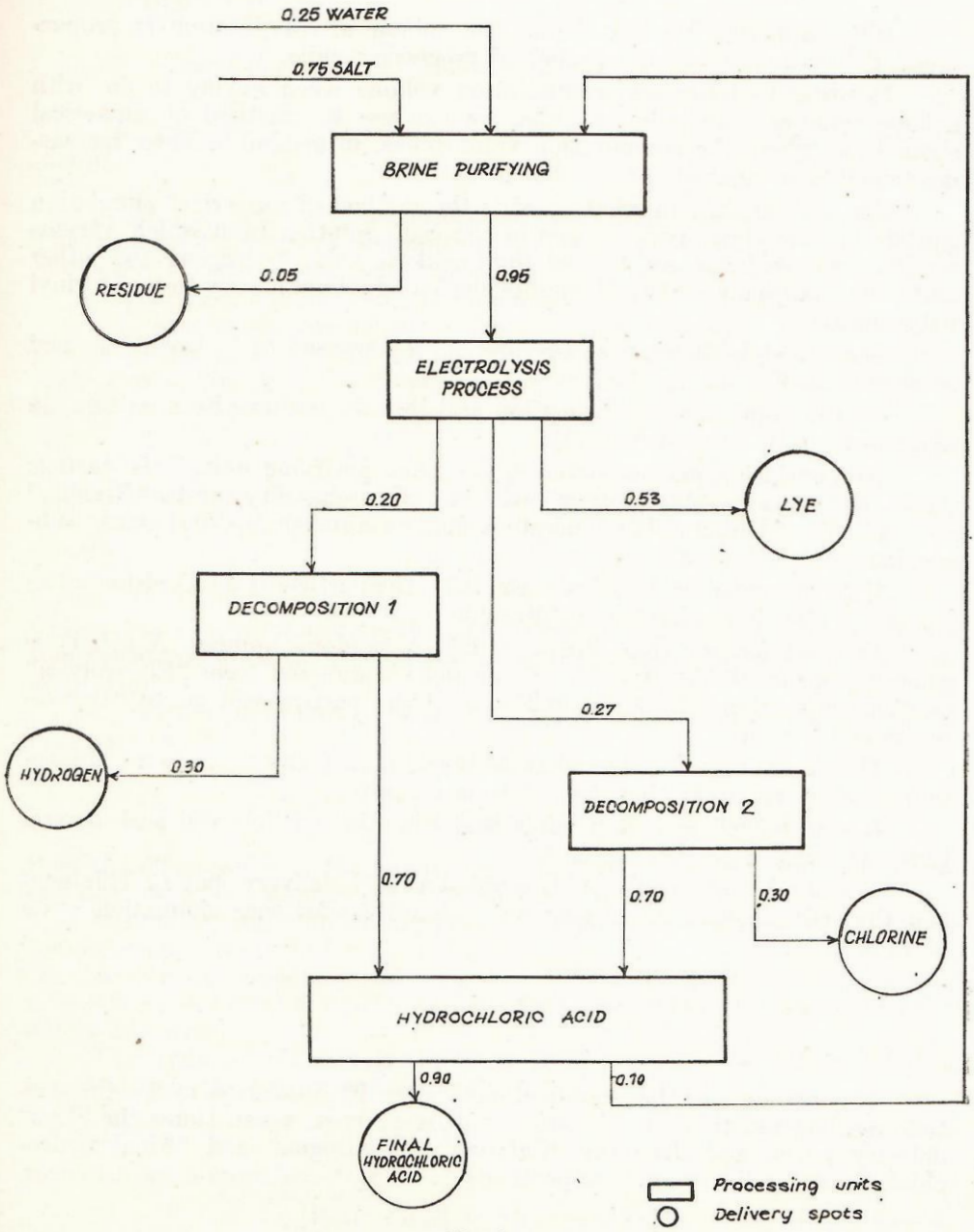

Fig. 2

4. SIMULATION THROUGH MONTE-CARLO METHOD OF THE TECHNOLOGICAL FLOW IN A PETROCHEMICAL PROCESSING UNIT

When applying Markov chains, the volume of computation is proportional to the square of the number of processing units.

In order to lessen the computation volume when having to do with a large number (hundreds) of units, we can use the method of numerical simulation, where the computation volume has, in general, a linear increasing tendency as against the number of units.

We shall explain in what consists the method of numerical simulation considering the electrolysis process of the NaCl solution from which various semifinished products are derived (hydrochloric acid, hydrogene and other materials that make up the 11 semifinished products necessary to make vinyl polychloride.

