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Determination of Demand Elasticity by the Indices Method*

In the framework of a group of goods or of a product grouping various qualities or articles there is a necessary link between the structure of the quantities and the assortment structure of their prices. This relation can be quantified by means of an elasticity coefficient specially obtained by applying the weighted correlation to the indices method.

Elasticity as a function of price characterizes processes occurring in time. However, in the course of the various time periods, the structures, respectively the correlation systems, change both as regards the proportions between the quantities of products purchased and the proportions between their prices.

Since there is a causal link between the structure of the quantities by which demand is expressed and the structure of their prices, it is necessary to determine elasticity for the changes of the first structure as a function of the changes of the second. In other words, it is a matter of the elasticity of the change in correlation between the quantities purchased, as against the change in the correlation between the prices of the products in the group of goods analyzed.

The characterization of the mean change in the structure of product quantities from a group studied can be made by means of the index of the aggregate physical volume of purchases effected by the consumer, and the characterization of the mean change of price structure by means of the aggregate price index.

The study in time of the reciprocal relations among the quantities of individual goods under the influence of the reciprocal relations among their

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prices should start with the proper definition of the basis of comparison, both for the price indices and for the indices of the physical volume of purchases (indices of quantities).

The characterization of the mean change in the exchange processes which occurs unceasingly among the reciprocal relations mentioned, raises from the very beginning, for the two indices, the problem of the prices at which the quantities in the two time periods compared should be estimated. To this end, we must start from the relations of prices in the basic period and those of the current period. If we choose the prices of the basic period the indices of the aggregate physical volume of purchases will be

$$\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}, \tag{1}$$

whilst, if we choose the prices of the current period,

$$\frac{\Sigma q_{i1} p_{i1}}{\Sigma q_{i0} p_{i1}}, \tag{2}$$

The difference between these two aggregate indices consists in the use of the *absolute* prices in the *basic* period or in the *current* period.

This differentiation is of great theoretical and practical interest.

When the prices of the various products in the current period differ by the same percentage of change in comparison with the corresponding prices of the basic period, then both aggregate indices offer the same result. Consequently the differences between the two aggregate indices are determined by the differences between the system of the existing relations between the prices of the basic period and the system of these relations in the current period. The cause of the difference between the two aggregate indices therefore lies in the diference between the effective, absolute, magnitudes of the prices.

Let us take as an example three assortments subjected to price changes.

Table 1

Assortment	Basic period			Current period		
	Unit price Lei	Quantity *k*	Expenses Lei	Unit price Lei	Quantity *k*	Expenses Lei
A	0.40	20	8	0.30	30	9
B	0.60	10	6	0.54	12	6.48
C	0.50	10	5	0.55	10	5.50
Total	—	—	19	—	—	20.98

The aggregate index of the physical volume described above is

$$\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}} = \frac{3 \times 0.4 + 12 \times 0.6 + 10 \times 0.5}{19} = 1.27$$

or

$$\frac{\Sigma q_{i1} p_{i1}}{\Sigma q_{i0} p_{i1}} = \frac{20.98}{20 \times 0.3 + 10 \times 0.54 + 10 \times 0.55} = 1.24$$

The difference in results — 127 per cent and 124 per cent — proceed from the different correlations, both as regards the prices and quantities of

the basic period and those of the current period. Thus, product A was, in comparison to product C, much more valuable in the basic period, when it represented 80 per cent of the latter's price, and less valuable in the current period, when it only represents 54.54 per cent of the price of product C. Likewise, product B was more valuable than product C in the basic period, namely, by 20 per cent, but in the current period the situation changes and product C is more valuable than product B. Thus, the correlations between the purchased quantities have also changed. Whereas in the basic period twice as large a quantity of A was purchased, as of C, in the current period three times as large a quantity of A was purchased, as of C.

A comparison between the two aggregate indices allows the changes having occurred in the population demand to be interpreted. Thus, if index (1) shows a larger magnitude than that of the index based on the prices in the current period, there results that, in the current period, those goods that have undergone a greater price reduction have been bought to a greater extent.

Similar problems are raised by the aggregate index of prices:

$$\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \tag{3}$$

$$\frac{\Sigma p_{i1} q_{i1}}{\Sigma p_{i0} q_{i1}} \tag{4}$$

The numeric characterization of changes in prices offered by this aggregate index only has a concrete significance when it refers to a determined quantity of goods. For example, in the case of price decreases the mean change of prices will be larger if the goods for which the prices have decreased represent an important proportion of the population's purchases, and, smaller, if these goods represent a smaller proportion.

In consumption problems relative magnitudes are of particular importance. An exemple of relative prices is the individual price index

$$i_{p1/0} = \frac{p_{i1}}{p_{i0}} \tag{5}$$

The advantage afforded by the individual price index is that it represents clearly and in a simplified manner the relative price change, independently of its absolute magnitude. A comparison of price changes is thus possible between various goods.

