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Splitorial Concepts Used in Scheduling

The author presents in this paper the main definitions, properties relations and laws on the concepts connected to the change of certain values (temporal, spacial, etc.) called splitorial concepts. The definition was suggested by the terminology used for some standard programs concerning the production efficient programming. Finally, it is shown the correspondence between the defined concepts and phenomena occuring in the production efficient programming, as well as how several practical applications can be performed.

1. INTRODUCTION

The evolution of production system may be described with the aid of an events succession $E^i_{\alpha j}(t^i_{\alpha j})$, $E^i_{\omega j}(t^i_{\omega j})$ $(i = 1, 2...)$ meaning the beginning and respectively the end of a condition parameter transformation of C_j/1/. No transformations of component C_j take place between two intervals $(t^i_{\omega j}, t^{i+1}_{\alpha j})$; $t^{i+1}_{\alpha j} \geqslant t^i_{\omega j}$. Thus, the time interval sets under the following form may be attached to each component C_j:

$$\tau = \{ (t^1_{\alpha j}, t^1_{\omega j}), (t^2_{\alpha j}, t^2_{\omega j}), ..., (t^n_{\alpha j}, t^n_{\omega j}) ...\} \tag{1}$$

where

$$t^i_{\alpha j} \leqslant t^1_{\omega j} \leqslant t^2_{\alpha j} \leqslant t^2_{\omega j} \leqslant ... \leqslant t^{n-1}_{\omega j} \leqslant t^n_{\alpha j} \leqslant t^n_{\omega j} ...$$

Intervals under the form $(t^i_{\alpha j}, t^i_{\omega j})$, $i = 1,2,...$ represent active periods for component C_j. On the other hand, intervals $(t^i_{\omega j}, t^{i+1}_{\alpha j})$, $(t^{i+1}_{\omega j}, t^{i+2}_{\alpha j})$ ... represent breaks in processing the components of production system, breaks caused by various constraints regarding certain available resources. The main characteristic of production process evolution reflected by intervals set (τ) consists

in splitting the activities length. Therefore, it was considered that the adequate name of this set is that of "splitor". This name was suggested by some methods of PERT-ICL program (for example MIN SPLIT) and by the terminology used for CAPOSS program package (IBM Company).

Consequently, all concept connected to the split of certain values (time, space, etc.) agreed to be called splitorial concepts. These concepts can be applied to studying the critical path analysis under different conditions of work in shifts [4] and to solving more general problems of efficient production programming [5].

2. DEFINITIONS

2.1. Directed Connected Pseudosplitor ($P\widetilde{S}$)

We call a connected pseudosplitor positively directed ($P\widetilde{S}$), a set of real number pairs under the form:

$$P\widetilde{S} = \begin{cases} \{(\alpha_i, \beta_i)\}; \ i = 1, 2, ..., n; \ n \in N \\ \alpha_j \in R, \ \beta_i \in R \end{cases} \tag{2}$$

subjected to: $\alpha_i \leqslant \beta_i$, $(\forall)_i$.

If $\beta_i \leqslant \alpha_i$, $(\forall)_i$ then, the pseudosplitor is negatively directed as follows:

$$-P\widetilde{S} = \{(\alpha_i, \beta_i)\} \quad i = 1,2, ..., n.$$

Pairs $(\alpha_i \beta_i)$ are called *components* of pseudosplitor $P\widetilde{S}$. These components can be considered as coaxial vectors $\overline{V}_i$, positively directed to a given axis O_x. The modulus of vector $\overline{V}_i \cdot |\overline{V}_i| = \beta_i - \alpha_i$ $(i = 1, 2, ..., n)$ is called the operative length of components (α_i, β_i).

Numbers α_i denote the *abscissa* of initial points for applying vectors V_i, and numbers β_i are the abscissa of the final points for the same vectors.

Point $\alpha = min\ \alpha_i$; $i = 1, 2, ..., n$ is called *initial point* of pseudosplitor $P\widetilde{S}$. Similarly, we define the *final point* of pseudosplitor $P\widetilde{S}$: $\beta = \max_i \beta_i$, $i = 1, 2, ... n$. The operative length $B(\widetilde{S})$ of pseudosplitor $P\widetilde{S}$ is:

$$B(P\widetilde{S}) = \sum_{i=1}^{n} (\beta_i - \alpha_i) \geqslant 0. \tag{3}$$

The length $L\,(P\widetilde{S})$ of a directed pseudosplitor $P\widetilde{S}$ is the difference between the final abscissa and the initial one:

$$L(P\widetilde{S}) = \beta - \alpha. \tag{4}$$

The splitorial density $\rho(P\widetilde{S})$ of pseudosplitor $P\widetilde{S}$ is denoted by the relation between the operative length $B\,(P\widetilde{S})$ and the length $L(P\widetilde{S})$, that is:

$$\rho(P\widetilde{S}) = \frac{B(P\widetilde{S})}{L(P\widetilde{S})}. \tag{5}$$

A number x having the property $\alpha_i \leqslant x \leqslant \beta_i$ and $\alpha_j \leqslant x \leqslant \beta_j$, $i \neq j$, is called a common point of pseudosplitor $P\widetilde{S}$.

