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Distribution of Optimum in Stochastic Linear Programming

In the economic models of production-inventory [17], of micro-economic or macroeconomic planning [13], [18], where the input elements — prices, specific consumptions, available resources — are usually non-negative random variables, the utilization of stochastic programming methods leads to making the decisions corresponding to the real states.

The paper presents two methods for calculating and well bounding the cumulative distribution function of the optimum (profit, production cost).

At the same time, certain results got with the aid of the computer for testing the presented algorithms, and possibilities of economic interpreting the results, are shown.

1. INTRODUCTION

The linear programming hypothesis that the coefficients are constant is sometimes not proved into the practice. In macroeconomic models or even in certain microeconomic ones, to point out the random character of linear programming coefficients, means to make the decisions corresponding to the real state.

From a practical point of view, the problem is not complicated, because if the density function of the random coefficients are not known, they can be evaluated according to a selection from the model's random variables, using the techniques of mathematical statistics. It is obvious that these density functions hold a main place in stochastic programming because they induce a density function for the optimum of the linear program and at the same time, a density function for the vector of the optimal solution.

2. DISTRIBUTION PROBLEMS

Let the linear program be written under a matrix form:

$$\begin{cases} Ax \leqslant b \\ x \geqslant 0 \\ max\ z(x) = c'x \end{cases} \tag{1}$$

where A is a real matrix $(m \times n)$, c and x are real vectors $(n \times 1)$ and b is a real vector $(m \times 1)$.

$$\text{Let } X = \{x \mid Ax \leqslant b,\ x \geqslant 0\} \tag{2}$$

be the set of feasible solutions of the linear program (1).

We suppose that vector $\theta = (A, b, c)$ is a random vector having random variable components defined on the probability space (Ω, K, P). By using these denominations, (1) and (2) become:

$$\begin{cases} A(\omega)\ x \leqslant b(\omega) \\ x \geqslant 0 \\ max\ c'(\omega)\ x \end{cases} \tag{1'}$$

$$X(\omega) = \{x \mid A(\omega)\ x \leqslant b(\omega),\quad x \geqslant 0\},\quad \text{for } \omega \in \Omega. \tag{2'}$$

If for every elementary event $\omega \in \Omega$ problem (1′) has a finite solution and if $\gamma(\omega)$ is the optimal value of the objective function and $x^*(\omega)$ is the optimal solution, then we can show that these are still random variables defined on (Ω, K, P).

Definition 1. If $\gamma(\omega) = \underset{x \in X(\omega)}{max}\ c'(\omega)\ x$ is a random variable, then the stochastic linear program (1′) has an optimal solution.

In this case, $\gamma(\omega)$ being a random variable, we have to find out its characteristcs: mean value, variance, probability to take values into a given interval, etc.

Definition 2. We shall call distribution problems in stochastic linear programming those problems having as an aim the statement of the density function (or cumulative distribution function) of the random variable $\gamma(\omega)$, as well as its several moments [2], [3], [4], [6], [7].

Obviously, if the random variables appear only in the vector c and the set X is non-empty and bounded, then program (1′) has always an optimal solution. But, in the general case, random variables appear also in the matrix A and (or) in the vector b; under these conditions, $X(\omega)$ can be the empty set or (and) $\gamma(\omega)$ can be non-bounded. Hence, the space is truncated (not for any ω (1′) has a solution) and to know the cumulative distribution function of vector $\theta = (A, b, c)$ is not enough for stating the cumulative distribution function of the random variable $\gamma(\omega)$ or of the random vector $x^*(\omega)$.

At the same time, Tintner [7] has showed that if the elements of vector θ are normal random variables, then $\gamma(\omega)$ and $x^*(\omega)$ are not always normally distributed; and Prekopa [12] has stated the conditions under which $\gamma(\omega)$ has an asymptotic normal distribution.

Although the main problem is to find out the cumulative distribution function of the random variable $\gamma(\omega)$ or its moments, in certain cases, when this is not possible because of problem nature, we shall only evaluate certain limits (bounds) for the cumulative distribution function of the variable $\gamma(\omega)$, bounds which must to be as sharp as possible and valid for a wider class of random variables.

Further on, we shall deal with the following groups of problems:

(A) Calculating the cumulative distribution function of $\gamma(\omega)$ — in the case of linear parametric programs;

(B) Estimating certain non-parametric bounds for the cumulative distribution function of $\gamma(\omega)$, when $\theta = (A, b, c)$ has components belonging to D.F.R. (Decreasing Failure Rate) class.