Another relative magnitude is, for instance, the individual quantity index.

$$i_{q1/0} = \frac{q_{i1}}{q_{i0}} \tag{6}$$

We can follow the link existing between the individual quantity indices on the one hand and the individual price indices, on the other hand, by determining the elasticity of the correlation between the change in quantity of a product from one group of products and the mean change of the group of products in physical volume as against the correlation between the change in the individual price index of the respective product and the mean change of prices in the group of products. Thus, this elasticity coefficient will show

us the percentage by which the correlation between an individual quantity index and the aggregate group index will change when the correlation between the individual price index and the aggregate index of prices for the group of products changes by 1 per cent.

The quantitative relations required to deduce this elasticity coefficient are obtained by applying the weighted correlation to the statistical indices method.

The conception of weighting can be applied and is even suited for the domain of regression analysis since one must not grant equal importance to the positions where the various points represented in a diagram of scatterings are situated. The regression line should take into account the importance of the position of certain points which require the assignment of different weights.

We note by p_i the value of the independent variable, by q_i the values of the dependent variable and by w_i the weights to be applied in the regression analysis to a magnitude, either of the variable p or of the variable q, in category i.

In the analysis of the weighted regression the deviations, either of p_i or of q_i are taken from the weighted means of p_i, respectively q_i, using as weights w_i in both cases. When computing the variances the co-variances *, the correlation and regression coefficients, — the squared weighted deviations of the individual values with respect to the mean, for the two variables and for the respective products of these deviations, are again weighted by w_i, before being totalized.

The weighted simple arithmetic mean is

$$\frac{\Sigma q_{i0} p_{i0}}{\Sigma q_{i0}} = \Sigma \frac{q_{i0}}{\Sigma q_{i0}} \times p_{i0}$$

the weighting element being $q_{i0}/\Sigma q_{i0}$. In case the weighting is made with the values in the basic period $w_0 = p_0 q_0$, then the weighted arithmetic mean of the individual indices

$$\frac{p_i}{p_0} \quad \text{is} \quad M\left(\frac{p_1}{p_0}\right) = \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}},$$

and that of the individual indices

$$\frac{q_1}{q_0} \text{ is } M\left(\frac{q_1}{q_0}\right) = \frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}.$$

The formula of the weighted variance is

$$s_w^2 = \frac{\Sigma w \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}\right)^2}{\Sigma w};$$

* The co-variance is the first order momentum of the product $[X - M(X)]\,[Y - M(Y)]$. Co-variance is a typical value which gives a measure of the relation of dependence between the two variables x and y (*Dicționar statistic economic*, under the editorship of professor M. Biji, Central Department of Statistics, 1962, p. 82).

since $w = p_0 q_0$, the formula may be written:

$$s_w^2 = \frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right)}{\Sigma p_{i0} q_{i0}};$$

The formula of the weighted standard deviation is $s = \sqrt{s_w^2}$, and the formula of the weighted co-variance * is:

$$Cov_w = \frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right) \left(\frac{q_{i1}}{q_{i0}} - \frac{\Sigma p_{i0} q_{i1}}{\Sigma p_{i0} q_{i0}} \right)}{\Sigma p_{i0} q_{i0}}$$

The coefficient of weighted regression is $\frac{w(Cov\, pq)}{w(s^2 p)}$ and its formula is

$$\frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right) \left(\frac{q_{i1}}{q_{i0}} - \frac{\Sigma p_{i0} q_{i1}}{\Sigma p_{i0} q_{i0}} \right)}{\Sigma p_{i0} q_{i0}} \times \frac{1}{\frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right)^2}{p_{i0} q_{i0}}}$$

The weighted correlation coefficient is $\frac{w(Cov\, pq)}{w(sp) \times w(sq)}$ and its formula is

$$\frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right) \left(\frac{q_{i1}}{q_{i0}} - \frac{\Sigma p_{i0} q_{i1}}{\Sigma p_{i0} q_{i0}} \right)}{\Sigma p_{i0} q_{i0}} \times$$

$$\times \frac{1}{\sqrt{\frac{\Sigma p_{i0} q_{i0} \left(\frac{p_{i1}}{p_{i0}} - \frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} b_{i0}} \right)^2}{\Sigma p_{i0} q_{i0}}}} \times \frac{1}{\sqrt{\frac{p_{i0} q_{i0} \left(\frac{q_{i1}}{q_{i0}} - \frac{\Sigma p_{i0} q_{i1}}{\Sigma p_{i0} q_{i0}} \right)^2}{\Sigma p_{i0} q_{i0}}}}$$

We use the application of the weighting conception to correlational analysis in studying the meaning of the difference between the two aggregate indices which use different weighting systems.

Thus, the aggregate price index may be of the form $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ or of the form $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$. In both cases the aggregate indices contain the arithmetic means of the individual price indices; it is only the weighting which differes.

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* The simple co-variance is $\frac{\Sigma (x - \bar{x})(y - \bar{y})}{n}$

A difference between the two weighted means does not necessarily result from this difference in weighting, not even if a great number of $\frac{q_{i1}}{\Sigma q_{i1}}$ differs from the corresponding $\frac{q_{i0}}{\Sigma q_{i0}}$. Some weight changes will act to increase the mean, and others to decrease the mean, and the two types of effects may tend to balance each other. The difference depends on the degree of statistical association between the magnitudes of the values which are weighted and the magnitudes of the changes in weights applied. If the correlation coefficient is zero or very small there will be no difference, or a very small one, between magnitudes of the changes in weights and in value. The most general condition for the absence of any difference is for the number of the correlation coefficient to be equal to zero.