2.2. Directed Connected Splitor ($\widetilde{S}$)

We call a directed connected splitor ($\widetilde{S}$) a pseudosplitor having n components characterized by the fact that the maximum number of the common point is $n-1$. Consequently, if the series of abscissas α_i, β_i is increasingly ordered, it follows:

$$\alpha_1 \leqslant \beta_1 \leqslant \alpha_2 \leqslant \beta_2 \leqslant \ldots \beta_{n-1} \leqslant \alpha_n \leqslant \beta_n. \tag{6}$$

Obviosuly, the splitors set is included into pseudosplitors set.

If $\alpha_{i+1} - \beta_i$ and $\beta_i - \alpha_i$ are constant, a *repeatitive splitor* is followed.

If $\beta_i = \alpha_{i+1}$, $1 = 1, 2, \ldots n-1$, we get a *chain splitor* denoted by ($\widetilde{S}_c$). The splitorial density of chain splitor is $\rho(\widetilde{S}_c) = 1$. Consequently, $\rho(\widetilde{S}) \leqslant 1$.

2.3. Initial extension ($\widetilde{S}'$) and final extension ($\widetilde{S}''$)

Splitors $\widetilde{S}' = \{(\alpha_i', \beta_i')\}$ and $\widetilde{S}'' = \{(\alpha_i'', \beta_i'')\}$, $i = 1, 2, \ldots n$ are called *initial extension* and *final extension* satisfying the following constraints:

$$\begin{aligned}
&-\infty \leqslant \alpha_1' \leqslant \beta' = \alpha_1;\ \beta_1 = \alpha_1'' \leqslant \beta_1'' \leqslant \alpha_2 \\
&\beta_1 \leqslant \alpha_2' \leqslant \beta_2' = \alpha_2;\qquad \beta_2 = \alpha_2'' \leqslant \beta_2'' \leqslant \alpha_3 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\beta_{n-1} \leqslant \alpha_n' \leqslant \beta_n' = \alpha_n;\ \beta_n = \alpha_n'' \leqslant \beta_n'' \leqslant \infty.
\end{aligned} \tag{7}$$

A directed splitor to which an initial and/or a final extension is attached is called a partial connected splitor. The general form of a partial connected splitor is ($\widetilde{S}'$, $\widetilde{S}$, $\widetilde{S}''$) or ($\widetilde{S}'$, $\widetilde{S}$) or ($\widetilde{S}$, $\widetilde{S}''$).

2.4. Conjugated Internal Splitor ($\widetilde{S}_*$) of a Connected Splitor ($\widetilde{S}$)

A splitor $\widetilde{S}_* = \{(\alpha_i^*, \beta_i^*)\}$, $i = 1, 2, \ldots, n-1$, having the property of $\alpha_i^* = \beta_i$ and $\beta_i^* = \alpha_{i+1}$ $i = 1, 2, \ldots n-1$, $n \geqslant 1$, is *a conjugated internal splitor.*

By adding a number $a \leqslant \alpha_i$ and a number $b \leqslant \beta_n$, we may define the external conjugated splitor ($\widetilde{S}^*$) of splitor $\widetilde{S}$. Therefore, by adding the intervals (a, α_1) and $(\beta_n b)$ to splitor ($\widetilde{S}_*$) we obtain:

$$\widetilde{S}^* = \{a, \alpha_1), (\alpha_1^*, \beta_1^*), \ldots, (\alpha_n^*, \beta_n^*), (\beta_n, b)\} \tag{8}$$

It follows: $(\widetilde{S}^*)_* = \widetilde{S}$.

The internal conjugated splitor of a chain splitor is

$$\widetilde{S}_{*c} = \{(\alpha_2, \alpha_2), (\alpha_3, \alpha_3), ..., (\alpha_n, \alpha_n)\} \tag{9}$$

and $B(\widetilde{S}_{*c}) = 0$.

2.5. The m Order Primal Multipseudosplitor

The $(M\widetilde{P}P)$ order primal multipseudosplitor is a set of pseudosplitors with m components:

$$M\widetilde{P}P = \{\{P\widetilde{S}_i\}\} = \{\{(\alpha_{ij}, \beta_{ii}\}\}, \; i = 1, 2, ..., m; \; j = 1, 2, ..., n. \tag{10}$$

Pseudosplitors $P\widetilde{S}_i$ are component elements of the multisplitor $(MP\widetilde{P})$. if these component elements are splitors, then a m $(M\widetilde{S}P)$ order primal multisplitor under the following form is obtained:

$$M\widetilde{S}P = \{\widetilde{S}_i\} = \{\{(\alpha_{ij}, \beta_{ij}\}\}, \; i = 1, 2, ..., m; \; j = 1, 2, ..., n. \tag{11}$$

where $\widetilde{S}_i$ is subjected to constraint (6).