The principle diagram of the preparation process of hydrochloric acid is shown in Fig. 2.

On the graph arcs the transition probabilities p_{ij} have been written, in agreement with relationship (4).

We consider a particle entering the brine purifying unit. This particle passes on either to "Electrolysis" with a 0.95 probability, or to "Residue" with a 0.05 probability. We generate a random number $N_1 \in [0, 1]$ with subroutine RANDU in SSP *).

If N_1 is comprised between 0 and 0.95, the particle is directed to "Electrolysis"; else, it is directed to "Residue".

Now we assume that $N_1 \in [0; 0.95]$. A random number $N_2 \in [0, 1]$ is generated again. If $N_2 \in [0; 0.20]$, the particle is directed from "Electrolysis" to "Decomposition 1". If $N_2 \in (0.20; 0.57]$ the particle will go to "Decomposition 2", a.s.o.

The described proceeding shall be repeated as many times as an absorption point is reached; then the cycle is resumed.

Due to recycling, it is possible that the same particle will pass several times through a processing unit.

The number of outputs N^j is counted at each delivery spot I_j. This enables the estimation of the absorption probabilities for each simulation cycle of rank N.

$$\tilde{p}_j^N = \frac{N^j}{N}.$$

For instance, in the examined case, after 10 simulation cycles the particle reached ten times the "Residue" delivery spot, seven times the "Lye" delivery place, and once the "Chlorine", "Hydrogen" and "Final hydrochloric acid" delivery spot, respectively.

*) Scientific Subroutine Package (program deck for IBM 360 computer).

That means that the estimated probabilities after 10 cycles will be:

$\tilde{p}^{10}$ (Residue) = 0

$$\tilde{p}^{10}\,(\text{Lye}) = \frac{7}{10} = 0.7$$

$$\tilde{p}^{10}\ (\text{Chlorine}) = \tilde{p}^{10}\ (\text{Hydrogen}) = \tilde{p}^{10}\ (\text{Hydrochloric acid}) = \frac{1}{10} = 0.1.$$

In agreement with the weak law of large numbers, for $N \to \infty$ we have $\lim_{N\to\infty} \tilde{p}_j^N = p_j$

where p_j will be the absorption probability at delivery spot I_j. This probability shows a relationship between inputs and outputs, having, obviously, a certain precision degree, previously determined.

In practice, a number N_0 of simulation cycles is determined.

The absorption probabilities are estimated for each cycle and probability $\tilde{p}_j^N 0$ of the last cycle is considered. Having determined an admissible relative error u_j from the first, we compute the admissible absolute value $\varepsilon_{j,a}^{N_0}$ with the aid of relationship:

$$\varepsilon_{j,a}^{N_0} = u_j \cdot \tilde{p}_j^{N_0}.$$

Likewise, we compute the effective error

$$\varepsilon_{j,e}^{N_0} = \frac{k \cdot \sigma_j}{\sqrt{N_0}}$$

where: k is a verisimilitude coefficient (inferred by means of Laplace's function depending upon the probability to secure an unsurpassing of the admissible error).

σ_j' is the square mean deviation of absorption probability for the delivery spot I_j.

If

$$\varepsilon_{j,a}^{N_0} \geqslant \varepsilon_{j,e}^{N_0}$$

then the effectuation of the simulation cycles stops and the probabilities of the last cycle are assumed to be absorption probabilities.

Otherwise, the number of cycles is supplemented with a value δ and computation is resumed for $N_1 = N_0 + \delta$.

In the examined example we chose $N_0 = 30$ cycles; $\delta = 10$ cycles; $u_j = 0.05$; $k = 2$ (which secures a 97 per cent probability of unsurpassing the admissible error).

The hydrochloric acid is the main final product. We obtained for it (see Annex 1) the absorption probability

$$p(\text{Hcl}) \cong 0.255$$

to which an admissible error

$$\varepsilon_a(\text{Hcl}) = 0.05 \times 0.255 = 0.02175$$

is ascribed. After 110 simulation cycles, an effective error

$$\varepsilon_e(\mathrm{Hcl}) = 0.012440 < \varepsilon_a(\mathrm{Hcl})$$

was obtained.

If we had considered a larger number of important products, the checking tests should have been effected for each product.

On the basis of the obtained relationships between inputs and outputs, the quantities of raw material corresponding to given outputs (and viceversa) can be computed. Likewise, on the basis of the possibilities of passing from a processing unit to another we can determine the loading degree of the units (see Annex 1).