In the case of a close correlation between the changes in magnitude of quantity weights and between the changes in magnitude of the prices, the diference between the two means will be great. Hence, if there is a link of dependence or causality, the difference mentioned will exist. Likewise, if the weights scattering is large, i.e. if the variance that expresses the extent to which these magnitudes differ from each other is large, then the difference between the two means (indices) will also be large.

In the above case, of the relative prices weighted by the quantities in the current period or by the quantities in the basic period, this difference is of particular importance *.

The difference between the two indices will be large if the association between the changes in price and the changes in the quantity of purchased goods is large.

Let us express the magnitude of the difference sought

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} - \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

in the form of a proportion of this difference vs. the price indices weighted by the quantities in the basic period, as demonstrated by Marris.

Noting the above difference by δ, we have

$$\delta = \frac{\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} - \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}}{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}} \tag{7}$$

We notice that

$$p_{i1} q_{i1} = p_{i0} \frac{p_{i1}}{p_{i0}} \times q_{i0} \frac{q_{i1}}{q_{i0}} = p_{i0} q_{i0} \frac{p_{i1}}{p_{i0}} \times \frac{q_{i1}}{q_{i0}}$$

* The first formulation of this problem is to be found in W. Bortkiewicz, *Zweck und Strucktur einer Preisindexzahl*, Nordisk Statistik Tidskrift, 1923, B II, p. 369.

and, in accordance with the original notation,

$$p_{i1}q_{i1} = p_{i0}q_{i0}p_{i1/0}q_{i1/0}.$$

Hence

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = \frac{\Sigma p_{i0}q_{i0}p_{i1/0}q_{i1/0}}{\Sigma p_{i0}q_{i0}q_{i1/0}}$$

However

$$\Sigma p_{i0}q_{i0}q_{i1/0} = \frac{\Sigma p_{i0}q_{i0}q_{i1/0}}{\Sigma p_{i0}q_{i0}} \times \Sigma p_{i0}q_{i0} = \frac{\Sigma p_{i0}q_{i1}}{\Sigma p_{i0}q_{i0}} \times \Sigma p_{i0}q_{i0}$$

From the above, it is seen that $\frac{\Sigma p_{i1}q_{i1}}{\Sigma p_{i0}q_{i1}}$ may be written

$$\frac{\frac{\Sigma p_{i0}q_{i0}p_{i1/0}q_{i1/0}}{\Sigma p_{i0}q_{i0}}}{\frac{\Sigma p_{i0}q_{i1}}{\Sigma p_{i0}q_{i0}}} \times \Sigma p_{i0}q_{i0}$$

which is equal to

$$\frac{1}{\frac{\Sigma p_{i0}q_{i1}}{\Sigma p_{i0}q_{i0}}} \times \frac{\Sigma p_{i0}q_{i0}p_{i1/0}q_{i1/0}}{\Sigma p_{i0}q_{i0}} \tag{8}$$

From (8) and (7) we obtain

$$\delta = \frac{1}{\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} \left[\frac{1}{\frac{\Sigma p_{i0}q_{i1}}{\Sigma p_{i0}q_{i0}}} \times \frac{\Sigma p_{i0}q_{i0}p_{i1/0}q_{i1/0}}{\Sigma p_{i0}q_{i0}} - \frac{p_{i1}q_{i0}}{p_{i0}q_{i0}} \right]$$

which can also be written

$$\delta = \frac{1}{\frac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}} \times \frac{1}{\frac{\Sigma q_{i0}p_{i0}}{\Sigma q_{i0}p_{i0}}} \left[\frac{\Sigma p_{i0}q_{i0}p_{i1/0}q_{i1/0}}{\Sigma p_{i0}q_{i0}} - \frac{\Sigma q_{i0}p_{i1}}{\Sigma q_{i0}p_{i0}} \times \frac{\Sigma p_{i0}q_{i1}}{\Sigma p_{i0}q_{i0}} \right] \tag{9}$$

Let us analyze the contents of the parenthesis. We notice that in the first part of the parenthesis we have the product between the individual price indices ($ip = p_{i1/0}$) and the individual weighted indices ($iq = q_{i1/0}$), this product being noted $p_{i1/0} \times q_{i1/0}$, that is weighted by $\Sigma q_{i0}p_{i0}$ (values in the basic period). In the right hand part of the parenthesis we have the product between the arithmetic mean of the individual price indices weighted by the quantities in the basic period $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ and the arithmetical mean of the individual quantity indices, weighted by the prices in the basic period $\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0}$.

Indeed, $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ may be written $\frac{\Sigma \frac{p_1}{p_0} p_0 q_0}{\Sigma p_0 q_0}$ and $\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0}$ may be written $\frac{\Sigma \frac{q_1}{q_0} q_0 p_0}{\Sigma q_0 p_0}$.