At the same time, we may say that the multipseudosplitor $(M\widetilde{P}P)$ or the multisplitor $(M\widetilde{S}P)$ are $m \times n$ hypermatrices:

$$M\widetilde{P}P = \| (\alpha_{ij}, \beta_{ij}) \|, \; i = 1, 2, ..., m; \; j = 1, 2, ..., n. \tag{12}$$

We call a n-order dual multipseudosplitor of a m-order primal multisplitor, a multipseudosplitor with n components obtained by changing the lines and columns within matrix (12), namely:

$$M\widetilde{P}D = \{P\widetilde{S}_j\} = \{(\alpha_{1j}, \beta_{1j}), (\alpha_{2j}, \beta_{2j}), ..., (\alpha_{nj}, \beta_{nj})\} \tag{13}$$
$$j = 1, 2, ..., n$$

or $M\widetilde{P}D = M\widetilde{S}P^T$; where $M\widetilde{S}P^T$ is the transposed hypermatrix of $\| \alpha_{ij}, \beta_{ij} \|$. If components $(\alpha_{ij}, \beta_{ij})$, $i = 1, 2, ... n$ are splitors $\widetilde{S}_j$ $(j = 1, 2, ... m)$ subjected to constraints (6), then a n — *order dual multisplitor* of the m-order primal multisplitor under the following form is obtained:

$$M\widetilde{S}D = \{\widetilde{S}_j\}, \quad j = 1, 2, ... n. \tag{14}$$

In this case, the operation of getting the dual multisplitor from a primal multisplitor, is an operation of transition to dual. We notice that by this operation of transition to dual of the dual multisplitor, we get the primal multisplitor.

2.6. Relations among Defined Splitorial Concepts

2.6.1. *Equality of Two Directed Connected Splitors* $\widetilde{S}_1$ and $\widetilde{S}_2$ is denoted by:

$$\widetilde{S}_1 = \widetilde{S}_2 \tag{15}$$

implying the moduli equality, equality of the abscissas of application points of component vectors and direction coincidence.

2.6.2. *Equality of Two Partially Connected Directed Splitors* is denoted by

$$(\widetilde{S}_1', \widetilde{S}_1, \widetilde{S}_1'') = (\widetilde{S}_2', \widetilde{S}_2, \widetilde{S}_2'') \tag{16}$$

implying the subjection of relation (15) between $\widetilde{S}_1$ and $\widetilde{S}_2$.

2.6.3. *Equimobility of two Partially Connected Directed Splitors* is denoted by:

$$\widetilde{S}_1 \sim \widetilde{S}_2, \tag{17}$$

Implying: moduli equality, equality of abscissas of application points of initial extension components, equality of final point abscissas of initial components and direction coincidence. Obviously, if two splitors are equimobile, then:

$$B(\widetilde{S}_1') + B(\widetilde{S}_1'') = B(\widetilde{S}_2') + B(\widetilde{S}_2''). \tag{18}$$

2.6.4. *Identity of Two Partially Connected Directed Splitors* is denoted by:

$$(\widetilde{S}_1', \widetilde{S}_1, \widetilde{S}_1'') \equiv (\widetilde{S}_2', \widetilde{S}_2, \widetilde{S}_2'') \tag{19}$$

and implies: $\widetilde{S}_1 = \widetilde{S}_2'$ and $\widetilde{S}_1 \sim \widetilde{S}_2''$.

2.6.5. *Iso-operative Length of Two Splitors* is denoted by:

$$\widetilde{S}_1 \cong \widetilde{S}_2 \tag{20}$$

and implies the equality of operative lengths, namely:

$$B(\widetilde{S}_1) = B(\widetilde{S}_2).$$

2.6.6. *Isotropy of Two Splitors* is denoted by:

$$\widetilde{S}_1 \simeq \widetilde{S}_2 \tag{21}$$

implying the same structure of the component vectors, excepting the sequence and application points. This means that a modulus $\overline{V}_{j2} \subset \widetilde{S}_2$ corresponds to every modulus of vectors $\overline{V}_{i1} \subset \widetilde{S}_1$, so that:

$$|V_{i1}| = |\overline{V}_{j2}| \text{ or } \beta_{i1} - \alpha_{i1} = \beta_{j2} - \alpha_{j2}$$
$$i = 1, 2, \ldots n; \; j = 1, 2, \ldots m.$$

Obviously, the isotropy relation implies $\widetilde{S}_1 = \widetilde{S}_2 \Rightarrow \widetilde{S}_1 \cong \widetilde{S}_2$.

2.6.7. *Equidimensionality of Two Splitors* $\widetilde{S}_1$ and $\widetilde{S}_2$ is denoted by:

$$\widetilde{S}_1 \approx \widetilde{S}_2 \tag{22}$$

and implies $L(\widetilde{S}_1) = L(\widetilde{S}_2)$.

2.6.8. *Inclusion of Two Splitors* is denoted by:

$$\widetilde{S}_1 \subseteq \widetilde{S}_2 \tag{23}$$

and implies: $\alpha_{j2} \leqslant \alpha_{i1} \leqslant \beta_{j2}$; $\beta_{i1} \leqslant \beta_{j2}$.

Remark

The above defined relations are reflexive, symmetrical and transitive.

