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A Markovian Stochastic Model of the Profit's General Rate Formation

In this work, the author tries, by using the Markovian stochastic processes theory, to model the formation of the general profit's rate through the transformation of the individual profit's rate in a mean rate, as exposed by Marx in the second section of the IIIrd volume of "The Capital" (see [1], first part, pp. 145—213).

The first part of the present work includes the exposition of Marx ideas in a simpler algebraic formulation, and the second part — the model proper.

1. Preliminaries

In the first section of the IIIrd volume of "The Capital" Marx studies the formation of the profit, of the profit's rate and of the cost price. Denoting by V the value of the commodities produced by a capital invested in a certain branch, we have:

$$V = c + CP^v + p \tag{1.1}$$

where c represents the share from the constant capital CP^c included in the commodities, CP^v — the variable capital, p — the surplus value.

Denoting by pc the cost price and by pr the profit, we obtain:

$$V = pc + p = pc + pr. \tag{1.2}$$

If $p' = \dfrac{p}{CP^v}$ is the surplus value, $pr' = \dfrac{p}{CP}$ — the profit's rate, CP being the total capital (sum of the constant capital CP^c and of the variable

one CP^v) and $k = \frac{CP^c}{CP^v}$ — the organic (value) composition of the capital(on condition that the commodity be sold at their value), it results

$$pr' = \frac{p}{CP^c + CP^v} = p' \frac{CP^v}{CP} = \frac{p'}{1+k}. \tag{1.3}$$

Hence, in these conditions, the profit's rate depends on two factors: the surplus value rate and the organic composition of the capital.

In the second section, Marx points to that, if the surplus value rate and the working day duration are the same for all production branches of a capitalist economy, the profits' rates might be, according to formula (1.3), different in each branch, taking into account the different organic compositions of the capitals, according to the branch wherein they are invested. But as a matter of fact, things occur as if a mean profit's rate pr'_M would be built up, and each capital would receive a profit, proportional to the magnitude of the invested capital.

Let us have an economy with the production branches $i (i = 1, ..., m)$, capitals $CP_j (j = 1, ..., n)$ and p' — surplus value rate, the same for the whole economy; denoting by k_i the organic composition of branch i, by CP_{ij} the capital CP_j invested in branch i, and by CP^c_{ij} the constant part or by CP^v_{ij} the variable part of that capital, respectively, it results (when supposing the equality of the organic composition for all capitals in the same branch) that:

$$k_{ij} = \frac{CP^c_{ij}}{CP^v_{ij}} = k_i. \tag{1.4}$$

If V_{ij} is the value of the commodities produced by capital CP_{ij} in a time interval, then

$$V_{ij} = pc_{ij} + CP_{ij} \frac{p'}{1+k_i} \tag{1.5}$$

where pc_{ij} is the cost price (value consumed in producing the commodities):

$$pc_{ij} = c_{ij} + CP^v_{ij} \tag{1.6}$$

and c_{ij} is the value of the consumed share from the constant capital CP^c_{ij}.

The total capital invested in branch i will be:

$$CP_i = \sum_j CP_{ij} = \sum_j CP^c_{ij} + \sum_j CP^v_{ij} = CP^c_i + CP^v_i \tag{1.7}$$

where CP^c_i and CP^v_i are the total volumes of constant and variable capital, respectively, in branch i, and the total value V_i produced in branch i is:

$$V_i = \sum_j V_{ij} = \sum_j c_{ij} + \sum_j CP^v_{ij} + \sum_j p_{ij} =$$

$$= c_i + CP^v_i + p_i = c_i + (1 + p')\, CP^v_i. \tag{1.8}$$

Moreover, the conservation of the organic composition of the capital by branch is also verified, since:

$$\frac{CP_i^c}{CP_i^v} = \frac{\sum_j CP_{ij}^c}{\sum_j CP_{ij}^v} = \frac{\sum_j k_i CP_{ij}^v}{\sum_j CP_{ij}^v} = k_i \cdot \tag{1.9}$$

The total capital CP invested in the whole economy will be:

$$CP = \sum_i CP_i = \sum_i CP_i^c + \sum_i CP_i^v = CP^c + CP^v, \tag{1.10}$$

and the value of all produced commodities:

$$V = \sum_i V_i = c + CP^v + p; \tag{1.11}$$

where CP^c and CP^v are the total constant and the total variable capital, respectively, c — the share from CP^c consumed for the goods production, p — the total produced surplus value.

We denote by k_M the mean organic composition (organic composition of total capital) i.e.:

$$k_M = \frac{CP^c}{CP^v}; \tag{1.12}$$

and we obtain the mean rate pr'_M of the profit:

$$pr'_M = \frac{p}{CP} = p' \frac{CP^v}{CP} = \frac{p' CP^v}{(1 + k_M) CP^v} = \frac{p'}{1 + k_M} \cdot \tag{1.13}$$

Things happen so, as if each capitalist would get for the produced commodities, not their value V_{ij}, equal, according to (1.8), with the sum of the cost price pc_{ij} of the commodities and the surplus value p_{ij}, but as if the achieved profit would be proportional to the weight of capital CP_{ij} in the total capital CP, i.e. for any i and j:

$$\overline{pr}_{ij} = pr'_M \cdot CP_{ij} = p \frac{CP_{ij}}{CP} \cdot \tag{1.14}$$

Thus the production price PPR_{ij} of the commodities produced by capital CP_{ij} is achieved:

$$PPR_{ij} = pc_{ij} + \overline{pr}_{ij} = pc_{ij} + pr'_M CP_{ij} \tag{1.15}$$

We verify the condition pointed to by Marx ([1] first part, pp. 176) that "the sum of the profits of all spheres of production must be equal with the sum of the surplus values, and the sum of the production prices of the total social product, must be equal with the sum of its values":

$$\sum_i \sum_j PPR_{ij} = \sum_i \sum_j (pc_{ij} + pr'_M CP_{ij}) =$$

$$= \sum_i \sum_j pc_{ij} + pr'_M \sum_i \sum_j CP_{ij} = c + CP^v + \frac{p}{CP} CP = V. \tag{1.16}$$

When (1.14) is taken into account, the production price can be also expressed thus:

$$PPR_{ij} = pc_{ij} + \frac{p'}{1 + k_M} CP_{ij}. \tag{1.17}$$

By comparing (1.5) to (1.17), it results that, for the branches having an upper organic composition ($k_i > k_M$), the production prices of the goods are higher than their values ($PPR_{ij} > V_{ij}$), and for those having a lower organic composition ($k_i < k_M$), the production prices are lower than the value of the goods ($PPR_{ij} < V_{ij}$).