However, the whole parenthesis in (9) only represents the formula of co-variance between $p_{i1/0}$ and $q_{i1/0}$, i.e. between the individual price index and the individual quantity index. As known, any co-variance between X and Y can take the form $\frac{\Sigma XY}{N} - \overline{X} \times \overline{Y}$.

since

$$cov\ XY = \frac{\Sigma(X - \overline{X})(Y - \overline{Y})}{N} = \frac{\Sigma XY}{N} - \overline{X}\frac{\Sigma Y}{N} - \overline{Y}\frac{\Sigma X}{N} +$$

$$+ \overline{X} \times \overline{Y} = \frac{\Sigma XY}{N} - \overline{X} \times \overline{Y} - \overline{Y} \times \overline{X} + \overline{X} \times \overline{Y} = \frac{\Sigma XY}{N} - \overline{X} \times \overline{Y}$$

This is, obviously, also valid for weighted co-variance.

However, the co-variance in X and Y constitutes the numerator of the ratio in the formula of the correlation coefficient

$$r_{xy} = \frac{\overline{xy} - \overline{x} \times \overline{y}}{s_x \times s_y}.$$

Thence

$$\overline{xy} - \overline{x} \times \overline{y} = r_{xy} \times s_x \times s_y$$

These relations are also valid for the weighted co-variance, in which case we have

$$(cov\ p_{i1/0}\, q_{i1/0}) = w(rp_{i1/0}\, q_{i1/0}) \times w(sp_{i1/0}) \times w(sq_{i1/0})$$

Introducing this relation in (9), we have

$$\delta = \frac{1}{\frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}} \times \frac{1}{\frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}} [w(rp_{i1/0} q_{i1/0}) \times w(sp_{i1/0}) \times w(sq_{i1/0})]$$

The formula obtained may, however, be written

$$\delta = w(rp_{i1/0} q_{i1/0}) \times \frac{w(sp_{i1/0})}{\frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_0}} \times \frac{w(sq_{i1/0})}{\frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}} \qquad (10)$$

An equality has thus resulted between the relative or proportional difference between the two aggregate price indices differently weighted (by the quantities in the current period and by the quantities in the basic period) and

the weighted correlation coefficient between the individual price indices $p_{i1/0}$ and individual quantity indices $q_{i1/0}$ multiplied by the variation coefficient of $p_{i1/0}$ and multiplied by the weighted variation coeficient of $q_{i1/0}$.

We have seen that the weighting element at the numerator is $w = q_{i0}p_{i0}$, and at the denominator the mean of the individual price indices is weighted by the same element, as

$$\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = \frac{\Sigma p_0 q_0 \frac{p_1}{q_0}}{\Sigma p_0 q_0}$$

and the mean of the individual quantity indices has the same weighting element

$$\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} = \frac{\Sigma q_0 p_0 \frac{q_1}{q_0}}{\Sigma q_0 p_0}$$

The weighted correlation coefficient $w(rp_{1/0}q_{1/0})$ measures the degree in which the variables (prices), displaying changes in value above the mean, from basic period to the current period, also tend to reach changes higher than the mean, in weights (quantities).

Relation (10) describes the relative difference between the two price indices in general terms, the difference being easy to interpret and being independent of even the magnitude of the indices themselves.

δ can also be expressed in the terms of the weighted regression coefficient of $q_{i1/0}$ according to $p_{i1/0}$. In this case, all the other conditions being equal, the larger the regression coefficient b is, the larger will δ be. By means of a small transformation, in view of the fact that we can obtain the regression coefficient by means of the formula

$$b_{q_{i1/0}\,p_{i1/0}} = \frac{cov\, p_{i1/0}q_{i1/0}}{s^2 p_{i1/0}}$$

formula (10) may be written

$$\delta = \left[\frac{w(cov.\, p_{i1/0}q_{i1/0})}{w(sp_{i1/0} \times w(sq_{i1/0})} \times \frac{w(sp_{i1/0})}{\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} \times \frac{w(sq_{i1/0})}{\frac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}}\right]$$

which, when simplified, becomes:

$$\delta = \left[\frac{w(cov.\, p_{i1/0}q_{i1/0})}{w(sp_{i1/0})} \times \frac{w(sp_{i1/0})}{\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} \times \frac{1}{\frac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}}\right]$$

We multiply by $w(sp_{i1/0})$ the first term in the parenthesis, both numerator and denominator, and thus the value of the ratio remains unchanged. We

also multiply the last term in the parenthesis by the price index, weighted by the quantities in the basic period $\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}$, both numerator and denominator, and we obtain

$$\delta = \left[\frac{w(cov. p_{i1/0} q_{i1/0})}{w(sp_{i1/0})} \times \frac{w(sp_{i1/0})}{w(sp_{i1/0})} \times \frac{w(sp_{i1/0})}{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}} \times \frac{1}{\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} \times \frac{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}} \right]$$