2.7. Conditioned Splitor S_k

A conditioned splitor S_k is a partially connected splitor, equimobil as against a chain splitor $\widetilde{S}_c$, that is:

$$(\widetilde{S}_k'', \widetilde{S}_k, \widetilde{S}_k'') \sim (\widetilde{S}_c', \widetilde{S}_c, \widetilde{S}_c'') \tag{24}$$

where:

$$\widetilde{S}_c' = \{(\alpha_{ic}', \beta_{ic}')\}, \widetilde{S}_c'' = \{(\alpha_{ic}'', \beta_{ic}'')\}$$

$$\alpha_{ic}' = \beta_{ic}' = \alpha_0 + \sum_{h=1}^{i-1} |V_h|, \; i = 2, 3, ..., n;$$

$$\alpha_1' = \beta_1' = \alpha_0, \widetilde{S}_c'' = \{(\alpha_{ic}'', \beta_{ic}'')\}; \alpha_{ic}'' = \alpha_0 + \sum_{h=1}^{i} |\overline{V}_k|, \tag{25}$$

$$\beta_{ic}'' = \infty, \; i = 1, 2, ..., n.$$

Therefore, a conditioned splitor S_k is perfectly determined of moduli, of component vectors and abscissa α_0 of initial application points are known that is:

$$S_k = \{\alpha_0, |V_i|\}. \tag{26}$$

The dual of a conditioned splitor S_k is a multisplitor $\hat{S}_k = \{(\alpha_k^i | V_i|)\}$ where:

$$\alpha_k^i \geqslant \alpha_k^{i-1} + |\overline{V}_{i-1}|, \; i = 2, 3, ..., n; \; \alpha_k' > \alpha_0 \tag{27}$$

If

$$\alpha_k^i = \alpha_k^{i-1} + |\overline{V}_{i-1}|, \; i = 2,3, ..., n; \; \alpha_k^i = \alpha_0$$

then, the dual of the chain splitor is obtained being denoted by: $(\alpha_0, \hat{S}_c)$. If the sign of the chain splitor is negative, then the final point β_m is specified and denoted by:

$$(-\hat{S}_c, \beta_m) = -(\hat{S}_c, \beta_m).$$

A conditioned multisplitor has the form:

$$MS_k = \{(\alpha_k^{ij}, S_k^{ij})\},\ i = 1, 2, ..., n \text{ and } j = 1, 2, ..., m. \tag{28}$$

The dual conditioned multisplitor of MS_k is:

$$\hat{MS}_k = \{(\alpha_k^{ji}, S_k^{ji})\},\ j = 1, 2, ..., n;\ i = 1, 2, ..., m. \tag{29}$$

where:

$$\alpha_k^{j+1,\,i} \geqslant \alpha_k^{ji} + S_k^{ji}. \tag{30}$$

If relation (30) is an equality then, a dual chain multisplitor of the following form is got:

$$\hat{MS}_c = \{(\alpha_c^i, \tilde{S}_c^i)\},\ i = 1, 2, ..., m. \tag{31}$$

2.8. A Connected Intersplitor of a Given Splitor

A connected intersplitor $\tilde{I}_s$ of a given splitor $\tilde{S}$ is a connected splitor included in $\tilde{S}$, namely: $\tilde{I}_s \subseteq \tilde{S}$.

A Connected Extrasplitor $\tilde{E}_s$ of a given splitor $\tilde{S}$, is a connected splitor having the characteristics $\tilde{S} \subseteq \tilde{E}_s$.

2.9. A Connected Subsplitor of a Given Splitor $\tilde{S}$ is an intersplitor $\tilde{s}$ of a splitor $\tilde{S}$, having the abscissa of the initial point $\alpha_{1s} \in [\alpha_i, \beta_i] \subset \tilde{S}$ the final point of abscissa $\beta_{ms} \in [\alpha_j, \beta_j] \subset \tilde{S}'$ $(j \geqslant i)$, characterized by the fact that every point x belonging to a component of splitor $\tilde{S}$ and satisfying the double constraint $\alpha_{1s} \leqslant x \leqslant \beta_{ms}$ belongs to $\tilde{s}$. The operative length of subsplitor $\tilde{S}$ is denoted by $B(\tilde{s}) = B_s$ satisfying the constraint $B_s \leqslant B(\tilde{s})$.

If the sign of $\tilde{s}$ is opposite to $\tilde{S}$, then we consider it as being negative (opposite) and denote it by $-\tilde{s}$. In this case, $B(-\tilde{s}) = -B_s$.

A connected subsplitor can be determined in the following three ways;

$$\tilde{s} = \tilde{s}(\tilde{S},\ \alpha_{1s}, B_s) = \tilde{s}(\tilde{S},\ B_s,\ \beta_{ms}) = \tilde{s}(\tilde{S},\ \alpha_{1s},\ \beta_{ms}) \tag{32}$$

To this end, algorithms are worked out, with whose aid the intervals are followed: $(\alpha_i, \beta_i) \subset \tilde{S}$.

If within a multisplitor the subsplitors are defined, then a *connected multisplitor* is obtained, denoted by: $M\tilde{s}P \subseteq M\tilde{S}P$, respectively $M\tilde{s}D \subseteq M\tilde{S}D$.