3. PARAMETRIC LINEAR PROGRAMS

3.1. *One-dimensional Case*

(a) The most simple case of one-dimensional parametric linear program is that one when vector c linearly depends on a random variable ξ and therefore $c = c^0 + \xi c^1$, where:

$$c = (c_1, c_2, \ldots, c_n).$$

This type of one-dimensional stochastic program is treated by B.Bereanu [2]. It is solved by taking into account the random variable ξ (having F as cumulative distribution function) as a parameter. Let (α, β) be the variation interval of ξ and let $\lambda_0, \lambda_1, \ldots, \lambda_p$ be the $p+1$ characteristic values of the parameter in this interval (with $\lambda_0 = \alpha$, $\lambda_p = \beta$).

Denoting by $x^0, x^1, \ldots, x^p$ the corresponding optimal solutions, by $s_0, s_1, \ldots, s_p$ the corresponding optimums and by

$$W_j(z) = \frac{z - c^{0\prime} x^j}{c^{1\prime} x^j} \quad (j = 1, \ldots, p)$$

we have the following:

Theorem. If G is the cumulative distribution function of the random variable $\gamma(\xi) = max\ (c^0 + \xi c^1)'\ x$, then:

$$G(z) = \sum_{j=1}^{p} H_j(z)$$

where H_j, $j = 1, \ldots p$ are given by the following expressions:

$$H_j(z) = \begin{cases} 0, & \text{if } \begin{cases} c^{1\prime} x^j > 0 \text{ and } W_j(z) \leqslant \lambda_{j-1} \\ c^{1\prime} x^j < 0 \text{ and } W_j(z) \geqslant \lambda_j \\ c^{1\prime} x_j = 0 \text{ and } c^{0\prime} x^j \geqslant z \end{cases} \\ F(\lambda_j) - F(\lambda_{j-1}), & \text{if } \begin{cases} c^{1\prime} x_j > 0 \text{ and } W_j(z) \geqslant \lambda_j \\ c^{1\prime} x^j < 0 \text{ and } W_j(z) \leqslant \lambda_{j-1} \\ c^{1\prime} x^j = 0 \text{ and } c^{0\prime} x^j < z \end{cases} \\ F(W_j(z)) - F(\lambda_{j-1}) & \text{if } \lambda_{j-1} < W_j(z) < \lambda_j \text{ and } c^{1\prime} x^j > 0 \\ F(\lambda_j) - F(W_j(z)) & \text{if } \lambda_{j-1} < W_j(z) < \lambda_j \text{ and } c^{1\prime} x^j < 0. \end{cases} \tag{3}$$

Remarks

1. A similar theorem is got when vector b is random, linearly depending on a single random variable. Set $X(\omega) = \{x \in R^n \mid x \geqslant 0,\ Ax = b^0 + \xi(\omega)\, b^1\}$ is supposed to be bounded and non-empty for any $\omega \in \Omega$.

2. In certain cases it is usefull to calculate the probability of the optimal solution got for some particular values of the coefficients.

In the one—dimensional case with random variables in vector c, we can easily join to each optimal solution a probability, namely:

$$X^* = \begin{pmatrix} x^1 & x^2 \dots x^p \\ p_1 & p_2 \cdots p_p \end{pmatrix} \tag{4}$$

where p_i is the probability for x^i to be the optimal solution and therefore $p_i = F(\lambda_i) - F(\lambda_{i-1})$, and x^i is the optimal solution corresponding to interval $[\lambda_{i-1},\ \lambda)]$.

3.2. *Multi-dimensional Case*

When vector b or (and) vector c linearly depends on more random variables $\xi_1,\ \xi_2, \dots\ \xi_s$ defined on $(\Omega,\ K,\ P)$, the approach of a distribution problem, similar to that one in the previous paragraph becomes practically impossible.

For instance, in the case of two random variables $\xi_1,\ \xi_2$ appearing only in vector c, therefore:

$$c = c^0 + c^1\xi_1 + c^2\xi_2$$

and each ξ_i $(i = 1, 2)$ has a variation domain $[\alpha_i, \beta_i]$, the common variation domain $T = [\alpha_1,\ \beta_1] \times [\alpha_2,\ \beta_2]$ has to be divided into convex areas $[T_1, \dots, T_m]$, where:

$T_j = \{(t_1, t_2) \in T \mid x^j(t_1, t_2)$ is the optimal solution corresponding to base $B_j\}$.

