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Optimum Solutions of Water Management for Irrigations

The authors present a linear programming model for calculating the water management in the irrigation networks with a view to achieve ever more efficient improvement works, both as regards the investment and working expenses. At the same time, a model of sizing four water reservoirs which can be carried out within a river basin, is presented.

INTRODUCTION

The designing, building and management of a networks complex of chanals, pipes, pumping stations, water reservoirs, are an important base of a water management system for irrigations and involve a series of technical-economic problems, as follows:

a) what storages and to what extend they will be used in order to irrigate certain surfaces;

b) which are the flows and the volumes within the supply networks by taking into account the availability of water sources, the consumption graph, etc.

The obtained results lead to optimum technical-economic solutions subjected to the calculation constraints. If the obtained designing solution is more economic, its achieving in practice, both as execution and particularly as working, requires a higher training degree for specialists and demands for introducing and using more exact management means, automatic if it's possible.

The solving of the problems concerning the water management within the irrigation systems is based on the calculation in the "water distribution node", representing the basic element for the balancesheet of water volumes or energy quantities. By a "water distribution node" (N.H.) we mean a storage (of any kind) a water source (energy), a pumping station, wells, etc., and generally any point where a balancesheet of inputs and outputs can be settled, as for example that one presented in Fig. 1, satisfied by the relation:

$$\Sigma IN = \Sigma IE \pm \varepsilon \tag{1}$$

where:

IN represents the possible inputs;

IE represents the possible outputs;

ε represents the exceedings for both directions.

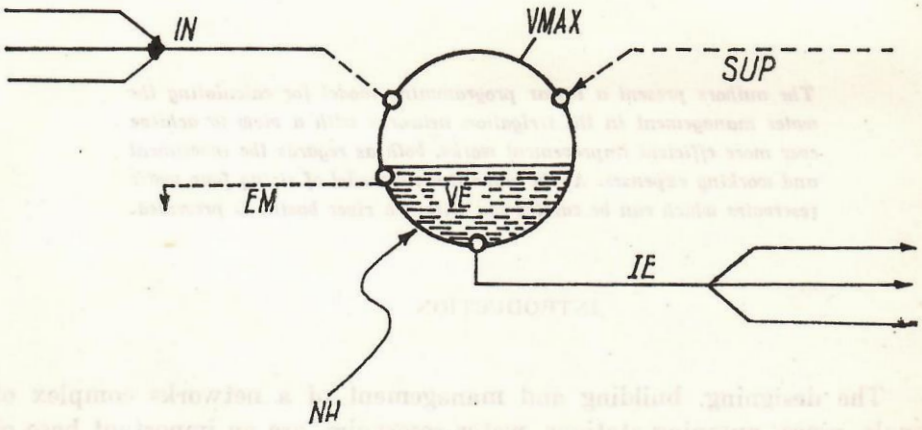

Fig. 1

For the water distribution node in Fig. 1, the balancesheet equation may be:

$$a_i + \sum_{j=1}^{J_i} x_{i,j} + y_i - \sum_{l=1}^{L_i} z_{i,l} - v_i - w_i = 0 \tag{2}$$

$$(i = 1, 2, ..., N)$$

where:

a_i — is the quantity existing in node i at the beginning of balancesheet period;

$x_{i,j}$ — is the water quantity entered by inflow j in node i;

y_i — is the input supplementary quantity for compensating the balancesheet in node i;

$z_{i,l}$ — water quantity come out through sluice l in node i;

v_i — emergency outflows in node i;

w_i — water quantity left in node i;

J_i — number of inputs of water volumes in node i;

L_i — number of outputs of water volumes in node i.