A complementary subsplitor $c\tilde{s}$ of a given subsplitor $\tilde{s}$ $(\tilde{S},\ \alpha_{1s},\ B_s)$ is a subsplitor of the same splitor $\tilde{S}$, having the operative length B_c, yielded by the relation:

$$B_c = B(c\tilde{s}) = B(\tilde{S}) - B_s. \tag{33}$$

Let α_{1c} be the critical point of subsplitor $c\widetilde{s}$ and β_{pc} the final point of the same subsplitor. If $\alpha_{1c} = \alpha_1$, then the complementary subsplitor $c\widetilde{s}_0$ is obtained and is called subsplitor of an initial definition of pair $(\widetilde{s}, \widetilde{S})$ stated by the relation:

$$c\widetilde{s}_0 = c\widetilde{s}_0(\widetilde{S}, \alpha_1, B(\widetilde{S}) - B_s).$$

If $\beta_{pc} = \beta_n (p \leqslant n)$, then we deal with a complementary subsplitor $c\widetilde{s}_f$ called subsplitor of a final definition following from the relation:

$$c\widetilde{s}_f = c\widetilde{s}_f(\widetilde{S}, B(\widetilde{S}) - B_s, \beta_n).$$

If the initial point α_{1s} of a subsplitor $\widetilde{s}$ of a given splitor $\widetilde{S}$ belongs to the subsplitor of an initial definition or if the final point β_{ps} belongs to the subsplitor of a final definition, we may say that the subsplitor $\widetilde{s}$ is *compatibly defined on the splitor* $\widetilde{S}$. On the other hand, we may say that the subsplitor $\widetilde{s}$ is *incompatibly defined* on the splitor $\widetilde{S}$, if we take into consideration an opposite direction.

12.10. An Independent Subsplitor of a Given Splitor

An independent subsplitor $\widetilde{s}$ of a splitor $\widetilde{S}$ is a subsplitor of an operative ength B_s, compatibly defined under the $\widetilde{s}$ $(\widetilde{S}, B_s)$ form. The initial point α_{1s} (or the final point β_{ms}) can be slided in the subsplitor field of initial definition, (respectively final definition). Let be two initial points α^1_{1s} and α^2_{1s} denoting two connected subsplitors $\widetilde{S}_1$ and $\widetilde{S}_2$ (therefore α^1_{1s} and α^2_{1s} belong to subsplitor field of initial definition). These two points define a subsplitor of splitor $\widetilde{S}$, whose operative length is $\delta\alpha_{1s}$. Consequently, the sliding of an independent splitor from a point α^1_{1s} is completely defined through operative length $\delta\alpha_{1s}$. With the aid of a rather simple algorithm, we may compute the elements of subsplitor $\widetilde{S}_2$, by knowing $\widetilde{S}_1$ and the operative length $\delta\alpha_{1s}$.

Similarly, we may consider the final points β^1_{ms} and β^2_{ms} defining a subsplitor of splitor $\widetilde{S}$ with the operative length $\delta\beta_{ms}$. If the initial point α_{1s} is included into the intersplitor $\widetilde{I}_{se} \subseteq \widetilde{S}$, characterized by: $\delta\alpha_{1s} = \delta\beta_{ms}$, $(\forall)\alpha_{1s} \in \in \widetilde{I}_{se}$, then, the length of the connected subsplitor $\widetilde{s}(\alpha_{1s}, \widetilde{S}, B_s)$ is invariable if α_{1s} is changed by $\alpha_{1s} + \delta\alpha_{1s} \in \widetilde{I}_{se}$. The intersplitor $\widetilde{I}_{se}$ having the characteristic: $L[\widetilde{s}(\alpha_{1s}, \widetilde{S}, B_s)] = L[\widetilde{s}(\alpha_{1s} + \delta\alpha_{1s}, \widetilde{S}, B_s)]$, $\alpha_1 . \in I_{se}$, $\alpha_{1s} + \delta\alpha_{1s} \in \widetilde{I}_{se}$ is called invariability intersplitor or equidimensionality field of an independent subsplitor. The initial and final points delimiting the invariability intersplitor are called critical points. It can be proved that the intersplitor I_{se} is a subsplitor. Thus, using the method of complete enumeration, a set of connected subsplitors are folowing: $D\widetilde{s}^\circ_1$, $D\widetilde{s}^\circ_2$, ..., $D\widetilde{s}^\circ_h$, defined by two critical succesive

points. Similarly, subsplitors of equidimensionality $D^f_{\tilde{S}_1}$, $D^f_{\tilde{S}_2}$, ..., $D^f_{\tilde{S}_h}$ $(h \leqslant n)$ characterized by:

$$L(\tilde{S}, \beta_{ms}, B_s) = L(\tilde{S}, \beta_{ms} + \delta\beta_{ms}, B_s) = L_i,$$

$$\beta_{ms} \in D^f_{\tilde{S}_i}, \ \beta_{ms} + \delta\beta_{ms} \in D^f_{\tilde{S}_i}, \ i = 1, 2, ..., h.$$

are obtained.