The expression of the cumulative distribution function of the random variable

$$\gamma = \max_{x \in X} c'x$$

is:

$$G(z) = \sum_{j=1}^{m} \int_{S_j(z)} \mathrm{d}\, f(t)$$

where F is the cumulative distribution function of the random vector $\xi = (\xi_1,\ \xi_2)$ and $S_j(z) = \{(t_1, t_2) \in T_j \mid \gamma(t_1, t_2) < z\}$.

Although T_j are plane convex polygons, the evaluation of the "area" of $S_j(z)$ is very difficult.

The succes of parametric techniques in the one-dimensional case has lead to getting the cumulative distribution function G in the case of more random variables using numerical methods for computing the integrals, namely the Cartesian integration method [4].

Denoting by: $\xi = (\xi_1, \dots, \xi_s)$ the random vector and by $[\alpha, \beta]$ its variation domain in R^s;

by: $F(u_1, \dots\ u_s)$ its cumulative distribution function and $f(u_1, \dots, u_s)$ its density function;

by: $\xi^r = (\xi_1, ..., \xi_r)\ r < s$; f_r the marginal density function of the vector ξ^r;

by $F\gamma(z|\, u_1 ..., u_r)$ the conditional cumulative function of the optimal value $\gamma(\omega)$ for $\xi^r = (\xi_1, ..., \xi_r)$; taking into account the following conditions:

(a) $[\alpha, \beta]$ is a compact domain in R^S, the vector ξ is absolutely continuous and f is a continuous density function;

(b) the conditions for the existence of the optimal solution are satisfied [4];

(c) the elements $A(\omega)$, $b(\omega)$, $c(\omega)$ of the linear program are linear functions of the components of ξ; then we have the following:

Theorem. If $A_i^{(n)}$ and $t_i^{(n)}$, $i = 1, 2, ..., n$ are the coefficients and nodes of Gauss quadrature formulae with n nodes in $[-1, 1]$. Then we have the approximation:

$$F_\gamma(s) \simeq M \sum_{i=1}^{n} ... \sum_{p=1}^{n} A_i^{(n)} ... A_p^{(n)} F\gamma(z|\, x_{1i}^{(n)}, ... x_{rp}^{(n)}) \cdot f(x_{1i}^{(n)}, ..., x_{rp}^{(n)}) \qquad (5)$$

where:

$$M = \prod_{k=1}^{r} (\beta_k - \alpha_k) \qquad (6)$$

and

$$x_{k1}^{(n)} = 1/2\ (\beta_k - \alpha_k)\ t_1^{(n)} + 1/2\ (\alpha_k + \beta_k), \quad 1 = 1, ... n,\ k = 1, ..., s. \qquad (7)$$

The proof is given in [4].

Remark:

Let $\xi = (\xi_1, ..., \xi_s)$ be the random vector; we separate one of the random variables, let it be ξ_s, that will be further taken into account as a parameter in calculations. The other $(s - 1)$ random variables are iteratively fixed on nodes of Gauss type; as a result we get problems of parametric linear programming that are solved according to the method from 3.1.; then applying formulas (5—7) from the previous theorem, we calculate the cumulative distribution function of the o, timum.

3.3. *Numerical Results*

We have performed a computing program for the cumulative distribution function of the random variable $\gamma(\omega)$ based on numerical integration method presented in 3.2.

The following example was used for program affirmation:

$$max\ Z(x_1, x_2) = (1 + \xi_1 + 2\xi_2)\ x_1 + (-1 + 3\xi_1 + 11\xi_2)\ x_2$$

$$\begin{cases} x_1 + 2x_2 \leqslant 6 \\ 2x_1 + x_2 \leqslant 6. \end{cases}$$

In order to facilitate the direct calculation of the cumulative distribution function G, we suppose that the random variables ξ_1, ξ_2 are uniform in $[0, 1]$.