Going on, Marx points to that "the truly difficult problem is that follows: how this equalization of the profit is produced, since this is obviously a result and cannot be a starting point" ([1], first part, pp. 177).

In the first place, he makes a distinction between the *market value* and the *individual value* of the goods produced by various capitalists within the same branch. If, according to the definition in volume I of "The Capital", the value is equal with the socially necessary working time, i.e. "the working time required to produce a certain use value, within the existing production conditions, normal from a social viewpoint and by the mean social skill degree and labour intensity" (Works, vol. 23, pp. 53), the *individual* value of a commodity must be understood as the working time required for its production, in the same mean social labour intensity, but in the *specific* (individual) production conditions of the respective capitalist. If the production conditions are better, a higher individual value corresponds to worse production conditions. In its turn, "the market value must be considered as the mean value of the commodities produced in a certain sphere" ([1], first part, pp. 181).

By supposing hence that, from the total volume Q_i of commodities from branch i, the capitalist j produces Q_{ij} commodity units of individual value VI_{ij} ($j = 1, ..., n$), the market value of the commodity unit i will be the weighted mean of the individual values:

$$VM_i = \frac{\sum_{j=1}^{n} Q_{ij} VI_{ij}}{Q_i} = \sum_{j=1}^{n} VI_{ij} \pi_{ij} \tag{1.18}$$

where the weight $\frac{Q_{ij}}{Q_i}$ of the goods volume Q_{ij} in the total goods volume was denoted by π_{ij} (see Fig. 1, where the index i was omitted). Similarly, if VI_i is defined as a discrete random variable, which takes values VI_{ij} with probabilities π_{ij} ($j = 1, ..., n$), the market value will be the mean value of this random variable:

$$VM_i = \sum_{j=1}^{n} VI_{ij} \pi_{ij}. \tag{1.19}$$

or, in the continuous case, Φ_i being the distribution function of VI_i:

$$VM_i = \int_{-\infty}^{+\infty} VI \, d\Phi_i. \tag{1.20}$$

The influence of the goods produced in the best and in the worst conditions, upon the market value of the commodities in each branch, are then studied. In most cases, the weight of the commodities of low individual value is approximately equal with that of the goods of high individual value; in this case, as results also from (1.18), "the market value must be considered as individual value of the goods produced in the mean conditions of the respective sphere, goods that make out the most of the products of the respective sphere. Only in exceptional combinations, the commodities produced in the worst or in the best conditions regulate the market value which, in its turn, makes up the oscillation centre of the market prices" ([1], first part, pp. 181).

Marx studies then the building up of the *market price*; the price deviations from the market values are due to the variable ratio between the demand volume and the supply volume and to the different distribution of demand and supply. Regardless of this ratio, the equilibrium between demand and supply is achieved in any case. Marx means that the equilibrium is the formation of a single market price, for selling all commodities of the same kind. The market price is set up at the level at which the volume of demand at that price isequal to the volume of supply at the same price (equal to the individual value of the respective commodity).

Let us assume, as above, that the capitals CP_{ij} produce Q_{ij} units of commodity i with individual value VI_{ij} ($j = 1, ..., n$), the total volume of the produced commodities being Q_i, and that, within the demand, the buyers requires, at the respective moment, q_{is} goods units at the unit price PD_{is}; the total demand volume is VD_i (the prices PD_{is} are calculated like the individual values, i.e. also in working times).

The market price PM_i will be equal with that PD_{is}, for which:

$$PD_{is} = VI_{ij} \text{ and } q_{is} = Q_{ij} \tag{1.21}$$

or a part of q_{is} is equal with a part of Q_{ij} (see Fig. 2, where all indexes i were omitted, for ease).

The supplementary profit, momentarily achieved by capitalist j in branch i will be equal with the difference between the price obtained for the sold commodity and its individual value:

$$prs_{ij} = Q_{ij}(PM_i - VI_{ij}) = \frac{V_{ij}}{VM_i}(PM_i - VI_{ij}) =$$

$$= \frac{V_{ij}}{VM_i}[(PM_i - VM_i) - (VM_i - VI_{ij}). \tag{1.22}$$

If the commodities are sold at their market value ($PM_i = VM_i$) (and this happens in the case when we take into account a longer time period, since the market price PM_i does oscillate round the market value VM_i), the supplementary profit is reduced to Q_{ij} ($VM_i - VI_{ij}$). Then it results, from the definition formula of VM_i (1.20) that for the whole branch i, the total supplementary profit (i.e. the sum of the supplementary profits of capitals j invested in that branch), will be null.

At a given moment, the profit obtained by a capital is equal with the difference between the total price of the products and of the elements of the non-consumed capital, and the price of the total capital (constant and variable) before starting the production. We will give the mathematical formula of the individual profit, in the second part of this work.

Marx also indicates that the equalization of the profit's rate, and hence the formation of the production price is carried out through the movement of the capital from the branches having lower profit rates to those having higher profit rates; this permanent movement "determines such a ratio between demand and supply, that the mean profit is the same for different production spheres", see [1] first part pp. 198 for capitals of equal size.

The results of that process were exposed at the beginning of these preliminaries.

2. Stochastic Model

We will try further down, to describe the formation process of the genera profit's rate, in a free-competition capitalist economy, as a discrete time Markovian stochastic process, with a finite number of states.

The utilization of the probabilistic methods asserted itself, on the one hand, due to the mass character of the studied phenomena, and on the other hand, due to the multiple states of the system in each moment of its evolution; likewise, the dynamic treatment was necessary, since the deep reality of the system is revealed only by studying its trends in time.

We undertook to follow thoroughly Marx' s thinking mode, as we are convinced of the rigour of his reasoning and of the consequently systemic character of the Marxist approach of the economic reality.

For any moment whatever of the process, we shall show firstly how the market value of the goods in each branch is built up as the mean of the individual values of the commodities produced by the various capital investments in the respective branches; then, the formation of the market price through the interaction between the stuctures and the volumes of the demand, and of the supply.

We shall show how the general profit's rate is formed, by studying the dynamics of the described system, i.e. of the structure by branches of the capital investments.

2.1. *System Description*

We can distinguish two levels within the system: 1) the production and consumption level, to which the momentary market value is formed; 2) the market level, to which the market price is formed. Within the commodity production, each branch consumes some resources, which are either own products or products of other branches, or labour-power. The labour-power consumes resources for its reproduction. We take into consideration the simple reproduction case, all surplus value being consumed unproductively. Thus,

on the one hand, a supply is born — the produced commodities, on the other hand there is a demand for consumption.

Of course, the demand and supply are not simultaneous, as the demand precedes the production, hence the supply. Nevertheless, in a certain time interval, they may be considered as simultaneous.