2.11. Coloured Splitor

A coloured splitor (connected or conditioned) is a splitor whose components $\bar{V}_i$ are characterized by a code q_i ("colour"), namely:

$$\tilde{S}^c_q = \{(\bar{V}_i, q_i)\} \text{ respectively,} \tag{34}$$

$$\tilde{S}^c_q = \{|V_i|, q_i\}, \text{ where } q_i \in \{1, 2, ..., n_q\}$$

$$i = 1, 2, ..., n; \ n_q \leqslant n.$$

If a matrix $\|\gamma_{qi, qj}\|$ of $n_1 \times n_q$ size is attached to a coloured splitor, then a splitor with transitions is obtained. The elements of this matrix $\gamma_{qi,qj}$ represents a similarity degree between vector of "colour q_i" and vector of "colour q_j".

Obviously, the maximum degree of similarity is achieved when $q_i = q_j$. The operative length of a coloured splitor is:

$$B_{i_1 i_2 ... i_h}(\tilde{S}^c_q) = \Sigma |\bar{V}_i|. \tag{35}$$

$$i/[(q_i = i_1) \vee ... \vee (q_i = i_h), \ h \leqslant n].$$

The coloured connected or independent subsplitor is defined, and also 2.9. and respectively 2.10.

2.12. Independent Coloured Stochastic Subsplitors

An independent coloured stochastic subsplitor is the conditioned coloured subsplitor, whose modulus is a random variable, each colour being achieved with a certain probability.

Obviously, we can define similarly, the *coloured stochastic splitor* and the *coloured stochastic multisplitor.*

2.13. Coloured Fuzzy-Splitors.

The coloured fuzzy-splitors are those coloured splitors which perform at least one of the following conditions:

1. For each component V_i it is attached a membership degree μ_{ij} to colour j;

2. According to fuzzy concepts, relations (6) are satisfied and therefore the difference between pseudosplitor and splitor is no longer strict;

3. From a fuzzy point of view, the modulus $|\overline{V}_i|$ is equal to a certain given value (constant);

4. According to a fuzzy meaning the initial point of application α_i (or final, point β_i) is equal to a given value a_i (respectively b_i).

A set of q coloured fuzzy splitors, with N components, represents a *coloured fuzzy multisplitor.*

2.14. Bidimensional Directed Connected Pseudosplitor

A bidimensional directed connected pseudosplitor is a set of surfaces (parallelograms) generated by two pairs of connected vectors having a positive projection and a common application point, namely:

$$\widetilde{PSB} = \{[(a_i, b_i), (\overline{V}_{i1}, \overline{V}_{i2})\}, \; i = 1, 2, ..., n. \tag{36}$$

The operative length of this pseudosplitor is

$$B(\widetilde{PSB}) = \sum_{i=1}^{n} |\overline{V}_{i1} \times \overline{V}_{i2}| = \sum_{i=1}^{n} |V_{i1}| \cdot |\overline{V}_{i2}| \, sin \, (\overline{V}_{i1}, \overline{V}_{i2}).$$

The surfaces (parallelograms) generated by V_{i1} and $\overline{V}_{i2}$ are called components of the bidimensional pseudosplitor $\widetilde{PSB}$. If the operative length of intersection of these components is nought, then a bidimensional splitor $\widetilde{SB}$ is obtained.

Similarly, we can define the bidimensional directed independent splitors, bidimensional multisplitors, etc. We can also extend this notion to m-dimensional spaces.

Remark

If we do not take into consideration the sense, we got the *undirected splitorial* concepts, unidirected connected pseudosplitor, undirected connected splitor, chain undirected connected splitor, etc.

3. OPERATIONS WITH SPLITORS

3.1. Splitorial Reunion of More Connected Splitors

This splitorial reunion of m-component splitors $\widetilde{S}_1, \widetilde{S}_2, ..., \widetilde{S}_m (m \geqslant 2)$ directed to a positive direction is a splitor $\widetilde{SE}$ of a minimum operative length which forms an extrasplitor for each splitor $\widetilde{S}_i$, namely:

$$\widetilde{SE} = \widetilde{S}_1 \cup \widetilde{S}_2 \cup \ ... \ \cup \widetilde{S}_m \supseteq \widetilde{S}_i, (\forall) \; i, \; i = 1, 2, ..., m \tag{37}$$

3.2. **The Splitorial Intersection** of m-connected splitors $\widetilde{S}_1, \widetilde{S}_2 ... \widetilde{S}_m$, $(m \geqslant 2)$ directed to a positive direction, is a splitor $\widetilde{SI}$ of a maximum operative length,

which forms an intersplitor for each splitor $\widetilde{S}_i$, namely:

$$\widetilde{S}I = \widetilde{S}_1 \cap S_2 \cap \ldots \cap \widetilde{S}_m \subseteq \widetilde{S}_i, \ (\forall) i \ i = 1, 2 \ldots m. \tag{38}$$

It was proved that the third form $(\widetilde{S}, \cup, \cap)$ is a lattix.