The cumulative distribution function G of the random variable $\gamma = \max z(x_1, x_2)$ is:

$$G(u) = \begin{cases} 0 & \text{for } u \leqslant 3 \\ \left(\frac{u}{6} - \frac{1}{2}\right)^2 & \text{for } 3 < u \leqslant \frac{39}{10} \\ \frac{1}{156} u^2 - \frac{1}{6} u + \frac{2}{5} & \text{for } \frac{39}{10} < u \leqslant \frac{27}{5} \\ \frac{1}{416} u^2 & \text{for } \frac{27}{5} < u \leqslant 8 \\ \frac{1}{26} (u - 4) & \text{for } 8 < u \leqslant \frac{78}{7} \\ -\frac{49}{51480} u^2 + \frac{128}{2145} u - \frac{389}{1430} & \text{for } \frac{78}{7} < u \leqslant \frac{108}{9} \\ \frac{1}{22}\left(\frac{2u}{3} - 1\right) & \text{for } \frac{108}{9} < u \leqslant 30 \\ 1 - \frac{3}{22}\left(\frac{13}{3} - \frac{u}{9}\right)^2 & \text{for } 30 < u \leqslant 39 \\ 1 & \text{for } 39 < u. \end{cases}$$

We present in table 1 the results obtained both through the quadratic method and the exact values for the cumulative distribution function G.

Kolmogorov's test was applied for testing the difference (in absolute value) between the empirical distribution and the exact (theoretical) one. "Test value" is $\sup_x |F^*_\gamma(x) - F_\gamma(x)|$, selection volume being of 40, where $F^*_\gamma(x)$ is the approximation of the cumulative distribution function and $F_\gamma(x)$ is the theoretical cumulative distribution function. The hypothesis concerning the fact that the difference between the two functions is not important, is proved.

4. NON-PARAMETRIC BOUNDS FOR THE CUMULATIVE DISTRIBUTION FUNCTION OF THE OPTIMUM

Let (1') be the stochastic linear program and let suppose that:

(i) $X(\omega)$ is non-empty and bounded;

(ii) the elements of the vector $\theta = (A, b, c)$ are continuous random variables, mutually independent, belonging to the D.F.R. class.

We denote by F^* the cumulative distribution function of the random variable $\gamma(\omega)$. Further on, we shall present certain results concerning the estimation of some bounds for F^* and some aspects of computing these bounds.

4.1. *Random Variables of D.F.R Class. Definitions, Properties* [1]

Let X be a continuous random variable having the cumulative distribution function $F(x)$ and the density function $f(x)$.

Definition. Failures rate function r for the random variable X' is:

$$r(x) = \frac{f(x)}{\bar{F}(x)}$$

for the value of x for which $F(x) < 1$, where $\bar{F}(x) = 1 - F(x)$.

The function $r(x)$, very frequently used in the theory of reliability is the probability that an equipment of age x to die in the time interval $[x, x + dx]$.

Definition. We say that X belongs to D.F.R.class or to I.F.R. (Increasing Failure Rate) class respectively, if r is a decreasing (respectively increasing) for those $x \geqslant 0$, for which $F(x) < 1$, and if $F(0^-) = 0$.

Examples:

1. *The Exponential Distribution*, having the density function $f(x) = \lambda e^{-\lambda x}$, $x > 0$, $\lambda > 0$, has a constant failure rate $r(x) = \lambda$.

2. *Weibull Distribution*, having the density function $f(x) = \lambda\alpha x^{\alpha-1}e^{-\lambda x^{\alpha}}$ and $\lambda, \alpha > 0$, $x \geqslant 0$, has the failure rate $r(x) = \lambda\alpha x^{\alpha-1}$ belonging to D.F.R. class for $0 < \alpha < 1$ and to the I.F.R. class for $\alpha \geqslant 1$.

4.2. *Bounding Theorems*

Let (1') be the stochastic linear program under the working hypothesis (i) and (ii). We define the following transformation for (1'):

$$y = \lambda x \quad \text{where} \quad \lambda = 1 \Big/ \sum_{j=1}^{n} x_j > 0 \tag{8}$$

for which (1') becomes:

$$\begin{cases} Ay \leqslant \lambda b \\ \sum_{j=1}^{n} y_j = 1 \\ y_j \geqslant 0, \quad j = 1, \ldots, n \\ max\ g(y, \lambda) = \sum_{j=1}^{n} c_j y_j / \lambda. \end{cases} \tag{9}$$

We have the following:

Theorem 1 [10]. If F_u is the cumulative distribution function of the random variable $g(y, \lambda_u)$ (λ_u is a fixed value for λ) ,then:

$$[\bar{F}_u(\mu_0^u)]^{t/\mu_0^u} \leqslant \bar{F}_u(t) \leqslant \mu_0^u e^{-1}/t \quad \text{for} \quad t > \mu_0^u \tag{10}$$

$$\bar{F}_u(\mu_0^u) \leqslant \bar{F}_u(t) \leqslant exp\,(-t/\mu_0^u) \quad \text{for} \quad t < \mu_0^u$$

where μ_0^u is the optimal solution of problem (9), where the random variables are replaced by their mean values, and $\bar{F}_u(t) = 1 - F_u(t)$.