By simplifying and reducing we obtain

$$\delta = \left[\frac{w(cov. p_{i1/0} q_{i1/0})}{w(sp_{i1/0})^2} \times \frac{w(sp_{i1/0})^2}{\left(\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}\right)^2} \times \frac{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} \right] \qquad (11)$$

Hence, equation (10) is

$$\delta = [w(b_{pi1/0\, qi1/0})] \times \left[\frac{w(sp_{i1/0})^2}{\left(\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}\right)^2} \right] \times \left[\frac{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} \right] \qquad (11\ a)$$

The regression coefficient weighted by the values in the basic period $q_0 p_0$, noted by w_b, expresses the value of the regression line slope between q_i and p_i, the sensitivity of the relation between them. The higher this sensitivity, the higher the typical magnitude of the products between the deviations of $p_{i1/0}$ and the deviation of $q_{i1/0}$ from their respective means, hence, the greater the co-variance and the greater δ.

It is well known that there is an economic link between the quantities q_i and the prices p_i: when prices decrease quantities increase and vice-versa (e.g. unorganized market). Hence, changes in the p_i series causes a series of changes in q_i, generating a certain series of $q_{i1/0}$. This causes a substantial co-variance between $p_{i1/0}$ and $q_{i1/0}$, resulting in the appearance of a large increase for δ.

The second parenthesis represents the square of the variation coefficient of $p_{i1/0}$ and has, with respect to the variation coefficient, the same properties as those of variance with respect to the mean square deviation. This square of the variation coefficient, which in fact is a relative magnitude, shows the independent character of the variations of $p_{i1/0}$. In case this coefficient is equal to zero the relative difference is also zero; we are then in the situation where the prices change in the same proportion between the two periods, i.e. all the $p_{i1/0}$ are equal. The third term shows the mean magnitude

of $p_{i1/0}$ with respect to $q_{i1/0}$. A feature of the basic mathematical network implied in the arrangement of formula (11) is that δ also be larger, the larger the value of the ratio

$$\frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}}{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0}}$$

Thus, the size of the difference between the price index weighted by the quantities in the basic period results from the existing association between changes in price and changes in quantity of goods sold. The goods whose prices decrease more will be purchased in greater o rsmaller quantities according to the laws of demand. The total results of the forces acting on changes in price and quantity of the goods, changes that have been expressed by means of two price indices $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$ and $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$ will be reflected in a more involved manner, but they will nevertheless be reflected in the magnitude of coefficient b in equations (11) and (11 a).

On basis of the regression coefficient b we can find the elasticity coefficient.

The formula of the elasticity coefficient e is:

$$e = \frac{\Delta q}{q} : \left(- \frac{\Delta p}{p} \right)$$

or

$$e = - \frac{p}{q} \times \frac{\Delta q}{\Delta p}$$

We notice that $\frac{\Delta q}{\Delta p}$ is nothing but the coefficient b of the regression of q according to p. Hence, the formula of elasticity may be written $e = - \frac{p}{q} b$. In this expression p and q are correspondent values, either observed or computed by means of equation $q = a + bp$. After computing b, elasticity is obtained by introducing in the equation the observed or computed value of p and q. In most cases the mean values of p and q are used, so that, practically, $e = - \frac{\bar{p}}{\bar{q}} b$.

According to the above

$$- e = \frac{w(cov.\, p_{i1/0}\, q_{i1/0})}{w(s p_{i1/0})^2} \times \frac{\frac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{\frac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}}$$

Let us again take the example of the three assortments of goods which constitute a group of related goods (such as the assortments in the group fats, meat, footwear, etc.).

Table 2

Assortment	Basic period			Current period		
	Unit price lei p_{i_0}	Quantity kg q_{i_0}	Sales lei $p_{i_1}q_{i_1}$	Unit price lei p_{i_1}	Quantity kg q_{i_1}	Sales lei $p_{i_1}q_{i_1}$
A	0.40	20	8	0.30	30	9
B	0.60	10	6	0.54	12	6.48
C	0.50	10	5	0.55	10	5.50
Total	—	—	19	—	—	20.98

Table 3

Individual indices

	$p_{i1/0} = \frac{p_{i_1}}{p_{i_0}}$	$q_{i1/0} = \frac{q_{i_1}}{q_{i_0}}$
A	0.75	1.5
B	0.90	1.2
C	1.10	1.0

The computations further develop as follows

	$q_{i_0}p_{i_1}$	$q_{i_1}p_{i_0}$
A	6	12
B	5.40	7.2
C	5.50	5.0
Total	16.90	24.2

$$\frac{\Sigma q_{i_1}p_{i_0}}{\Sigma q_{i_0}p_{i_0}} = \frac{24.2}{19} = 1.27368 \qquad \frac{\Sigma q_{i_0}p_{i_1}}{\Sigma q_{i_0}p_{i_1}} = \frac{20.98}{16.90} = 1.24142$$

$$\frac{\Sigma p_{i_1}q_{i_0}}{\Sigma p_{i_0}q_{i_0}} = \frac{16.90}{19} = 0.8894732;$$

$$\frac{\Sigma p_{i_1}q_{i_1}}{\Sigma p_{i_0}q_{i_1}} = \frac{20.98}{24.2} = 0.8669422$$