TABLE OF THE ELECTROLYSIS SIMULATION CYCLES

No.	Brine	Electrolysis	Decomposition 1	Decomposition 2	Hydrochloric acid import	Residue	Lye	Hydrogen	Chlorine	Pure Hydrochloric acid	Probability of having a residue	Probability of obtaining lye	Probability of obtaining hydrogen	Probability of obtaining chlorine	Probability of obtaining hydrochloric acid	Probability of having recirculations
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	47.	79.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	1.000	0.0	0.0	0.0	0.0
2	56.	24.	0.	40.	24.	0.	0.	0.	0.	1.	0.0	0.500	0.0	0.0	0.500	0.0
3	83.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.667	0.0	0.0	0.333	0.0
4	52.	69.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.750	0.0	0.0	0.250	0.0
5	45.	49.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.800	0.0	0.0	0.200	0.0
6	89.	95.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.833	0.0	0.0	0.167	0.0
7	70.	69.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.857	0.0	0.0	0.143	0.0
8	77.	45.	0.	78.	0.	0.	0.	0.	1.	0.	0.0	0.750	0.0	0.125	0.125	0.0
9	60.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.778	0.0	0.111	0.111	0.0
10	2.	1.	86.	0.	0.	0.	0.	1.	0.	0.	0.0	0.700	0.100	0.100	0.100	0.0
11	5.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.727	0.091	0.091	0.091	0.0
12	1.	93.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.750	0.083	0.083	0.083	0.0
13	51.	65.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.769	0.077	0.077	0.077	0.0
14	32.	8.	57.	0.	72.	0.	0.	0.	0.	1.	0.0	0.714	0.071	0.071	0.143	0.0
15	18.	62.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.733	0.067	0.067	0.133	0.0
16	8.	97.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.750	0.063	0.063	0.125	0.0
17	4.	54.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.765	0.059	0.059	0.118	0.0
18	92.	63.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.778	0.056	0.056	0.111	0.0
19	50.	27.	0.	19.	65.	0.	0.	0.	0.	1.	0.0	0.737	0.053	0.053	0.158	0.0
20	20.	38.	0.	49.	47.	0.	0.	0.	0.	1.	0.0	0.700	0.050	0.050	0.200	0.0
21	42.	28.	0.	93.	0.	0.	0.	0.	1.	0.	0.0	0.667	0.048	0.095	0.190	0.0
22	4.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.682	0.045	0.091	0.182	0.0
23	65.	41.	0.	65.	23.	0.	0.	0.	0.	1.	0.0	0.652	0.043	0.087	0.217	0.0
24	49.	85.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.667	0.042	0.083	0.208	0.0
25	74.	74.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.680	0.040	0.080	0.200	0.0
26	78.	4.	25.	0.	11.	0.	0.	0.	0.	1.	0.0	0.654	0.038	0.077	0.231	0.0
27	44.	61.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.667	0.037	0.074	0.222	0.0
28	73.	87.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.679	0.036	0.071	0.214	0.0
29	62.	90.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.690	0.034	0.069	0.207	0.0
30	81.	73.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.700	0.033	0.067	0.200	0.0