These bounds are sharper than Tchebyshef's bounds [10] which for $t < \mu_0$ are only $\bar{F}_u(t) \geqslant 1 - \sigma_g^2(\mu_0^u - t)^{-2}$ and for D.F.R. distribution we know that $(\sigma_g/\mu_0^u) > 1$, hence Tchebyshef's bound is 0 and this is not sharp. (σ_g^2 is the variance of random variable g).

We notice that bounds (10) are valid only for λ constant.

But, if we consider λ being a discrete random variable (as really happens in the general case, when the random variables of vector $\theta = (A, b, c)$ take values in a large variation domain) and if we take random variables only in vector c, then we get the following bounding theorem:

Theorem 2. If $\lambda = \sum_{j=1}^{n} x_j$ is a discrete random variable having the distribution

$$\lambda: \begin{pmatrix} \lambda_1 & \lambda_2, \ldots, \lambda_s \\ p_1 & p_2, \ldots, p_s \end{pmatrix}$$

where $p_u = P(\lambda = \lambda_u)$, then $F^*(t) = \sum_{u=1}^{s} p_u F_u(t)$, therefore the cumulative distribution function F^* is a discrete mixture of the cumulative distribution functions of the associated problems (9), according to the random variable λ; thus the bounding limits for F^* are

$$\sum_{u=1}^{s} p_u l_s^u \leqslant \bar{F}^*(t) \leqslant \sum_{u=1}^{s} p_u l_d^u \tag{11}$$

where

$$l_s^u = \begin{cases} \bar{F}_u(\mu_0^u)^{t/\mu_0^u}, & \text{if } t \geqslant \mu_0^u \\ \bar{F}_u(\mu_0^u), & \text{if } t < \mu_0 \end{cases}$$

and

$$l_d^u = \begin{cases} \mu_0^u e^{-1}/t, & \text{if } t \geqslant \mu_0^u \\ exp\ (-t/\mu_0^u) & \text{if } t < \mu_0^u. \end{cases}$$

4.3. *Numerical Results*

A program for estimating the limits of the cumulative distribution function of the optimum was carried out. The program was tested for the case when the random variables appearing only in vector c, are Weibull and belong to D.F.R. class.

Elements $\bar{F}_u(\mu_0^u)$ and p_u required for calculating the limits l_s^u and l_d^u and finally in the formulae (11) are estimated with the aid of a selection from the random variables γ and λ; selection is resulted after simulating the random variables of Weibull type .For the following example:

$$\begin{cases} max\ z = (\xi_1 x_1 + \xi_2 x_2 + \xi_3 x_3 + \xi_4 x_4) \\ 5x_1 + 4x_2 + 2x_3 + x_4 \leqslant 10 \\ 5x_2 + 2x_2 + x_3 + 6x_4 \leqslant 8 \\ x_1 + 8x_2 + 4x_3 + 5x_4 \leqslant 20 \\ x_i \geqslant 0 \quad i = 1, \ldots, 4 \end{cases}$$

$$\xi_1 \in W(0;1;0,8);\quad \xi_2 \in W(0;1;0,6);$$
where: $\xi_3 \in W(0;1;0,8);\quad \xi_4 \in W(0;1;0,8)$

the bounding limits for F^* are presented in Table 2.

Weibull random variable $W(\alpha, \lambda, \nu)$ was generated with the aid of the formulae:

$$W = \alpha + \left(\frac{x}{\lambda}\right)^{1/\nu}$$

where x is an exponential random variable of mean value 1.

As we notice in table 2, the bounding limits calculated with the aid of formulae (11) bound the empirical cumulative distribution function got by simulation. The upper (right)limit is much more sharper than the lower (left) one, that is very close to zero.

CONCLUSIONS

The deterministic treatment of an economic problem, modelled with the aid of a linear program of type (1), may lead to "optimal" solutions whose probability of practical achievement is very small. On the other hand, by solving a stochastic model, we get for each solution (for instance, plan structure) its probability to be achieved under the conditions imposed by problem's constraints.

At the same time, the objective function for most cases represent a production cost or a profit; being analysed as a random variable, we can calculate its probability, as for instance the probability that the profit exeeds a certain fixed value, or the probability that the production cost is established between certain values, taking into account the problem constraints.