On basis of the indices thus computed we reach

Table 4

	$p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}$	$\left(p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}\right)^2$	$p_{i_0} q_{i_0}$	$p_{i_0} q_{i_0} \left(p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}\right)^2$
A	−0.1394737	0.019453	8	0.155624
B	0.0105263	0.000111	6	0.000666
C	0.2105263	0.044321	5	0.221605
Total				0.377895

Since

$$[w(sp_{i1/0})]^2 = \frac{\Sigma p_{i_0} q_{i_0} \left[p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}\right]^2}{\Sigma p_{i_0} q_{i_0}}$$

There results

$$[w(sp_{i1/0})]^2 = \frac{0.377895}{19} = 0.019889211$$

Similarly:

	$q_{i1/0} - \frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}$	$\left(q_{i1/0} - \frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}\right) \times \left(p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}\right)$
A	0.2263157	−0.0315651
B	−0.0736843	−0.0007756
C	−0.2736843	−0.0576177

After which

	$q_{i_0} p_{i_0}$	$q_{i_0} p_{i_0} \left(q_{i1/0} - \frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}\right) \times \left(p_{i1/0} - \frac{\Sigma p_{i_1} q_{10}}{\Sigma p_{i_0} q_{i_0}}\right)$
A	8	−0.252508
B	6	−0.0046536
C	5	−0.2880885
Total	19	−0.5452629

We can thus obtain

$$w(cov.\, p_{i1/0} q_{i1/0}) = \frac{\Sigma q_{i_0} p_{i_0} \left(q_{i1/0} - \frac{\Sigma q_{i_1} p_{i_0}}{\Sigma q_{i_0} p_{i_0}}\right) \times \left(p_{i1/0} - \frac{\Sigma p_{i_1} q_{i_0}}{\Sigma p_{i_0} q_{i_0}}\right)}{\Sigma q_{i_0} p_{i_0}}$$

$$w(cov.\, p_{i1/0} q_{i1/0}) = \frac{-0.5452629}{19} = -0.028698048$$

$$wb_{q_{i1/0} p_{i1/0}} = \frac{w\, cov.\, p_{i1/0} q_{i1/0}}{[w(sp_{i1/0})]^2} = \frac{-0.028698048}{0.019889211} = -1.442896$$

The regression coefficient of the physical volume index according to the price index is —1.442896 and shows that when the aggregate price index decreases by one unit the aggregate physical volume index decreases by —1.442896. Based on the above we compute the elasticity coefficient.

$$-e = b\frac{\bar{p}_{i1/0}}{\bar{q}_{i1/0}} = -1.442896 \times \frac{\dfrac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}}{\dfrac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}} = -1.442896 \times$$

$$\times \frac{0.8894737}{1.27368} = -1.007645$$

The regression coefficient shows by how many units of $q_{i1/0}$ (measured in $q_{i1/0}$ units measurement), $q_{i1/0}$ is modified when $p_{i1/0}$ changes by one unit (measured in $p_{i1/0}$ units of measurement). The elasticity coefficient expresses these relations in percentages showing by how many percentages $q_{i1/0}$ changes when $p_{i1/0}$ changes by one per cent. The regression coefficient is not to be mistaken for the elasticity coefficient. From the formula of the latter $-e = \frac{\Delta q}{\Delta p} \times \frac{p}{q}$ the regression coefficient only represents the first ratio in the right hand part of the equation.

There is, however, another manner of deducing the elasticity coefficient by directly applying the analysis of regression to the indices method. This method uses mainly δ, the relative difference between price indices differently weighted, as already shown.

The regression equation $Y = a + bx$ may be written $\overline{Y} = a + b\bar{x}$. From the second equation we deduce the value of a which is $a = \overline{Y} - b\bar{x}$ and introduce it in the first equation $Y = \overline{Y} - b\bar{x} + bx$, which means $Y - \overline{Y} = b(x - \bar{x})$, wherefrom $b = \frac{Y - \overline{Y}}{x - \bar{x}}$.

We hereunder accept the premise that there exists perfect correlation between q and p and hence the relative quantities observed are placed exactly on the regression line of the magnitudes $q_{i1/0}$ according to $p_{i1/0}$. If from the above assortments, q_i, we follow up assortment a, then

$$b = \frac{q_{a1/0} - \bar{q}_{i1/0}}{p_{a1/0} - \bar{p}_{i1/0}}.$$

This is also valid for the weighted regression coefficient which becomes

$$w(bq/p) = \frac{q_{a1/0} - \dfrac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}}{p_{a1/0} - \dfrac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} \qquad (12)$$

We now rewrite formula (11 a), substituting (12) to the weighted regression coefficient

$$\delta = \left[\frac{q_{a1/0} - \dfrac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}}{p_{a1/0} - \dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}} \right] \times \left[\frac{[w(s p_{i1/0})]^2}{\left(\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right)^2} \right] \times \left[\frac{\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{\dfrac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} \right]$$