In order to solve the problems specific to water management in the irrigation systems with the aid of linear programming, we can write the following constraints considered from a technical point of view:

of storage

$$c_i \leqslant w_i \leqslant d_i, \qquad (i = 1, 2, ..., N) \tag{3}$$

where c_i, d_i are minimum and maximum of the storage in node i;

of inflow

$$x_{im} \leqslant e_{im}$$

where:

$$m \in J_i'; \quad J_i' \subset \bar{J}_i; \quad \bar{J}_i = \{1, 2, ..., J_i\} \tag{4}$$

$\bar{J}_i$ is the inflows set in node i;

e_{im} is the brought volume; $(i = 1, 2, ..., N)$.

of outflow

$$z_{in} \leqslant f_{in} \tag{5}$$

where:

$$n \subset L_i'; \quad L_i' \subset \bar{L}_i; \quad \bar{L}_i = \{1, 2, ..., L_i\}$$

$\bar{L}_i$ is the outflows set in node i;

f_{in} is the outflowed volume; $(i = 1, 2, ..., N)$.

System variables are quantities in thousand m³/month which enter $(x_{i,j})$ and come out $(z_{i,j})$ in nodes from 1 to N of a system, and also the above shown values (2) denoted by y, v and w.

Under these conditions the objective function will be:

$$FO = \sum_{i=1}^{N} \left(\sum_{j=1}^{J_i} x_{ij} K_J^{\circ} + \sum_{l=1}^{L_i} z_{il} K_l' + (y_i + v_i)\, K_i'' + w_i K_i''' \right) = min \tag{6}$$

where $K^{(\circ,','',''')}$ is the complex cost:

$$K = (I + T \cdot C)/Q \tag{7}$$

where

I — investment for the work (thousand lei);

T — normalized time of work recovery (years);

C — yearly expenses of maintaining and managing the works (mil. lei/year);

Q — medium capacity of transport or storage; (thousand m.c./year) for which investment I and expenses C were calculated and indices of K are the complex costs:

K^0 — for inflows

K' — for outflows

K'' — for emergencies

K''' — for storage reservoirs.

For the analysed cases, because of the bounded intervals of values, we may consider, without doing too great mistakes, a linear dependence between the investment and volumes (5).

EXAMPLE

It is necessary to settle the surfaces which can be irrigated in a droughty year from an irrigation network of three water aqueducts by pumping and for storages from the river considered as interconnected to both directions.

We shall also consider the up-stream inflow, as well as the ensurance of the down stream water demands (Fig. 2).

As system variables, there were considered all the connections existing between storages and consumers denoted by $x_1 \ldots x_{27}$. Because of programming, all variables x, y, z, v, w, in the theoretic model, were denoted by X, starting from X_1 to the maximum number of variables.

The constraints system is:

I. **Balancesheet of storages**

a) storage BOT

$$1) \quad -x_4 + x_6 - x_{10} + x_{11} - x_{15} + x_{19} + x_{28} - x_{32} = a;$$

b) storage DRAG

$$2) \quad x_7 - x_{11} + x_{12} - x_{16} + x_{20} + x_{29} = b;$$

c) storage NAI

$$3) \quad -x_2 + x_8 - x_{12} + x_{13} + x_{16} - x_{17} + x_{30} = c;$$

d) storage HULUB

$$4) \quad -x_5 + x_9 - x_{14} + x_{17} + x_{23} + x_{27} + x_{31} = d;$$

where a, b, c and d are the water quantities existing in the storages considered at the beginning of calculation period.