EPS = 0.033408

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
31	14.	20.	0.	99.	0.	0.	0.	0.	1.	0.	0.0	0.677	0.032	0.097	0.194	0.0
32	13.	89.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.688	0.031	0.094	0.188	0.0
33	12.	74.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.697	0.030	0.091	0.182	0.0
34	32.	25.	0.	64.	57.	0.	0.	0.	0.	1.	0.0	0.676	0.029	0.088	0.206	0.0
35	72.	15.	42.	0.	16.	0.	0.	0.	0.	1.	0.0	0.657	0.029	0.086	0.229	0.0
36	23.	87.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.667	0.028	0.083	0.222	0.0
37	18.	25.	0.	93.	0.	0.	0.	0.	1.	0.	0.0	0.649	0.027	0.108	0.216	0.0
38	31.	51.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.658	0.026	0.105	0.211	0.0
39	28.	3.	72.	0.	0.	0.	0.	1.	0.	0.	0.0	0.641	0.051	0.103	0.205	0.0
40	1.	60.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.650	0.050	0.100	0.200	0.0
														EPS = 0.025169		
41	51.	61.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.659	0.049	0.098	0.195	0.0
42	12.	20.	11.	0.	92.	0.	0.	0.	0.	1.	0.0	0.643	0.048	0.095	0.214	0.0
43	48.	63.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.651	0.047	0.093	0.209	0.0
44	47.	10.	37.	0.	34.	0.	0.	0.	0.	1.	0.0	0.636	0.045	0.091	0.227	0.0
45	71.	19.	76.	0.	0.	0.	0.	1.	0.	0.	0.0	0.622	0.067	0.089	0.222	0.0
46	88.	43.	0.	61.	79.	0.	0.	0.	0.	1.	0.0	0.609	0.065	0.087	0.239	0.0
47	27.	51.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.617	0.064	0.085	0.234	0.0
48	61.	9.	3.	0.	43.	0.	0.	0.	0.	1.	0.0	0.604	0.063	0.083	0.250	0.0
49	26.	74.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.612	0.061	0.082	0.245	0.0
50	6.	73.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.620	0.060	0.080	0.240	0.0
														EPS = 0.024603		
51	84.	42.	0.	1.	26.	0.	0.	0.	0.	1.	0.0	0.608	0.059	0.078	0.255	0.0
52	44.	36.	0.	16.	71.	0.	0.	0.	0.	1.	0.0	0.596	0.058	0.077	0.269	0.0
53	85.	71.	0.	0.	0.	0.	0.	1.	0.	0.	0.0	0.604	0.057	0.075	0.264	0.0
54	61.	29.	0.	28.	4.	0.	0.	0.	0.	0.	0.0	0.593	0.056	0.074	0.259	0.019
55	72.	95.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.600	0.055	0.073	0.255	0.018
56	17.	53.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.607	0.054	0.071	0.250	0.018
57	59.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.614	0.053	0.070	0.246	0.018
58	64.	38.	0.	52.	72.	0.	0.	0.	0.	1.	0.0	0.603	0.052	0.069	0.259	0.017
59	63.	32.	0.	30.	88.	0.	0.	0.	0.	1.	0.0	0.593	0.051	0.068	0.271	0.017
60	59.	60.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.600	0.050	0.067	0.267	0.017
														EPS = 0.024602		
61	25.	13.	55.	0.	12.	0.	0.	0.	0.	1.	0.0	0.590	0.049	0.066	0.279	0.016
62	77.	55.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.597	0.048	0.065	0.274	0.016
63	42.	53.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.603	0.048	0.063	0.270	0.016
64	42.	72.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.609	0.047	0.063	0.266	0.016
65	53.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.615	0.046	0.062	0.262	0.015
66	32.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.621	0.045	0.061	0.258	0.015
67	9.	9.	72.	0.	0.	0.	0.	1.	0.	0.	0.0	0.612	0.060	0.060	0.254	0.015
68	51.	65.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.618	0.059	0.059	0.250	0.015
69	25.	70.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.623	0.058	0.058	0.246	0.014
70	91.	23.	0.	15.	83.	0.	0.	0.	0.	1.	0.0	0.614	0.057	0.057	0.057	0.014
														EPS = 0.019759		
71	61.	21.	0.	76.	0.	0.	0.	0.	1.	0.	0.0	0.606	0.056	0.070	0.254	0.014
72	65.	8.	63.	0.	9.	0.	0.	0.	0.	0.	0.0	0.597	0.056	0.069	0.250	0.028
73	85.	27.	0.	99.	0.	0.	0.	0.	1.	0.	0.0	0.589	0.055	0.082	0.247	0.027
74	49.	6.	96.	0.	0.	0.	0.	1.	0.	0.	0.0	0.581	0.068	0.081	0.243	0.027
75	22.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.587	0.067	0.080	0.240	0.027
76	10.	47.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.592	0.066	0.079	0.237	0.026
77	93.	32.	0.	55.	41.	0.	0.	0.	0.	1.	0.0	0.584	0.065	0.078	0.247	0.026
78	54.	57.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.590	0.064	0.077	0.244	0.026
79	52.	2.	43.	0.	40.	0.	0.	0.	0.	1.	0.0	0.582	0.063	0.076	0.253	0.025
80	57.	81.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.587	0.063	0.075	0.250	0.025
														EPS = 0.016512		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
81	75.	21.	0.	50.	8.	0.	0.	0.	0.	0.	0.0	0.580	0.062	0.074	0.247	0.037
82	2.	37.	0.	4.	91.	0.	0.	0.	0.	1.	0.0	0.573	0.061	0.073	0.256	0.037
83	10.	35.	0.	25.	32.	0.	0.	0.	0.	1.	0.0	0.566	0.060	0.072	0.265	0.036
84	66.	6.	45.	0.	16.	0.	0.	0.	0.	1.	0.0	0.560	0.060	0.071	0.274	0.036
85	90.	96.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.565	0.059	0.071	0.271	0.035
86	68.	42.	0.	41.	68.	0.	0.	0.	0.	1.	0.0	0.558	0.058	0.070	0.279	0.035
87	44.	51.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.583	0.057	0.069	0.276	0.034
88	4.	68.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.568	0.057	0.068	0.273	0.034
89	71.	19.	71.	0.	0.	0.	0.	1.	0.	0.	0.0	0.562	0.067	0.067	0.270	0.034
90	56.	96.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.567	0.067	0.067	0.067	0.033
														EPS = 0.016500		
91	76.	94.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.571	0.066	0.066	0.264	0.033
92	78.	20.	14.	0.	10.	0.	0.	0.	0.	0.	0.0	0.565	0.065	0.065	0.261	0.043
93	28.	83.	0.	0.	0.	0.	1.	0.	0.	0.	0.0	0.570	0.065	0.065	0.258	0.043
94	42.	7.	64.	0.	22.	0.	0.	0.	0.	1.	0.0	0.564	0.064	0.064	0.266	0.043
95	58.	44.	0.	48.	88.	0.	0.	0.	0.	1.	0.0	0.558	0.063	0.063	0.274	0.042
96	96.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.010	0.552	0.063	0.063	0.271	0.042
97	88.	66.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.557	0.062	0.062	0.268	0.041
98	1.	13.	68.	0.	95.	0.	0.	0.	0.	1.	0.010	0.551	0.061	0.061	0.276	0.041
99	54.	71.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.556	0.061	0.061	0.273	0.040
100	35.	73.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.560	0.060	0.060	0.070	0.040
														EPS = 0.015230		
101	25.	88.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.564	0.059	0.059	0.267	0.040
102	6.	42.	0.	100.	0.	0.	0.	0.	1.	0.	0.010	0.559	0.059	0.069	0.265	0.039
103	15.	92.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.563	0.058	0.068	0.262	0.039
104	21.	97.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.567	0.058	0.067	0.260	0.038
105	93.	85.	0.	0.	0.	0.	1.	0.	0.	0.	0.010	0.571	0.057	0.067	0.257	0.038
106	73.	72.	0.	0.	0.	0.	1.	0.	0.	0.	0.009	0.575	0.057	0.066	0.255	0.038
107	75.	1.	33.	0.	90.	0.	0.	0.	0.	1.	0.009	0.570	0.056	0.065	0.262	0.037
108	45.	60.	0.	0.	0.	0.	1.	0.	0.	0.	0.009	0.574	0.056	0.065	0.259	0.037
109	51.	64.	0.	0.	0.	0.	1.	0.	0.	0.	0.009	0.578	0.055	0.064	0.257	0.037
110	30.	98.	0.	0.	0.	0.	1.	0.	0.	0.	0.009	0.582	0.055	0.064	0.255	0.036
														EPS = 0.012440		