3.3. Sum of Two Subsplitors on a Given Splitor

The sum of two independent subsplitors on a given splitor $\widetilde{S}$ is an independent subsplitor having the operative length equal to the algebraic sum of two splitors operative length which are added, namely:

$$s_1(\widetilde{S}, B_{1s}) + s_2(\widetilde{S}, B_{2s}) = s(\widetilde{S}, \omega_1 B_1 + \omega_2 B_{2s}) \tag{39}$$

where $\omega_i = +1$ if the direction of subsplitor s_i coincides to the direction of subsplitor $\widetilde{S}$ and $\omega_i = -1$ on the contrary case.

This operation is true if:

$$|\omega_1 B_{1s} + \omega_2 B_{2s}| \leqslant B(\widetilde{S}). \tag{40}$$

If $B(\widetilde{S}) = \infty$ and B_{is} are finite, then the sum of two independent subsplitor is associative, commutative and completely defined.

The neutral element of this operation is an independent subsplitor of a nought operative length, namely:

$$s_e = s(\widetilde{S}, \ 0) \tag{41}$$

For every given independent subsplitor $s(\widetilde{S}, \ B_s)$ there is an opposite subsplitor $-s$ having the form: $-s \ (\widetilde{S}_1, B_s) = s(\widetilde{S}_1, -B_s)$.

In conclusion, the multitude of the independent subsplitors of finite operative length sliding on the splitor $\widetilde{S}$ of an infinite operative length, together with the operation "sum", form a group.

3.4. Sum of a Connected Subsplitor and an Independent Subsplitor on a Given Splitor

is a connected subsplitor having as an operative length the sum of the operative lengths of two subsplitors which is also the initial point, namely:

$$\begin{aligned} \widetilde{s}(\widetilde{S}, \alpha_{1s}, B_{1s}) + s(\widetilde{S}, B_{2s}) &= [s_1(\widetilde{S}, B_{1s}) + s_2 \ (\widetilde{S}, \ B_{2s})] \, \alpha_{1s} = \\ &= \widetilde{s}(S, \ \widetilde{\alpha_{1i}}, \ B_{1s} + B_{2s}). \end{aligned} \tag{42}$$

The sum of a connected subsplitor and an independent subsplitor on a given splitor $\widetilde{S}$ is a law of external composition. When the given splitor $\widetilde{S}$ is repeated the construction algorithm for this sum is relatively simple, as by applying the theory of m-modulo rest classes, $(m = \alpha_{i+1} - \alpha_i)$, we can avoid the difficult computations (4).

3.5. **Independent Primal Sum between a Connected Primal Multisplitor $M\tilde{s}P$ and a Conditioned Splitor S_k** is a connected multisplitor whose component subsplitors are obtained by adding the subsplitors $M\tilde{s}P$ to the independent multisplitor made up of the dual conditioned splitors S_k, namely:

$$M\tilde{s}P + S_k = \{\tilde{s}_i(\alpha^i_{1s},\ B_{1s}) + \hat{S}_k\} = \{[s_i(\alpha^i_{1s},\ B_{1s}) + (\alpha^i_k, s^i_k)]\} \tag{43}$$

so that the length of every resulted subsplitor to be minimum. We see that by computation, the sum has no sense. The number of component splitors must be equal to the number of S_k components. The following construction algorithm of this sum results:

$$\begin{aligned}
&\alpha^1_{2s} = max\ (\alpha_0,\ \alpha^1_{1s} + B^1_{1s});\ \beta^1_{2s} = \alpha^1_{2s} + s^1_k\\
&\alpha^2_{2s} = max\ (\beta^1_{2s},\ \alpha^2_{1s} + B^2_{2s});\ \beta^2_{2s} = \alpha^2_2 + s^2_k.\\
&\qquad\qquad\qquad

Thus, the connected splitor represents a succession of time intervals (α_i, β_i) $i = 1, 2, \ldots n$ in which the performer is working, intervals alternated by periods $(\beta_i \alpha_{i+1})$ in which he is not working [5].

The application initial point of a splitor represents the start of an activity, while the final point represents the end of an activity. The orientation of the splitor is chosen according to time direction. The splitors oriented to a direction opposite to the lapse of time (negative splitors) can be used in "reverse" calculations (from the final to the initial term). In case no such calculations appear, one may operate with undirect concepts.

The operative length of a splitor may represent the effectively worked total time. The operative length of the internal conjugated splitor may represent either the unworked or expectation time. The length of a splitor generally represents the calendar time. Splitorial density can be interpreted as the performer's activity degree. The "fuzzy splitors" represent intermediate states between the "worked" time at an optimum rate (100) and the "unworked" time (nought rate).

The components of the coloured splitors represent various activities made on certain jobs. A certain code (respectively "colour") is attached to each job. The pseudosplitor reflects the performer's possibility of carrying out simulataneous processings. Generally, due to a training and selftaining process, the splitors of an activities succesion are turned into a pseudosplitor [5]. At the begining, the simultaneous intervention of several performers in processing a job (or in assembling some jobs). The splitorial reunion represents the time when processings are operated on a set or subset (at least one processing).

Generally, the operations with splitors model most of the scheduling problems. The "independent" operations permit the modelling of those processes in which breaks between activities may occur (other breaks as those allowed by a working regime in shifts). The connected operations model the processes, where the continuity between jobs-operations must be ensured.