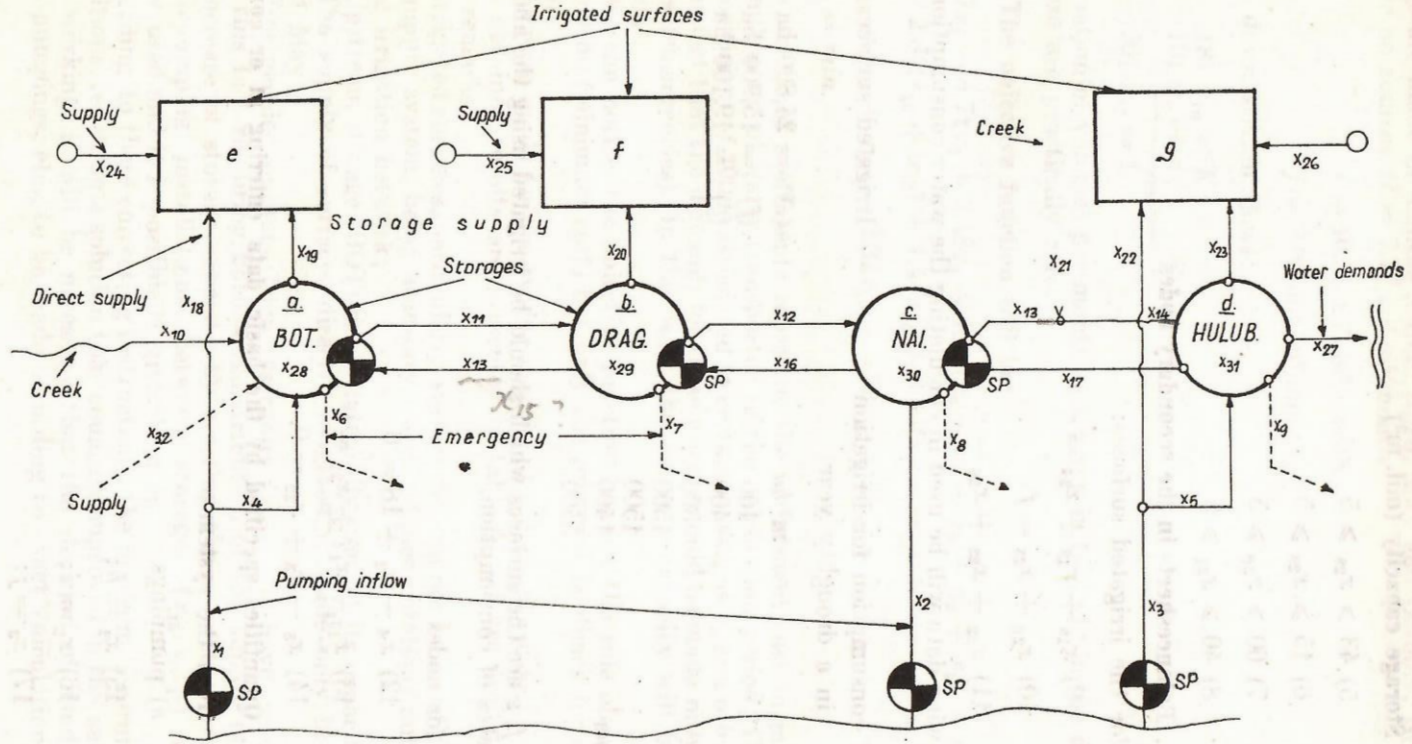
Irrigated surfaces
Supply
x_{24}
e
Supply
x_{25}
f
Creek
g
x_{26}
Storage supply
Direct supply
x_{18}
x_{10}
Creek
x_{19}
Storages
x_{20}
x_{21}
x_{22}
x_{23}
Water demands
a.
BOT.
x_{28}
x_{11}
x_{13}
b.
DRAG.
x_{29}
SP
x_{12}
x_{16}
c.
NAI.
x_{30}
SP
x_{13}
x_{17}
x_{14}
d.
HULUB.
x_{31}
x_{27}
x_{32}
Supply
x_4
x_6
Emergency
x_7
x_8
x_9
x_5
Pumping inflow
x_1
SP
x_2
SP
x_3
SP

Fig. 2

II. **Storage capacity** (mil. m^3)

5) $43 \geqslant x_{28} \geqslant 5$

6) $15 \geqslant x_{29} \geqslant 5$

7) $60 \geqslant x_{30} \geqslant 5$

8) $40 \geqslant x_{31} \geqslant 5$

III. **Balancesheets in the secondary nodes**

a) for the irrigated surfaces:

9) $x_{18} + x_{19} + x_{24} = e$

10) $x_{20} + x_{25} = f$

11) $x_{22} + x_{23} + x_{26} = g.$

The following data will be used in calculating the water consumption for irr gations:

Rates of consumption for irrigation in a droughty year

month	*m^3/ha*
April	100
May	400
June	500
July	1500
August	1500
September	1400
Total	5400

Irrigated surfaces

(e) = 24,800 ha
(f) = 15,900 ha
(g) = 19,100 ha

where e, f, g are the surfaces which should be irrigated using the above me tioned rates of consumption.

b) for nodes

12) $x_4 - x_1 + 18 = 0$

13) $x_{13} + x_{14} + x_{21} = 0$

14) $x_5 - x_3 + x_{22} = 0.$

IV. **Quantities specified by the basic data entering at or coming o from the system**

a) pumpings

15) $x_1 = h;$

16) $x_2 = i;$

17) $x_3 = j;$

Pumpings are made in months March, April, May and October, November, December on sources $H = 7.77$ mil m³/month

$$i = 10.36 \text{ mil. m}^3\text{/month}$$

$$j = 9.06 \text{ mil. m}^3\text{/month.}$$

b) down stream flows:

18) $x_{10} = K$;
19) $x_2 = m$;
20) $x_{27} = 1$.

Variation of volumes per months are given in table 1; in the droughty year, these are practically zero.

V. The objective function will be:

$$9.27x_1 + 4.71x_2 + 3.48x_3 + 100\,(x_6 + x_7 + x_8 + x_9 + x_{15} + x_{16} +$$
$$+ x_{17}) + 2.84(x_{18} + x_{19}) + 1.44x_{20} + 3.44(x_{22} + x_{23}) +$$
$$+ 100(x_{24} + x_{25} + x_{26}) + 2.32x_{28} + 5.53x_{29} + 2x_{30} + 4.32x_{31} +$$
$$+ 100x_{32} = min.$$

The values of coefficients represent the investment cost in million lei for one million m³ of water transported in the calculation period. The exaggerated coefficient of 100 is associated to certain works, as x_6 and others referring to the fact that the demands for these works should be made only in very rare cases (emergencies). In this way, the network capacity will be better pointed out.

As we can notice, the objective function has not the role of supplying an information (minimum cost) but only the role of a feedback for selecting a solution.

Runs were made for the twelve months of a droughty year.

The obtained results are presented in table 1 and are leading to the following remarks:

1. Irrigated surfaces, particularly surface "e" can not be satisfied by the foreseen supply system, being necessary water supplementations outside the considered irrigation network;

2. Upstream storage (BOT) is not integrally used at its capacity (x_{28});

3. The supply of surfaces directly irrigated is made only in months April and May;

4. Emergencies outputs from reservoirs ($x_{6,7,8 \text{ and } 9}$) have considerable volumes, this fact requiring both a redistribution of the supply (x_1, x_2, x_3) and an increase of storage capacity by erecting the respective reservoirs;

5. Repumping installations between storages ($x_{15,\ 16,\ 17}$) are not frequently used and it is possible to give them up.

According to these correcting instructions, the program is rerun for the new conditions, resulting a solution that could be improved in the same way.

For working, it will be necessary that the decision of handling the gateways, pumpings, etc., to be made according to exact calculations, as for