The quantities of raw materials and finite products corresponding to 1253 tons of hydrochloric acid

Raw material

Salt = 1185.8750 tons
Water = 3557.6250 tons

Finite products

Residue = 44.7500 tons
Lye = 2864.0000 tons
Hydrogen = 268.5000 tons
Chlorine = 313.2500 tons

Loading on processing units
Unit

Brine purifying = 5101.5000 tons
Electrolysis = 4877.7500 tons
Decomposition 1 = 895.0000 tons
Decomposition 2 = 1118.7500 tons
Hydrochloric acid = 1432.0000 tons.

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UNE APPLICATION DES CHAINES MARKOV ET DE LA SIMULATION DANS L'ETUDE DE CERTAINS PROCESSUS PETROCHIMIQUES

On présente une application de la théorie des chaînes Markov et de la simulation dans l'étude des processus pétrochimiques. Associant une chaîne Markov finie à un processus pétrochimique, on détermine les quantités de matières premières et demiproduits nécessaires à l'exécution d'un plan de production connu. Le problème présenté est traité de même, par la méthode de la simulation numérique, d'autan plus avantageuse que le nombre des installations devient plus grand.

ПРИМЕНЕНИЕ ЦЕПИ МАРКОВА И СИМУЛИРОВАНИЕ В ИССЛЕДОВАНИИ НЕКОТОРЫХ НЕФТЕ-ХИМИЧЕСКИХ ПРОЧЕССОВ

В работе излагается применение теории цепей Маркова и симулирование в изучении пефтехимических процессов. Ассоциируя конечную цепь Маркова с пефтехимическим процессом мы определяем количество сырья и полуфабрикатов, необходимое для выполнения заданного производственного плана. Данный вопрос трактуется также и при помощи метода числового симулирования который становится выгодной по мере роста количества оборудования.

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