2.1.1. *Formation of the Commodities' Momentary Market Value*

Let us consider an economy with $(m + 1)$ branches denoted by $i (i = 1, ..., m + 1)$, branch $(m + 1)$ corresponding to the labour-power (workers).

Let us assume that there are n capitalists, owners of the fixed means of production F_k^j $(j = 1, ..., n)$, produced by branches $k (k = 1, ..., m)$, measured in physical units. We suppose that there exist, for any capitalist, even for the production of the same commodity, various production conditions (hence various productive consumptions) and we shall define for each capitalist j, specific consumptions (by unit of product).

The components of the constant capital are also products of the economic branches $k (k = 1, ..., m)$. We denote, for the production of commodity $i (i = 1, ..., m)$ by the capitalist j, by f_{ki}^j the specific wear of the fixed, assets, by g_{ki}^j the specific consumption of the circulating assets (raw materials, materials, semimanufactured goods-for $k = 1, ..., m$ and labour-power for $k = m + 1$). Then, for the whole quantity Q_{ij} of commodity i, produced by capitalist j, the consumption of products of branch k will be: for the fixed assets (wear):

$$F_{ki}^j = Q_{ij} f_{ki}^j, \quad k = 1, ..., m \tag{2.1}$$

for the constant circulating assets:

$$G_{ki}^j = Q_{ij} g_{ki}^j, \quad k = 1, ..., m \tag{2.2}$$

and for the labour-power:

$$H_i^j = Q_{ij} g_{k+1,i}^j. \tag{2.3}$$

We denote by q_k^{m+1} the mean consumption for the reproduction of a labour power unit, and we obtain the labour-power H_i^j consumption:

$$I_{ki}^j = H_i^j \; q_k^{m+1}, \quad k = 1, ..., m. \tag{2.4}$$

After the complete wear of a unit of the component k of the fixed assets, the replacement of the respective equipment is done. Nevertheless, we should assume, for simplifying, that an amount equal to the sum of wears ΣF_{ki}^j at moment $(t - 1)$ is subtracted from the fixed assets, and that these assets are replaced by an equal quantity, at moment t, hence the total volume of the demand of products from branch k will be:

$$VD_k(t) = \sum_i \sum_j F_{ki}^j(t - 1) + G_{ki}^j(t) + I_{ki}^j(t) + q_k(t), \quad k = 1, ..., m \tag{2.5}$$

where q_k is the non-productive consumption of the capitalist class, and the total necessary labour-power will be:

$$VD_{k+1}(t) = \sum_i \sum_j H_i^j. \tag{2.6}$$

We denote by $F_{ki}^j(Q_{ij})$ the share of the fixed assets F_k^j used to produce the quantity Q_{ij} of commodities i by capitalist j, and:

$$\frac{F_{ki}^j(Q_{ij})}{F_k^j} = \alpha_{ki}^j(Q_{ij}), \quad k = 1, ..., m. \tag{2.7}$$

Within certain bounds, the ratio between $F_{ki}^j(Q_{ij})$ and Q_{ij} is kept unchanged; we shall put hence:

$$\frac{F_{ki}^j(Q_{ij})}{Q_{ij}} = \varphi_{ki}^j, \quad k = 1, ..., m. \tag{2.8}$$

Hence, for a capitalist j, the way he invests his capital may be expressed either by the amounts $Q_i^{r_j}(i = 1, ..., m)$, of course subjected to the constraints

$$\sum_{i=1}^{m} \varphi_{ki}^i Q_{ij}^r = F_k^j, \quad k = 1, ..., m, \; r_j \in R_j \tag{2.9}$$

(where r_j denotes an arbitrary numerotation of the possible amounts Q_{ij}), or through a square matrix $\alpha^{r_j} = (\alpha_{ki}^{r_j})\; i, k = 1, ..., m$, of components $\alpha_{ki}^{r_j} \leqslant 1$, with

$$\sum_{i=1}^{m} \alpha_{ki}^{r_j} = 1, \; k = 1, ..., m, \; r_i \in R_j, \tag{2.10}$$

which express, according to (2.7), the proportions of the utilization of the fixed assets F_k^j for producing the amounts of goods $Q_{ij}^{r_j}$. From relation (2.8) we can easily infer the amounts $Q_i^{r_j}$ and, further on, the productive consumption, determined by this investment structure. Reversely, the amounts $Q_{ij}^{r_j}$ being given, we can calculate the components of matrix α^{r_j}. We shall call α^{r_j} the structure by branches of the investment of capital j.

At a given moment, due to the various production conditions, in each branch i the commodities (labour-power) have not the same value, but each commodity unit (labour-power unit) has an individual value VI.

As pointed to in the first part of the work, VI will be given by the working time necessary to the production (reproduction) of the respective commodity unit (labour-power unit) in the particular production (reproduction) conditions, and by a mean labour intensity. In considering the probability of selecting, in a random manner, a commodity unit from the quantity of commodities produced in branch i, each commodity unit has the same probability as any other to be selected; a probability, equal to its weight in the total commodity volume produced in the respective branch, can be attached to an arbitrary quantity of commodity. In this way, we can define magnitude VI as a random variable. In the case of a mass production (of a great number of workers) the individual values cover an interval A_i of the real straight line (we can assume they varies continuously) and we can attach them a probability distribution Φ_i, possibly

defined with the help of a probability density. The mean value of this random variable will be the market value VP_i of the commodities from i (or the labour-power). Marx indicates that "only in uncommon combinations the commodities produced in the worst or in the best conditions regulate the market value" (see [1], first part, pp. 181); so we can obtain a faithful representation by considering that individual value of commodities in each branch i (the labour-power individual value) has a normal (Gaussian) distribution, of mean value $m_i^0 = VP_i$ and of dispersion $(\sigma_i^0)^2$ (see [4], pp. 307). The commodities produced by the assets F_{ki}^i will have their individual values included in a neighbourhood A_{ij} of the individual value $\overline{VI}_{ij}$ (the mean of the individual values from interval A_{ij}). More precisely, if Φ_{ij} is the distribution on the interval A_{ij}, then

$$\overline{VI}_{ij} = \int_{A_{ij}} VI \, \mathrm{d}\Phi_{ij}, \tag{2.11}$$

and the probability P_{ij}, for any event $B_{ij} \subset A_{ij}$ is defined as:

$$P_{ij}(VI \in B_{ij}) = P_i(VI \in B_{ij} \mid VI \in A_{ij}) =$$

$$= \frac{P_i(VI \in B_{ij},\ VI \in A_{ij})}{P_i(VI \in A_{ij})} = \frac{P_i(VI \in B_{ij})}{P_i(VI \in A_{ij})}. \tag{2.12}$$

