VIVEK PATKAR
S. P. AGGARWAL

Department of Operational Research
University of Delhi-India

On Parametric Geometric Programming*

Geometric programs are studied where a parameter is present either in the objective function or in the constraints or in both. The solution to such programs is shown to be a continuous function of the parameter.

INTRODUCTION

Geometric programming is a powerful tool for solving a class of highly nonlinear optimization problems occuring in the field of engineering design. The initial development of Duffin et al. [2] has been extended by different authors and this approach can now handle a wider class of problems which was not possible initially e.g. the coefficients may be negative, constraints may be of greater than or equal to type etc.

The problem of post optimality analysis for geometric programs was first studied by Duffin et al. [2] and was extended by Theil [7], while Rijckaert [6] and Phillips [5] considered sensitivity analysis for signomial programs. The geometric programs with variable coefficients were studied by Patkar and Aggarwal [4]. So far the idea of parametric geometric programming has not received any attention and this is precisely the object of this note.

A parametric geometric program is such that a parameter may occur in the objective function or in the constraint set or in both. For parametric linear or parametric linear fractional programming, the solution is a discrete function of the parameter whereas for geometric programs, the solution comes out to be a continuous function of the parameter. The parametric geometric programs with zero degree of difficulty and positive degree of difficulty are studied in the next two sections respectively.

*) Research work supported by U.G.C.

1. A PARAMETRIC GEOMETRIC PROGRAM WITH ZERO DEGREE OF DIFFICULTY.

We consider a geometric program with zero degree of difficulty and the presence of a parameter in any one of the following cases or in all of them.

(a) in the objective function,

(b) in the left hand side of the constraints,

(c) in the right hand side of the constraints.

(Case (c) can be merged with case (b)).

Since the degree of difficulty is zero [2], in all above cases, the dual variables corresponding to such primal program are determined uniquely and independent of the parameter. Then from the duality relations [2], the primal variables are found out as a function of the parameter.

Following example illustrates the method.

Example. We consider a geometric program where a parameter is present in the objective function as well as the constraint set.

$$\text{Minimize } G\, g(t) = (2+\lambda)\, t_1^{\frac{1}{3}} t_2^{\frac{1}{4}} + \left(\frac{1}{8}+\lambda\right) t_1^{\frac{1}{2}} t_2^{\frac{1}{2}}$$

$$\text{subject to } \frac{1}{1+\lambda}\, t_1^{\frac{2}{3}} t_2^{-1} \leqslant 1, \quad t_1 > 0,\ t_2 > 0$$

$$0 \leqslant \lambda < \infty.$$

Corresponding dual geometric program is

$$\text{Maximize } v(\delta) = \left(\frac{2+\lambda}{\delta_1}\right)^{\delta_1} \left(\frac{1+8\lambda}{8\delta_2}\right)^{\delta_2} \left(\frac{1}{(1+\lambda)\delta_3}\right)^{\delta_3} \delta_3^{\delta_3}$$

subject to

$$\delta_1 + \delta_2 = 1$$

$$\frac{1}{3}\delta_1 - \frac{1}{2}\delta_2 - \frac{2}{3}\delta_3 = 0$$

$$\frac{1}{4}\delta_1 + \frac{1}{2}\delta_2 - \delta_3 = 0$$

$$\delta_i \geqslant 0, \quad i = 1, 2, 3$$

$$\text{We have } \delta_1 = \frac{5}{6},\ \delta_2 = \frac{1}{6} \text{ and } \delta_3 = \frac{7}{24}$$

$$\text{and } v = 3 \cdot 2^{\frac{1}{2}} \cdot (2+\lambda)^{\frac{5}{6}} (1+8\lambda)^{\frac{1}{6}} / 5^{\frac{5}{6}} \cdot (1+\lambda)^{\frac{7}{24}}.$$

Then

$$t_1 = 5 \cdot (1 + 3\lambda)/8\,(2 + \lambda)(1 + \lambda)^{\frac{1}{4}}$$

and

$$t_2 = [5 \cdot (1 + 8\lambda)(1 + \lambda)^{\frac{5}{4}}/8 \cdot (2 + \lambda)]^{-\frac{2}{3}}.$$

2. A PARAMETRIC GEOMETRIC PROGRAM WITH POSITIVE DEGREE OF DIFFICULTY

We consider a parametric geometric program having the degree of difficulty not very large. Now the dual variables are expressed in terms of the independent dual variables (depending on the degree of difficulty) and then the substituted dual objective function is maximized over the independent dual variables. Hence the values of the dual variables so determined help in determining the primal variables as function of the parameter.

Following parametric geometric program with one degree of difficulty illustrates the procedure.