TABLE INCLUDING THE RESULTS OF OPTIMIZATION THE WATER NETWORK FOR 12 MONTHS OF A DROUGTY YEAR

SPECIFICATION	VARIABLE	MONTHS											
		X	XI	XII	I	II	III	IV	V	VI	VII	VIII	IX
AC BOTO	28	5,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0
ORAG	29	5,0	5,0	12,7	11,4	13,3	15,0	15,0	8,6	5,0	5,0	5,0	5,0
NAI	30	38,0	56,0	60,0	60,0	60,0	60,0	60,0	60,0	55,7	26,1	5,0	5,0
HULU	31	17,0	25,5	40,0	40,0	40,0	40,0	40,0	40,0	29,1	5,0	5,0	5,0
	1	7,77	7,77	-	-	-	7,77	7,77	7,77	-	-	-	-
	4	7,77	7,77	7,77	-	-	7,77	5,29	-	-	-	-	-
	18	-	-	-	-	-	-	2,48	7,77	-	-	-	-
	2	10,36	10,36	10,36	-	-	10,36	10,36	10,36	-	-	-	-
	3	9,06	9,06	9,06	-	-	9,06	9,06	9,06	-	-	-	-
	5	9,06	9,06	9,06	-	-	9,06	7,15	4,18	-	-	-	-
IRRIGATIONS	22	-	-	-	-	-	-	1,91	4,88	-	-	-	-
	20	-	-	-	-	-	-	1,58	6,35	7,93	23,8	-	-
	11	12,0	7,77	7,77	-	3,19	16,9	1,58	-	-	-	-	1,29
	15	-	-	-	-	-	-	-	-	-	-	-	1,29
	12	17,0	7,77	-	1,29	1,29	-	-	-	-	-	-	1,29
	16	-	-	-	-	-	-	-	-	4,28	23,8	-	-
	13	-	-	6,69	1,29	1,29	-	-	-	-	5,77	21,1	1,29
	17	-	-	-	-	-	-	-	-	-	-	-	-
	27	1,29	1,29	1,29	1,29	1,29	1,29	1,29	1,29	1,29	1,29	1,29	1,29
EMERGENCY	6	-	-	-	-	-	8,7	4,87	-	-	-	-	-
-"-	7	-	-	-	-	-	-	-	-	-	-	-	-
-"-	8	-	-	-	-	-	10,36	10,36	10,36	-	-	-	-
-"-	9	-	-	-	-	-	7,77	5,86	2,88				-
	24	-	-	-	-	-	-	-	2,15	12,4	37,7	32,2	9,9
	25	-	-	-	-	-	-	-	-	-	-	20,6	6,3
	26	-	-	-	-	-	-	-	-	-	-	4,9	7,6
	10	-	-	-	-	3,19	2,70	11,60	-	-	-	-	-
	21	-	-	-	-	-	-	-	-	-	-	-	-
	32	-	-	-	-	-	-	-	-	-	-	-	-
	14	-	-	6,69	1,29	1,29	-	-	-	-	5,77	21,1	1,29
	19	-	-	-	-	-	-	-	-	-	-	-	-
	23	-	-	-	-	-	-	-	-	26,5	23,5	19,6	-

example those mentioned above: this is necessary because the solutions to be applied in practice are refined, and are hardly to be achieved using the traditional methods of manual calculation or based on "inspiration".

* * *

From all those stated above, a series of aspects for the subsequent development of these calculation methods, are drown:

1. It is possible within the framework of scientific management of irrigation systems, to mathematize the functioning relations according to rigurous technical-economic criteria.

2. The technique of linear programming has a corresponding application field, but it is necessary to introduce the complete automation of the calculations, to exist a program of data preparation and another one of results ordering, all made on FELIX 256 computer, the county computing centres are endowed with.

3. The presented program may be extended to other types of problems, as for instance to settling the program at a pumping station for irrigations etc.

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SOLUTIONS OPTIMALES D'EXPLOITATION DE L'EAU POUR LES IRRIGATIONS

Les auteurs présentent un modèle de programmation linéaire pour le calcul de l'exploitation de l'eau dans les systèmes des irrigations en vue de la réalisation de certains travaux d'amélioration foncière de plus en plus efficaces, tant en ce qui concerne les investissements, que les dépenses pour l'exploitation.

Aussi, on présente un modèle pour établir les dimensions de quatre accumulations d'eau pouvant être exécutées dans le bassin d'un rivière.

ОПТИМАЛЬНЫЕ РЕШЕНИЯ ПО ВОДНОМУ ХОЗЯЙСТВУ ДЛЯ ОРОШЕНИЙ

Авторами статьи приведена модель линейного программирования работы по хозяйствованию воды в оросительных системах с целью осуществления мелиоративных работ с большей эффективностью, как по капитальным вложениям, так и по эксплуатационным затратам. Приведена также модель определения необходимых размеров четырёх водохранилищ, реализуемых в бассейне одной реки.

15. X. 1977.