Intervals A_{ij} form a cover for A_i. In these conditions and using (2.11) and then (2.12), we get:

$$\sum_{j=1}^{n} \overline{VI}_{ij} P_i(VI \in A_{ij}) = \sum_{j=1}^{n} P_i(VI \in A_{ij}) \int_{A_{ij}} VI \, \mathrm{d}\, \Phi_{ij} =$$

$$= \sum_{j=1}^{n} P_i(VI \in A_{ij}) \frac{1}{P_i(VI \in A_{ij})} \int_{A_{ij}} VI \, \mathrm{d}\, \Phi_i = \sum_{j=1}^{n} \int_{A_{ij}} VI \, \mathrm{d}\, \Phi_i. \tag{2.13}$$

When $A_{ij} \cap A_{ik} = \nsubseteq$, $j \neq k$, as $\bigcup_{j=1}^{n} A_{ij}, = A_i$ it results:

$$\sum_{j=1}^{n} \overline{VI}_{ij} P_i(VI \in A_{ij}) = \int_{A_i} VI \, \mathrm{d}\, \Phi_i = VP_i \tag{2.14}$$

hence the market value VM_i is the mean of the values $\overline{VI}_{ij}$.

We may also presume that the goods produced by assets F_{ki}^i have the individual values all equal to $\overline{VI}_{ij}$. If Q_{ij} is the quantity of commodity produced by capitalist j in branch i, then her total market value will be:

$$V_{ij} = Q_{ij} VM_i, \tag{2.15}$$

and the total goods amount produced in branch i:

$$Q_i = \sum_j Q_{ij}. \tag{2.16}$$

2.1.2. *Formation of the Market Price*

Demand and supply for each commodity i (labour-power inclusively) meet each other in the market. The demand is the expression of the needs of the future production in another distribution of the capitals and labour-power by branches. Up to the moment of actually concluding the sales contract, there is a permanent fluctuation of buyers' capitals in view of the best conditions for the future investment. From the moment the sales contract was concluded, the new distribution by branches of the capitals j is determined.

Of course, as a matter of fact, not all commodities arrive in the market at the same moment, neither within a branch, nor (so much the less) within the whole economy. Throughout the modelling, we disregard the differences of the capitals' rotation periods. On the other hand, all that happens within the time period between two successive moments t and $(t + 1)$ is considered as simultaneous (the time is discrete).

The supply is characterized, for a branch i, by the production volume $Q_i(t)$ and by the distribution Φ_i^s of the individual values of the produced goods (labour force), we can presume as normal, having the mean $m_i^s(t) = VM_i(t)$ and the dispersion $(\sigma_i^s(t))^2$ $(i = 1, ..., m + 1)$.

The demand will be characterized by the volumes $VD_i(t + 1)$. We shall suppose that the prices $PD_i(t + 1)$ the buyers are willing to pay for goods, and the labour-power prices (wages for workers) have also a distribution Φ_i^d (possibly normal), with the mean $m_i^d(t + 1)$ and the dispersions $(\sigma_i^d(t + 1))^2$, $(i = 1, ..., m + 1)$. (The supposition that supply and demand have normal distributions is not strictly necessary).

The market price $PM_i(t)$ will be placed in a "equilibrium interval" E_i, for which the volume Q_{E_i} of the commodities (labour-power) supply, whose individual value is situated within that interval, is equal to the volume VD_{E_i} of the commodities (labour-power) demand, whose price the buyer is willing to pay is also situated within that interval.

Taking into account the fact that the weight of Q_{E_i} within Q_i results from probability $P_i(VI \in E_i)$, i.e.

$$Q_{E_i} = Q_i \cdot P_i^s(VI \in E_i) \tag{2.17}$$

and acting similarly for the demand, we finally obtain following equations of the equilibrium intervals (the supply is taken for moment t and the demand for $(t + 1)$):

$$Q_i \cdot P_i^s(VI \in E_i) = VD_i \cdot P_i^d(PD_i \in E_i),\ i = 1, ..., m + 1. \tag{2.18}$$

Let be $E_i = [\alpha_i^1, \alpha_i^2]$; then the equation can be written for a certain i, when f_i^s, f_i^d are the probability densities, corresponding to distributions Φ_i^s, Φ_i^d (if these densities do exist):

$$Q_i \int_{\alpha_i^1}^{\alpha_i^2} f_i^s(x)\, \mathrm{d}x = VD_i \int_{\alpha_i^1}^{\alpha_i^2} f_i^d(x)\, \mathrm{d}x. \tag{2.19}$$

Applying Lagrange's theorem for the above integrals (the market price PM_i being placed within the interval E_i), we may say that this price can be approximated through a solution of the equation:

$$Q_i f_i^s(x) = VD_i f_i^d(x) \tag{2.20}$$

We say "approximated" because the probability of the event that the market price be *exactly* this solution is null; but PM_i will be situated certainly within the interval E_i, neighbourhood of the solution. In the case of the normal distributions we obtain (the lower indexes omitted):

$$\frac{Q}{\sigma^s \sqrt{2\pi}} e^{-\frac{(x-m^s)^2}{2(\sigma^s)^2}} = \frac{VD}{\sigma^d \sqrt{2\pi}} e^{-\frac{(x-m^d)^2}{2(\sigma^d)^2}} \tag{2.21}$$

We reduce by $\sqrt{2\pi}$, calculate the logarithms and group the terms and then we obtain a quadratic equation in x:

$$2[(\sigma^s)^2 - (\sigma^d)^2]x^2 - 2[m^s(\sigma^d)^2 - m^s(\sigma^d)^2]x + (m^s)^2 - (m^d)^2 + \\ + \ln\left[\left(\frac{VC \cdot \sigma^s}{Q \cdot \sigma^s}\right)^4 (\sigma^s)^2 (\sigma^d)^2\right] = 0. \tag{2.22}$$

If the volumes of demand and supply are equal ($Q = VD$) and so are the dispersions ($\sigma^s = \sigma^d$), it results that

$$x_1 = x_2 = \frac{m^s + m^d}{2}, \tag{2.23}$$

hence the market price PM_i is the mean of the average values of demand and supply. Consequently, in the case when the mean values coincide, the market price must be taken as $PM_i = VM$, i.e. equal to the market value. In general, the root of the equation (2.24) placed between the two means m_i^s and m_i^d (see Fig. 3), must be taken as market price PM_i.

2.2. *System's States and Evolution*

We shall call "primary" process, the process whose set of states is $S = \{s\}$, a state s being characterized by:

— the investments structure, $\alpha^{r_j} = (\alpha_{ki}^{r_j})$, with $r_j \in R_j$, of the capitals $j (j = 1, ..., n)$ in branches i of the economy $(i = 1, ..., m)$ (or the corresponding goods amounts $Q_{ij}^{r_j}$).