Example

$$\text{Minimize } g(t) = t_1^3 t_2^{-2} + 10 t_1^{-1} + 0 \cdot 592 t_1 t_2^{-3}$$

subject to

$$8 \cdot 62 t_1^{-1} t_2^3 \leqslant 1,\ t_1 > 0,\ t_2 > 0$$

$$0 < \lambda < \infty.$$

Corresponding dual geometric program is

$$\text{Maximize } v(\delta) = \left(\frac{\lambda}{\delta_1}\right)^{\delta_1} \left(\frac{10}{\delta_2}\right)^{\delta_2} \left(\frac{0 \cdot 592}{\delta_3}\right)^{\delta_3} \left(\frac{(8 \cdot 62}{\delta_4}\right)^{\delta_4} \cdot \delta_4^{\delta_5}$$

subject to

$$\delta_1 + \delta_2 + \delta_3 = 1$$

$$3\delta_1 - \delta_2 + \delta_3 - \delta_4 = 0$$

$$-2\delta_1 - 3\delta_3 + 3\delta_4 = 0$$

$$\delta_i \geqslant 0,\ i = 1, 2, 3, 4.$$

We choose to solve δ_2, δ_3 and δ_4 in terms of δ_1, then

$$\delta_2 = \frac{7}{3}\delta_1,\ \delta_3 = 1 - \frac{10}{3}\delta_1,\ \text{and}\ \delta_4 = 1 - \frac{8}{3}\delta_1 \cdot$$

On substituting the expression for δ_2, δ_3 and δ_4 into the dual function $v(\delta)$ and maximizing it over $\delta_1 \geqslant 0$, we have

$$\delta_1 = \frac{0 \cdot 5379}{3 + 1 \cdot 793\,\lambda^{0.3}},\quad \delta_2 = \frac{1 \cdot 2551}{3 + 1 \cdot 793\,\lambda^{0.3}}$$

$$\delta_3 = 1 - \frac{1 \cdot 793}{3 + 1 \cdot 793\,\lambda^{0.3}}, \delta_4 = 1 - \frac{1 \cdot 4344}{3 + 1 \cdot 793\,\lambda^{0.3}}$$

and these δ_i' s are optimal for $0 < \lambda < \infty$.

Hence we determine $v(\delta)$ and then by the duality relations [2], t_1, t_2 are expressed as continuous function of λ.

Remark

In linear and linear fractional programming problem, the solution occurs at the extreme point of the feasible region and thus, although the parameter varies continuously, the optimal solution varies discontinuously as one jumps from one extreme point to an adjacent extreme point. However, for geometric programs the concept of extreme point solution is not possible and hence a continuously varying solution is obtained.

Another interesting point is that for parametric geometric programs, the parameter may not necessarily be in linear form but may be in nonlinear form and still above procedure would work.

REFERENCES

1. AGGARWAL, S. P.: *Parametric Linear Fractional Functional Programming.* Metrika, 12 (1968), 106–114.
2. DUFFIN, R. J., PETERSON, E. L., ZENER, C. M.: *Geometric Programming.* John Wiley & Sons, New York, 1967.
3. HADLEY G.: *Linear Programming.* Addison-Wesley Pub. Co., Inc., 1962.
4. PATKAR, V., AGGARWAL, S. P.: *Geometric Programs with Variable Coefficients,* Communicated.
5. PHILLIPS, D. T.: *Sensitivity Analysis in Generalized Polynomial Programming.* AIIE Trans., 6 (1974), 114–119.
6. RIJCKAERT, M. J.: *Sensitivity Analysis in Geometric Programming.* In Mathematical Programs for Activity Analysis. Van Moescke (ed), North Holland, 1974, 61–72.
7. THEIL, H.: *Substitution Effects in Geometric Programming.* Management Science, 19 (1972) 25–30.

CONSIDÉRATIONS CONCERNANT LA PROGRAMMATION GÉOMÉTRIQUE PARAMÉTRIQUE

Cet ouvrage étudie des problèmes de programmation géométrique dans le cas où un paramètre est présent aussi bien dans les fonctions objectives, que dans les restrictions, ou bien dans chacune de celle-ci prise à part. La solution pour de tels programmes est une onction continue du paramètre.

СООБРАЖЕНИЯ ПО ПАРАМЕТРИЧЕСКОМУ ГЕОМЕТРИЧЕСКОМУ ПРОГРАММИРОВАНИЮ

В этой работе исследуются проблемы геометрического программированию в случае, когда один параметр выступает как в целевых, функциях, так и в ограничениях, или в каждом в отдельности. Решением для таких программ является непрерывная функция одного параметра.

1. XII. 1977.