We further effect the following permitted operations

$$\delta = \left[\frac{q_{a1/0} - \dfrac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}}{\dfrac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} \right] \times \left[\frac{\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}}{p_{a1/0} - \dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}} \right] \times \left[\frac{[w(s p_{i1/0})]^2}{\left(\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right)^2} \right] \qquad (13)$$

The first two parentheses in the second part of the equation may be written by reversing the second fraction and we then obtain

$$\delta = \left[\frac{q_{a1/0}}{\dfrac{\Sigma q_{i1} p_{i0}}{\Sigma q_{i0} p_{i0}}} - 1 \right] : \left[\frac{p_{a1/0}}{\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}}} - 1 \right] \times \left[\frac{[w(s p_{i1/0})]^2}{\left(\dfrac{\Sigma p_{i1} q_{i0}}{\Sigma p_{i0} q_{i0}} \right)^2} \right] \qquad (14)$$

We notice that the first parenthesis only represents the proportional change of the relative quantity compared to the volume change of all the quantities in the group or, in other words, a measure of the relative change in the quantity of assortment *a* sold. We have thus obtained a measure of the ratio in which the importance in quantitative sales of assortment *A* in comparison to the whole group has increased or decreased.

The second parenthesis shows us a measure of the proportional change in the relative price of assortment *a* as compared to the mean change of all the prices in the group, respectively the change of the relative price in comparison to the relative prices of all the assortments in the group.

Both the mean change of the quantities in the group and the mean change of the general level of prices are here expressed by indices with the weights of the basic period.

In the manner in which the first two parentheses in (14) were written, they represent the formula of the elasticity coefficient. Indeed, we notice that there is a relation between the correlation change in the increase in the purchases of product *a* in comparison with the increase in the purchases of the products of the whole group and between the correlation change between the change in price of product *a* in comparison with the change in the general price level for the entire group. In other words, the elasticity thus identified is the elasticity of the correlation change in quantities as a function of the correlation change in prices. Obviously, both correlations are those of product *a* in comparison to the group, a correlation of the variations among the quantities of goods (extent to which the purchase of product *a* has increased or decreased its quantitative weight in the framework of the group) and a correlation among the price variations (changes in price of *a* in com-

parison with the changes in the price level of the group). Undoubtedly, a change in the correlation between the price of a product and the remaining prices results in a change of the rank which the quantitative purchases of the respective product represent in the framework of the other products of the group.

This elasticity coefficient therefore shows us by how many percentages the proportion (correlation) between the increase in quantity of a product and the mean increase of all the quantities in the group changes when the proportion (correlation) between the change in price of this product and the mean change in prices for the group change by one per cent.

In equations (13) and (14) the last parenthesis represents, as already shown, the square of the weighted variation coefficient. From (14) there results:

$$\left[\frac{q_{a1/0}}{\frac{\Sigma q_{i1}p_{i0}}{\Sigma q_{i0}p_{i0}}} - 1\right] : \left[\frac{p_{a1/0}}{\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} - 1\right] = \frac{\delta}{\frac{[w(sp_{i1/0})]^2}{\left(\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}\right)^2}}$$

On basis of this formula we are able to compute elasticity for any product i, with the limiting condition that perfect correlation should exist between $q_{i1/0}$ and $p_{i1/0}$. Such a restriction does not exist for the group as a whole.

Using once more the data in the table

$$\delta = \frac{\frac{\Sigma p_{i1}q_{i1}}{\Sigma p_{i0}q_{i1}} - \frac{\Sigma p_{i1}q_i{}^0}{\Sigma p_{i0}q_{i0}}}{\frac{\Sigma p_{i1}q_{i0}}{\Sigma p_{i0}q_{i0}}} = \frac{0.8669422 - 0.8894737}{0.8894737} = -0.02533127$$

$$-e = \frac{-0.02533127}{\frac{0.019889211}{(0.8894737)^2}} = 1.0076403$$

as already obtained. It was to be expected that there should be differences between these two different results, consequently to rounding off ($e = -1.007645$ and $e = -1.0076403$).

In our example none of the magnitudes of the observed relative quantities is placed exactly on the regression line. In comparison with the elasticity coefficient 1.0076 that an assortment situated exactly on the regression line should have, assortment C has an elasticity coefficient 0.9079 and assortment A 1.13. The closeness of these elasticity coefficients to the actual one is explained by the location of the observed relative quantities very close to the computed regression line (for assortment A 1.4769 computed in comparison to 1.50 observed, and for assortment C 0.96992 computed in comparison with 1.000 observed, which represents a deviation of 1.564 per cent for assortment A and 0.310 per cent for assortment C). Assortment B presents deviations from the computed regression line (1.2585 computed in comparison

with 1.200 observed, which represents deviations of 4.65 per cent), which prove substantial through their consequences. In this latter case the premise of the position of the observed magnitudes on the computed regression line cannot be admitted and thus the procedure described is no longer valid.