There are given, in the same time:

— the intervals A_{ij} wherein the individual values of the goods produced as a result of these investments are placed; if $\alpha_{ki}^{r_j} = 0$, $(k = 1 ..., m)$, then $A_{ij} = \emptyset$. We denote $A_i = \bigcup_{j=1}^{n} A_{ij}$;

— the distributions Φ_i^s and Φ_i^d of the commodity (labour-power) supply and demand from branches i $(i = 1, ..., m + 1)$.

We suppose that the number m of the economy branches, the ratios φ_{ki}^{j}, as well as the specific consumptions are the same for any state s of the process and during its whole evolution.

We shall study only the case of the simple reproduction and of the stationary labour -power. We disregards the concentration and the centralization of the capitals and the technical progress. We neglect the differences between the capitals' turnover times in various branches (we consider that, between two successive moments t and $(t + 1)$ of the process, a single turnover takes place).

The profits obtained as a result of the investments from a certain state $s_t \in S$, wherein the system finds itself at moment t, are determined, as allready seen, by demand distribution and volume, characterizing state $s_{t+1} \in S$ of the "primary" system at moment $(t + 1)$. We suppose that the passage from a state to another of the process is done because each capitalist wants to invest his capital in the branches which would bring him the highest profit, taking into account only the last prices situation. Capitalist j will select for the moment $(t + 1)$ a structure α^{rj} of the capital investment (or he will decide to produce these quantities Q_{ij}^{r} of commodities, respectively) for which:

$$\sum_{i=1}^{m} Q_{ij}(s_{t+1})\ PM_i(s_t, s_{t-1}) - pc_j(s_{t+1}, s_t,\ s_{t-1}) \to max \tag{2.24}$$

where $pc_j(s_{t+1}, s_t, s_{t-1})$ is the cost of production, at moment $(t + 1)$, of the quantities Q_{ij} of commodities, if the process was at moment t in state s_t and before, in state (s_{t-1}) (we give further on its formula); prices PM_i depend on the ratio between demand and supply, hence on the totality of investment decisions at moment t and $(t - 1)$, which determine states s_t and s_{t-1}. Constraints:

$$\sum_{i} Q_{ij}(s_{t+1})\ \varphi_{ki}^{j} = F_k^j,\ k = 1, ..., m \tag{2.25}$$

must be added, and if need be, also the constraint that the total price of the necessary circulation assets and the price of replacement of the share of fixed assets, worn out at moment t, do not exceed the "liquid capital" disponibilitiee (if there is lack of credit). A linear programming problem is obtained, whoss optimal solutions represent the possible investment decisions of capitalist j. We obtain the corresponding investment structures through relations (2.24), (2.25). Even if the investment decisions are not optimal solutions, but only possible solutions of the above linear programming problem, we shall presume that the set of corresponding investment structures $\{\alpha^{rj}\}_{r_j \in R_j}$ is a finite set, hence that R_j is finite. Supposing that state s_t of the process at moment t is given by a random variable denoted by $X(t)$, and the investment structure α^{rj} of capital j — by a random variable $I_j(t)$, then the probability that its investment structure at moment $(t + 1)$ be α^{rj}, taking into account the previous states of the process is according to the above mentioned:

$$p(I_j(t + 1) = \alpha^{rj} \mid X(t) = s_t, ...,\ X(0) = s_0) =$$
$$= p(I_j)(t + 1) = \alpha^{rj}\ X(t) = s_t,\ X(t - 1) = s_{t-1}) \tag{2.26}$$

and this is valid for any moment t of the evolution. The transition of the process in a state s being determined by the investment decisions $I_j (j = 1, ..., n)$, it results that the transition probability of the system from a state to another one, will be:

$$p(X(t+1) = s_{t+1} \mid X(t) = s_t,\ S(t-1) = s_{t-1}, ...,\ X(0) = s_0) =$$

$$= p(I_1(t+1) = \alpha^{r_1}(s_{t+1}), ...,\ I_n(t+1) = \alpha^{r_n}(s_{t+1}) \mid X(t) =$$

$$= s_t, ...,\ X(0) = s_0). \tag{2.27}$$

Supposing that the investment decisions for various capitals are independent between them, at a given moment, we obtain, taking into account (2.26):

$$p(X(t+1) = s_{t+1} \mid X(t) = s_t, ...,\ X(0) = s_0) =$$

$$= \prod_{j=1}^{n} p(I_j(t+1) = \alpha^{r_j}(s_{t+1}) \mid X(t) = s_t, ...,\ X(0) = s_0) =$$

$$= \prod_{j=1}^{n} p(I_j(t+1) = \alpha^{r_j}(s_{t+1}) \mid X(t) = s_t,\ X(t-1) = s_{t-1}) =$$

$$= p(I_1(t+1) = \alpha^{r_j}(s_{t+1}), ...,\ I_n(t+1) = \alpha^{r_n}(s_{t+1}) \mid X(t) =$$

$$= s_t,\ X(t-1) = s_{t-1}) =$$

$$= p(X(t+1) = s_{t+1} \mid X(t) = s_t,\ X(t-1) = s_{t-1}). \tag{2.28}$$

Hence, the state of the "primary" process, at moment $(t+1)$, does not depend but by the states of the process in the last two previous moments, t and $(t-1)$.

We shall call "secondary" process, the process whose states σ are formed from the successive state pairs of the "primary" process evolution:

$$\sigma = (s_t,\ s_{t-1})\ s_t,\ s_{t-1} \in S; \tag{2.29}$$

consequently, the states' set $\mathcal{S}$ of the "secondary" system is the Cartesian product of set S by itself. The evolution moments of this system will be situated at time intervals $\Delta\tau = 2\Delta t$, where Δt are the intervals between two moments, t and $(t+1)$, of the "primary" process. We use relation (2.28) and obtain:

$$p\,(X(t+2) = s_{t+2},\ X(t+1) = s_{t+1} \mid X(t) = s_t, ...,\ X(0) = s_0) =$$

$$= p(X(t+2) = s_{t+2} \mid X(t+1) = s_{t+1} \mid X(t) = s_t, ...,\ X(0) = s_0) \times$$

$$\times\ p(X(t+1) = s_{t+1} \mid X(t) = s_t,\ X(t-1) = s_{t-1}, ...,\ X(0) = s_0) =$$

$$= p(s_{t+2} \mid s_{t+2},\ s_t) \times p(s_{t+1} \mid s_t,\ s_{t-1}). \tag{2.30}$$

It results that the pairs $(X(t+1),\ X(t+2))$ build up a Markov chain with the following transition probabilities:

$$p((s_{t+2},\ s_{t+1}) \mid (s_t,\ s_{t-1})) = p(s_{t+2} \mid s_{t+1},\ s_t) \times p(s_{t+1} \mid s_t,\ s_{t-1}). \tag{2.31}$$

Thus we proved that the "*secondary*" *process is a Markov chain* and the state of the process at a given moment $(\tau + 1)$ depends only on the process state at moment τ, whichever τ would be and the time is discrete.