The sum of two splitors on a given splitor can be used in analysing the critical path in the case of a differential regime of shifts. This problem was approached in a previous paper [4]. We noticed that by using the "sum" of two splitors on a given splitor (an operation generalizing in fact the "sum" operation used in the common sense of the word), the algorithm used for analysing the time in a network graph is not changed from a formal point of view.

Special advantages are obtained in case the working regime is a repeatitive splitor (which is frequently met in practics).

In case of a flowshop problem, the independent primal sum between a connected primal multisplitor and a conditioned splitor MS_k can be used. The component v_{ih} of MS_k represent the time of processing job i on installation h, where $v_{ih} > 0$. If the succesion of conditioned splitors $S_{k1}, S_{k2}, \ldots, S_{kn}$ is changing, then the value of another performance function CT (for example total cost) is changing too. In most cases, for each scheduling problem there is a set of values of parameters characterizing the production process for which the performance function expresses through the total cost, should be constant, namely:

$CT = c$, where c is a constant value. This relation represents an indifference variety in a multidimensional space. If $P(|CT - c) \leqslant \varepsilon) \geqslant \eta$, where ε is the given feasible error and η is the given risk coefficient, then we obtain

an indifference practical domain having the property that the performance is approximativelly constant when the sequence of jobs is changing. If for a given problem, the distance between the point defined by the parameters of the scheduling problem dwelt upon and the indifference practical domain is small, then the scheduling problem can be analysed using a Monte-Carlo simulation method. To this end, the random sequence are generated (RANDOM strategy), on whose basis the multisplitors MS_k^i results and the sum between $M\tilde{s}P$ and MS_k^i is performing.

The performance CT_j is calculated for each sequence j. *If* v_{ih} is a random variable upon N cycles of simulation, the average performance CT is estimated. Let $CT_{min} = \overline{CT}(1 - \varepsilon_1)$ and $CT_{max} = CT\ (1 + \varepsilon_2)$ where ε_1 and ε_2 are given feassible errors. Let β be a given risk coefficient. If $P(CT_{min} \leqslant \overline{CT} \leqslant CT_{max}) \geqslant \beta$, then we choose the sequence h having the property $\min\limits_j CT_j = CT_h$.

If $P\ (CT_{min} \leqslant \overline{CT} \leqslant CT_{max}) < \beta$, a simple heuristic algorithm can be applied to finding the succesion of lanching the jobs which should represent a suboptimal solution for the performance function. Here is a brief presentation of the steps of this algorithm.

Step 1: we perform only the sum:
$M\tilde{s}P + S_k^i = M\tilde{s}P(j), j = 1, 2, ..., n;\ n =$ number of jobs.

Step 2: we choose $\min CT_i = CT_h$. Job r_h is thus resulting and lunched in production.

Step 3: we perform the sum $M\tilde{s}P + S_k^h$.

Step 4: The conditioned splitor S_k^h is eliminated from the set of the conditioned splitors.

The algorithm is repeated from step 1 till all jobs are exhausted. The performance obtained from the sequence when applying this algorithm is compared to the best performance obtained through RANDOM strategy. If through this algorithm, a significantly better performance was obtained, the above algorithm can be implemented. Otherwise, it is necessary to try another algorithms, to chose the most adequate one, on the basis of suboptimal values analysis of the performance function.

In case of a scheduling problem with imposed terms t_i for the delivery of each job, the above algorithm can be relatively simple adapted. To this end, the difference is performed: $M\tilde{s}P - S_k^i = M\tilde{s}P_f(j)$ where the final point β_0 is the imposed term. This calculus is similar to the "backwards calculus" in the critical path analysis.

The association of the splitorial concepts with the graph theory, mathematical simulation, industrial dynamic techniques and other well known instruments open up wider prospects for an industrial production modelling.

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CONCEPTS SPLITORIAUX UTILISES DANS LA PROGRAMMATION OPERATIVE DE LA PRODUCTION

On présente les principales définitions, propriétés, relations et lois de la composition concernant les concepts sur le changement de certaines grandeurs (temporelles, spatiales, etc.), que l'on a nommé concepts splitoriaux. La dénomination a été suggérée par la terminologie de certains programmes standards de la programmation opérative de la production.

Dans la partie finale de l'article, on met en évidence la correspondance qui existe entre les concepts définis et les phénomènes intervenant dans la programmation, opérative de la production, ainsi que la manière dont on peut effectuer les différentes applications pratiques.

СПЛИТОРЯЛЬНЫЕ ПОНЯТИЯ, ИСПОЛЬЗОВАННЫЕ В ОПЕРАТИВНОМ ПРОГРАММИРОВАНИИ ПРОИЗВОДСТВА

В статье представлены главные композиционные определения, свойства, отношения и законы, относящиеся к понятиям, связанным с изменением некоторых величин (временных, пространственных и т.д.) и названным сплиторяльными понятиями. Это название было внушено терминологиеи, использованной в некоторых стандартных программах касающихся оперативного программирования производства.

В конце статьи показано ссотношение существующее между выше определёнными понятиями и явлениями, происходящими в оперативном программировании производства, а также показан способ реализации различных практических приложений.

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