This analysis is much more usefull for the decision maker, supplying also a risk measure to adopt an uncertain solution.

Table 1

No. crt.	Value of Objective Function	Probability	
		Quadrature	Exact
1	3.1527	0.0	0.01623
2	4.0503	0.03143	0.05155
3	4.9479	0.06244	0.05885
4	5.8456	0.08540	0.08214
5	6.7432	0.10835	0.10930
6	7.6408	0.13131	0.14034
7	8.5385	0.17370	0.17456
8	9.4361	0.21181	0.20908
9	10.3337	0.24893	0.24360
10	11.2313	0.28604	0.27812
11	12.1290	0.30972	0.31173
12	13.0266	0.33114	0.34380
13	13.9242	0.36241	0.37434
14	14.8219	0.40467	0.40334
15	15.7195	0.44526	0.43089

Table 1 (continued)

No. crt.	Value of Objective Function	Probability	
		Quadrature	Exact
16	16.6171	0.46789	0.45809
17	17.5147	0.48873	0.48530
18	18.4124	0.50957	0.51250
19	19.3100	0.53042	0.53970
20	20.2076	0.55156	0.56690
21	21.1053	0.59138	0.59410
22	22.0029	0.63127	0.62130
23	22.9005	0.66185	0.64850
24	23.7981	0.68089	0.67570
25	24.6958	0.69993	0.70290
26	25.5934	0.71897	0.73010
27	26.4910	0.74976	0.75730
28	27.3887	0.78275	0.78450
29	28.2863	0.81574	0.81171
30	29.1839	0.84890	0.83891
31	30.0815	0.86957	0.86610
32	30.9792	0.88997	0.89169
33	31.8768	0.91038	0.91458
34	32.7744	0.93079	0.93475
35	33.6721	0.95119	0.95221
36	34.5697	0.97160	0.96696
37	35.4673	0.98063	0.97899
38	36.3649	0.98708	0.98831
39	37.2626	0.99354	0.99492
40	38.1602	1.00000	0.99881

TEST VALUE 0.020117 (KOLMOGOROV)

Table 2

Left Limit	Distribution Empirical Function (by Simulation)	Right Limit	Value of Objective Function
0.000743	0.222500	0.229595	14.58
0.000502	0.201250	0.214641	15.60
0.000340	0.173750	0.201516	16.61
0.000231	0.165000	0.189903	17.62
0.000157	0.147500	0.179556	18.64
0.000107	0.135000	0.170278	19.66
0.000073	0.127500	0.161912	20.68
0.000050	0.117500	0.154330	21.69
0.000034	0.106250	0.147426	22.71
0.000023	0.102500	0.141113	23.72
0.000016	0.095000	0.135318	24.74
0.000011	0.082500	0.129981	25.76
0.000007	0.078750	0.125049	26.77
0.000005	0.071250	0.120477	27.79
0.000003	0.068750	0.116228	28.80

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REPARTITION DE L'OPTIMUM DANS LA PROGRAMMATION LINEAIRE STOCHASTIQUE

Dans les modèles économiques de stock-produits [17], de planification microéconomique ou macroéconomique [13], [18], dont les éléments d'entrée — prix,consommations spécifiques, disponible de ressources—sont communément des variables aléatoires non-négatives, on utilise les méthodes de la programmation stochastique.

L'auteur présente deux méthodes de calcul et d'encadrement en de limites aussi strictes que possibles de la fonction de répartition de l'optimum (bénéfice, coût de fabrication).

On présente aussi certains résultats obtenus à l'aide de l'ordinateur pour la vérification des algorithmes présentés et les possibilités d'interprétation économique des résultats.

РАСПРЕДЕЛЕНИЕ ОПТИМУМА В СТОХАСТИЧЕСКОМ ЛИНЕЙНОМ ПРОГРАММИРОВАНИИ

В экономических моделях типа запас-производство [17] микро- и макроэкономичекого планирования [13], [18], в которых входные элементы — цены, удельные расходы, количество наличных ресурсов, — являются обычно стохастическими неотрицательными переменными, используют методы стохастического программирования при принятии решений, соответствующих реальной ситуации.

В работе представлены два метода для расчёта и вмещения функции распределения оптимума (прибыли, расходов) в наиболее строгие лимиты.

Показаны, также, некоторые результаты, полученные при помощи ЭВМ, тестирования предлагаемых алгоритмов и существующие возможности экономического истолкования этих результатов.

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