We say that a Markov chain has stationary transition probabilities or, more briefly, that it is *stationary*, if for any pair of states σ, σ' and for any τ (denoting by $\mathscr{X}(\tau)$ the random variable which gives state τ at moment σ):

$$p(\mathscr{X}(\tau + 1) = \sigma \mid \mathscr{X}(\tau) = \sigma') = p(\sigma \mid \sigma'). \tag{2.32}$$

If we suppose the homogeneity in time of the behaviour of those who invest capitals, i.e.:

$$p(I_j(t + 1) = \alpha r_j \mid X(t) = u,\ X(t - 1) = v) = p(\alpha r_j \mid u, v) \tag{2.33}$$

which ever t and $u, v \in S$ would be, it results from relations (2.31) and (2.28) that the "secondary" process is a stationary (homogeneous) Markov chain. Further on, when not specifying anything else, we understand by process, the "secondary" one. We recall, applied to our process, some notions from the teory of discrete time, stationary Markov chains with a finite number of states (ser, for details, [3] — [5]).

The evolution of the process is determined by the matrix P of the transition probabilities:

$$P = (p(\sigma \mid \sigma'))_{\sigma,\sigma' \in \mathcal{S}} = (p(\mathscr{X}(\tau + 1) = \sigma \mid \mathscr{X}(\tau) = \sigma'))_{\sigma,\sigma' \in \mathcal{S}} \tag{2.34}$$

and by the initial distribution (at moment $\sigma = 0$) of the process states:

$$P_0 = (p_0(\sigma))_{\sigma \in \mathcal{S}} = (p(\mathscr{X}(0) = \sigma))_{\sigma \in \mathcal{S}}. \tag{2.35}$$

For two states σ and σ' of the process, we denote by $p^{(\tau)}(\sigma \mid \sigma')$ *the transition probability at step* τ (the probability that the system be at moment τ in state σ, if it was at moment zero in state σ'), hence:

$$p^{(\tau)}(\sigma \mid \sigma') = p(\mathscr{X}(\tau) = \sigma \mid \mathscr{X}(0) = \sigma'). \tag{2.36}$$

This probability can be calculated recursively, since:

$$p^{(1)}(\sigma \mid \sigma') = p(\sigma \mid \sigma') = p(\mathscr{X}(1) = \sigma \mid \mathscr{X}(0) = \sigma') \tag{2.37}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$p^{(\tau)}(\sigma \mid \sigma') = \sum_{\sigma'' \in \mathcal{S}} p^{(\sigma-1)}(\sigma \mid \sigma')\, p(\sigma'' \mid \sigma)$$

where σ'' are the states we come to from σ' through a step, and from where we come to σ through $(\tau - 1)$ steps.

We say that the transistion form state σ' to state σ is *possible* if there is a $\tau < \infty$ with

$$p^{(\tau)}(\sigma \mid \sigma') > 0. \tag{2.38}$$

We say that the two states σ and σ' *communicate*, if transitions $\sigma \to \sigma'$ and $\sigma' \to \sigma$ are possible. A subset $\mathscr{A}$ of states $(\mathscr{A} \subset \mathcal{S})$ is called *closed*, if starting from a state in $\mathscr{A}$, we remain still in $\mathscr{A}$, i.e.:

$$p(\mathscr{A} \mid \sigma) = 1. \tag{2.39}$$

We say that the evolution is *periodical* if there is one biggest integer number $d > 1$, called *chain's period*, such that the set of the states can be decomposed into d disjoint subsets $\mathcal{B}_1, \ldots, \mathcal{B}_d$, each of them being closed under the probability of transition at step d.

If we suppose that there is not a reason that once a capitalist j made the investment corresponding to α^{rj} at a given moment, he could not make any other investment with structure α^{rj} after a certain time, it results that *all system's states are communicating.*

In the case when all states are communicating between them and the evolution is non-periodical, one can prove that exist and are finite the limits:

$$\lim_{\tau \to \infty} p^{(\tau)}(\sigma \mid \sigma') = \bar{p}(\sigma), \ \forall \sigma, \sigma' \in \mathcal{S}. \tag{2.40}$$

The Markov chains having the property (2.40) are called *ergodic.* The $\bar{p}(\sigma)$ probabilities, called *limiting state probabilities*, express the probability that the system will be in state σ, after a sufficiently long time period from the beginning of the process. It is clear that they are independent of the initial state σ of the chain and they are the single solution of the linear equations system in $p(\sigma)$:

$$\sum_{\sigma \in \mathcal{S}} p(\sigma \mid \sigma')\, p(\sigma) = p(\sigma'), \sigma' \in \mathcal{S}$$

$$\sum_{\sigma \in \mathcal{S}} p(\sigma) = 1. \tag{2.41}$$

2.3. *Formation of the Profit's General Rate*

The mean rate of profit of a capitalist j is the limit toward which the mean of profit's rate of the respective capitalist trends in time.

Further on, when we shall speak of the investments from a state $\sigma = (s, s')$ of the "secondary" process, we shall refer to the investments from state s of the "primary" process. Similarly, the profit in state σ is the one obtained by selling the commodities produced in the conditions of state s. If the process was, at the initial moment, in state σ', the expected value of the profit by capitalist j at moment $\tau = 1$ will be:

$$pr_j(1, \sigma') = \sum_{\sigma \in \mathcal{S}} pr_j(\sigma)\, p(\sigma \mid \sigma')\sigma' \in \mathcal{S}. \tag{2.42}$$

After τ steps, the mean in time of the profits' expected values of capitalist j will be:

$$pr_j(\tau, \sigma') = \frac{1}{\sigma} \sum_{\sigma \in \mathcal{S}} pr_j(\sigma) \sum_{q=1}^{\tau} p^{(q)}(\sigma, \sigma'), \ \sigma' \in \mathcal{S}. \tag{2.43}$$

In order to see which are the trends of these magnitudes in a sufficiently great time interval, we let $\tau \to \infty$. We noticed that relations (2.40) take place if the system is ergodic.