The method of determining in this manner the elasticity coefficient of the proportion in the quantities of a given product, in comparison with the mean change of quantities in the framework of one group as a function of the ratio between the change in the respective price as compared to the mean change in the price level of the group, is very exacting and stringent, requiring the respective assortment to have its observed relative quantities exactly the same or very close in size to those computed by means of the regression line. In cases where the observed magnitudes are very close to the ones computed (it is well known that there often is a very high correlation between prices and quantities), we are able to obtain an approximate elasticity, respectively, an evaluation obtained from a linear simple regresion analysis.

If δ is a measure of the difference between the price indices of all the goods, weighted by the quantities in the current period, and the index of the price of all these goods, weighted by the quantities in the basic period, then the resulting elasticity for product A will be the elasticity in replacing the product by all the remaining products. In this case, however, δ refers to the case of product A or of other goods whose data fall exactly on the computed weighted regression line. However, not all the goods will find the same elasticity of replacement at the consumer's ; some will have closer replacements, others will be replaced by products not so close to them, while others will not be replaced at all. This explains the fact that the results for the entire series of goods will be scattered about the regression line, and this elasticity coefficient should only be considered a representative elasticity, as Marris found out.

In contrast, in view of determining mean elasticity for a group of products (the closer the products are related, the more evident the economic significance and the more practically can they be used in planning), these problems do not arise, and the second method proves to be very prompt, since the elasticity coefficient can be found from the ratio between δ and the square of the variation coefficient.

It can be seen from the above that, given the dispersion of prices, δ is larger or smaller according as the elasticity of quantity correlation, as a function of price correlation, is larger or smaller. Likewise, we may generally state that, all conditions remaining unchanged, the greater the representative substitution elasticity among goods, the greater will δ be.

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LE CALCUL DE L'ELASTICITE DE LA DEMANDE A L'AIDE DE L'APPLICATION DE LA REGRESSION A LA METHODE DES INDICES

L'élasticité par rapport au prix caractérise des processus qui se déroulent à travers le temps. Mais durant les diverses périodes de temps les structures, c'est-à-dire les systèmes des corrélations, changent aussi bien en ce qui concerne les proportions des quantités achetées de produits, que les proportions existentes entre les prix de ceux-ci.

Vu l'existence d'une liaison causale entre la structure des quantités qui exprime la demande et la structure des prix, il est nécessaire de déterminer l'élasticité de la première structure par rapport à la seconde. Autrement dit, il s'agit de l'élasticité de la corrélation des quantités par rapport à la corrélation des prix de ces produits.

Il est possible d'étudier la liaison entre les indices-rapport de la quantité et les indices-rapport de prix à l'aide de l'élast'cité de la corrélation entre le changement de l'indice-rapport de quantités d'une marchandise faisant partie d'un groupe de marchandises et le changement moyen de la quantité physique du groupe de marchandises, par rapport à la corrélation entre le changement de l'indice-rapport de prix du produit en cause et le changement moyen des prix du groupe entier. Ainsi ce coefficient d'élasticité sera destiné à montrer avec quel pourcentage se modifiera la corrélation entre un indice-rapport de quantités et l'indice agrégatif de quantités à la suite d'une modification de 1 % de la corrélation entre l'indice-rapport de prix et l'indice agrégatif des prix du même groupe de marchandises.

Les relations quantitatives nécessaires pour la déduction de ce coefficient d'élasticité sont obtenues par l'application de la corrélation pondérée à la méthodes des indices statistiques.

La notion de groupe peut s'élargir jusqu'à comprendre de nombreuses catégories de produits, comme par exemple, tous les produits alimentaires.

ОПРЕДЕЛЕНИЕ ЭЛАСТИЧНОСТИ СПРОСА ПО МЕТОДУ ПОКАЗАТЕЛЕЙ

Эластичность в зависимости от цены характеризует процессы, развивающиеся во времени. Еще в течение различных периодов времени структуры, соответственно, системы соотношения, изменяются, как с точки зрения пропорций между количеством купленных продуктов, так и с точки зрения пропорций между их ценами.

Так как существует причинная связь между структурой количеств, выражающей спрос, и структурой их цен, появляется необходимость определения эластичности для изменения

первой структуры в зависимости от изменения второй. Другими словами, речь идет об эластичности изменения соотношения между купленными количествами в зависимости от изменения соотношения между ценами продуктов в анализируемой группе товаров.

Можно проследить связь, существующую между индивидуальными показателями количеств, с одной стороны, и индивидуальными показателями цен, с другой стороны, путем определения эластичности соотношения между изменением количества продукта в определенной группе продуктов и средним изменением физического объема группы продуктов в зависимости от соотношения между изменением индивидуального показателя цены данного продукта и средним изменением цен группы продуктов.

Так, этот коэффициент эластичности будет показывать на сколько процентов изменится соотношение между индивидуальным показателем количеств и агрегированным показателем группы тогда, когда изменится на 1% соотношение между индивидуальным показателем цены и агрегированным показателем цен этой же группы продуктов.

Количественные отношения, необходимые для вывода этого коэффициента эластичности, получаются путем применения соотношения, взвешенного по методу статистических показателей.

Понятие группы может быть расширено до тех пор, пока не охватит многочисленные группы продуктов, как например, продукты пищевой промышленности.