In this case we have also:

$$\lim_{\tau \to \infty} \frac{1}{\tau} \sum_{q=1}^{\tau} p^{(q)}(\sigma \mid \sigma') = \bar{p}(\sigma), \ \sigma \in \mathcal{S} \tag{2.44}$$

hence we obtain the mean profit of capitalist j on a longlasting period of time, by passing to limit in (2.43):

$$\overline{pr_j} = \lim_{\tau \to \infty} pr_j(\tau, \sigma') = \sum_{\sigma \in \mathcal{O}} pr_j(\sigma)\, \bar{p}(\sigma) \tag{2.45}$$

which does not depend on the initial state of the system. Let us now define the magnitudes $p^{r_j}(\sigma)$. The profit obtained by capitalist j in state σ, if the process was at the previous moment in anyother state (which we denoted also by σ') is equal with the difference between the total price of the capital after finishing the production process (composed from the price of all produced commodities $\sum_i Q_{ij} PM_i(\sigma)$ and from the price of the left fixed assets $\sum_{k=1}^{m} \left[F_k^j(\sigma) - \right.$ $\left. - \sum_i F_{ki}'^j(\sigma)\right] PM_k(\sigma)$ and the capital's price before beginning the production (formed by the price of the existing fixed assets and of the constant and variable circulating assets, to which the price of the fixed assets, worn out in the previous state σ' and replaced in σ, as agreed, is added), hence

$$pr_j(\sigma, \sigma') = \sum_{i=1}^{m} Q_{ij}(\sigma)\, PM_i(\sigma) +$$

$$+ \sum_{k=1}^{m} \left[\sum_{i=1}^{m} F_{ki}^j(\sigma) - \sum_{i=1}^{m} F_{ki}'^j(\sigma)\right] PM_k(\sigma) - c(\sigma, \sigma') \tag{2.46}$$

where we denoted the capital's price from state σ, before the beginning of the production, by

$$c(\sigma, \sigma') = \sum_{k=1}^{m} \left[\sum_{i=1}^{m} F_{ki}^j(\sigma) + \right.$$

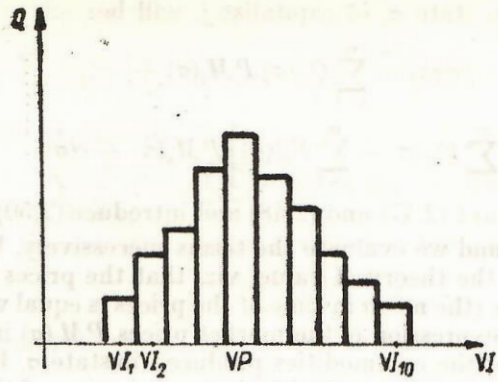
$$\left. + \sum_{i=1}^{m} G_{ik}^j(\sigma)\right] MP_k(\sigma') + \sum_{i=1}^{m} H_i^j(\sigma)\, PM_{m+1}(\sigma') \cdot \tag{2.47}$$

The profit $pr_j(\sigma, \sigma')$ depends on state σ' only through the agency of term $c(\sigma, \sigma')$. For a moment sufficiently distant from the beginning of the process, let $\bar{p}(\sigma' | \sigma)$ be the probability that the process would have been, at the previous moment, in state σ', if at the respective moment it is founding in state σ, hence:

$$\bar{p}(\sigma' \mid \sigma) = \frac{\bar{p}(\sigma')\; p(\sigma \mid \sigma')}{\bar{p}(\sigma)} \,. \tag{2.48}$$

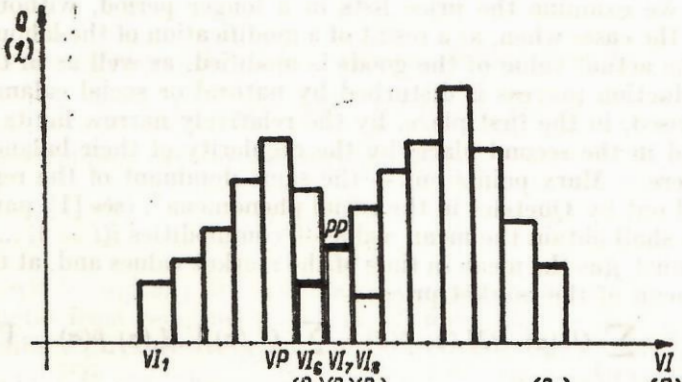
Then, the mean of the capital's price in state σ, after a sufficiently long time from the beginning of the process, will be:

$$c(\sigma) = \sum_{\sigma' \in \mathcal{O}} c(\sigma, \sigma')\; \bar{p}(\sigma' \mid \sigma) \tag{2.49}$$



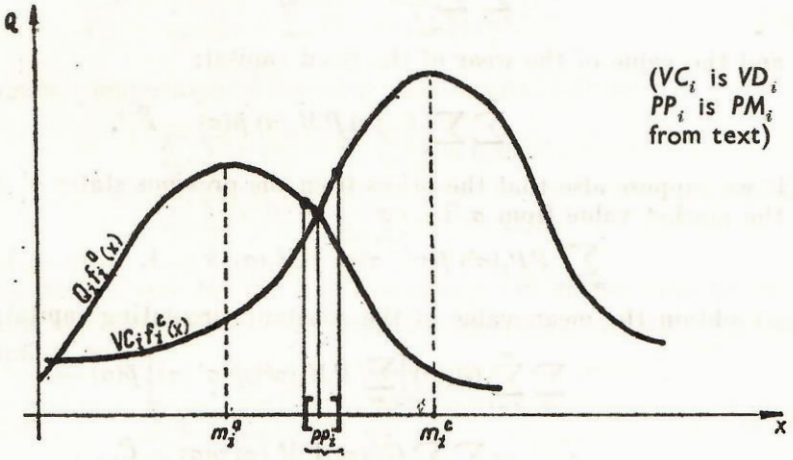
(VP is VM from text)

Fig. 1



(PP is PM, VP is VM from text)

Fig. 2



(VC_i is VD_i PP_i is PM_i from text)

Fig. 3

and the mean profit, in state σ, of capitalist j, will be:

$$pr_j(\sigma) = \sum_{i=1}^{m} Q_{ij}(\sigma)\, PM_i(\sigma) +$$

$$= \sum_{k=1}^{m} \left[\sum_{i=1}^{m} F_{ki}^{j}(\sigma) - \sum_{i=1}^{m} F_{ki}^{\prime j}(\sigma) \right] PM_k(\sigma) - c(\sigma) \cdot \tag{2.50}$$

We take into account (2.47) and (2.48) and introduce (2.50) in the expression of $\overline{pr_j}$ from (2.45) and we evaluate the terms successively. We shall use a hypothesis essential in the theory of value, viz. that the prices are oscillating in time, round the value (the mean in time of the prices is equal with the value) and hence that, — the expression of the market prices $PM(\sigma)$ in terms of the market value $VM_i(\sigma)$ of the commodities produced in state σ, being given — the mean in time of the prices is equal with the mean in time of the momentary values $V_{ij}(\sigma) = Q_{ij}(\sigma)\ VM_{ij}(\sigma)$.

"If we examine the price lists in a longer period, without taking into account the cases when, as a result of a modification of the labour's productive force, the actual value of the goods is modified, as well as in the cases when the production process is disturbed by natural or social calamities, we shall be surprised, in the first place, by the relatively narrow limits of the oscillations and in the second place, by the regularity of their balancing. We shall found here — Marx points out — the same dominant of the regulating mean signalled out by Quetelet in the social phenomena". (see [1], part II, pp. 398). Thus we shall obtain the mean value of commodities $i (i = 1, ..., m)$ produced by capitalist j(as the mean in time of the market values and, at the same time, as the mean of the market prices):

$$\sum_{\sigma \in \mathcal{S}} Q_{ij}(\sigma)\ PM_i(\sigma)\ \bar{p}(\sigma) = \sum_{\sigma \in \mathcal{S}} Q_{ij}(\sigma)\, VM_i(\sigma)\ \bar{p}(\sigma) = \bar{V}_{ij}; \tag{2.51}$$

alike-the value of the fixed capital used by the capitalist in branch i:

$$\sum_{k=1}^{m} \sum_{\sigma \in \mathcal{S}} F_{ki}^{j}(\sigma)\ PM_k(\sigma)\ \bar{p}(\sigma) = \bar{F}_{ij} \tag{2.52'}$$

and the value of the wear of the fixed capital:

$$\sum_{k=1}^{m} \sum_{\sigma \in \mathcal{S}} F_{ki}^{\prime j}(\sigma)\, PM_k(\sigma)\, \bar{p}(\sigma) = \bar{F}_{i}^{\prime j}. \tag{2.52''}$$

If we suppose also that the prices from the previous states σ' oscillate round the market value from σ, i.e.

$$\sum_{\sigma' \in \mathcal{S}} PP_k(\sigma')\ \bar{p}(\sigma' \mid \sigma) = VM_k(\sigma),\ k = 1, ...,\ m + 1 \tag{2.53}$$

we obtain the mean value of the constant circulating capital:

$$\sum_{\sigma} \sum_{k=1}^{m} G_{ik}^{j}(\sigma) \left[\sum_{\sigma'} PM_k(\sigma')\ \bar{p}(\sigma' \mid \sigma) \right] \bar{p}(\sigma) =$$

$$= \sum_{\sigma} \sum_{k=1}^{m} G_{ik}^{j}(\sigma)\ VM_k(\sigma)\, \bar{p}(\sigma) = \bar{C}_{ij}. \tag{2.54}$$

Similarly, for the variable capital:

$$\sum_{\sigma} H_i^j(\sigma)\left[\sum_{\sigma'} PM_{m+1}(\sigma')\ \bar{p}(\sigma' \mid \sigma)\right]\bar{p}(\sigma) = \overline{CP}_{ij}^{v} \tag{2.55}$$

and for the fixed capital:

$$\sum_{\sigma}\sum_{k=1}^{m} F_{ki}^{j}(\sigma)\left[\sum_{\sigma'} PM_k(\sigma')\,\bar{p}(\sigma' \mid \sigma)\right]\bar{p}(\sigma) = \bar{F}_{ij}. \tag{2.56}$$

The value of the constant capital invested in branch i by capitalist j will be:

$$\overline{CP}_{ij}^{c} = \bar{F}_{ij} + \bar{C}_{ij}, \tag{2.57}$$

the organic composition of the capital, as ratio of the constant and variable capital:

$$\bar{k}_{ij} = \frac{\overline{CP}_{ij}^{c}}{\overline{CP}_{ij}^{v}}, \tag{2.58}$$

the surplus value produced by the investment in that branch:

$$\bar{p}_{ij} = \overline{V} - (\overline{F_i'^j} + \overline{C}_{ij} + \overline{CP}_{ij}^{v}) = \overline{pr}_{ij} \tag{2.59}$$

and the corresponding rate of the surplus value:

$$\bar{p}'_{ij} = \frac{\bar{p}_{ij}}{\overline{CP}_{ij}^{v}}. \tag{2.60}$$

For the whole capital j, by summing according to index i the corresponding magnitudes from relations (2.51), (2.52) and (2.54) — (2.59), we obtain the fixed capital $\bar{F}_j$, respectively the constant circulating capital $\bar{C}_j$, the total constant capital $\overline{CP}_j^{c}$ and the variable capital $\overline{CP}_j^{v}$. The mean profit of capital j is formed from the sum of the surplus values produced on average in each branch, hence:

$$\overline{pr}_j = \sum_i \bar{p}_{ij} = \sum_i \overline{pr}_{ij} = \sum_i \bar{p}'_{ij}\frac{\overline{CP}_{ij}}{1+\kappa_{ij}} \tag{2.61}$$

The mean organic composition of the total social capital will be:

$$\bar{k}_M = \frac{\sum_j\sum_i \overline{CP}_{ij}^{c}}{\sum_j\sum_i \overline{CP}_{ij}^{v}} = \frac{\sum_j\sum_i \dfrac{\bar{k}_{ij}}{1+\bar{k}_{ij}}\overline{CP}_{ij}}{\sum_j\sum_i \dfrac{1}{1+\bar{k}_{ij}}\overline{CP}_{ij}} \tag{2.62}$$

and the mean profit's rate for the whole economy will be the ratio between the total mean profit (the sum of the profits achieved by capitals $\overline{CP}_j$), and the total capital, hence:

$$\overline{pr}'_M = \frac{\sum_j \overline{pr}_j}{\sum_j \overline{CP}_j} = \frac{\sum_j\sum_i \bar{p}_{ij}}{\sum_j \overline{CP}_j}. \tag{2.63}$$

If the surplus value rate is the same for the whole economy, irrespective of branch or capital, i.e.

$$\bar{p}'_{ij} = \bar{p}', \quad i = 1, ..., m; j = 1, ..., n, \tag{2.64}$$

if the organic composition of the capitals invested in a branch i, no matter which, is the same, irrespective of the capital owner, i.e.:

$$\bar{k}_{ij} = \bar{k}_i, \quad i = 1, ..., m; j = 1, ..., n. \tag{2.65}$$

and if the proportion β_{ij} between the capital $\overline{CP}_{ij}$, invested on average in a branch, and the whole capital $\overline{CP}_j = \overline{CP}_j^v + \overline{CP}_j^c$ of the owner is the same, irrespective of the capital, hence:

$$\beta_{ij} = \frac{\overline{CP}_{ij}}{\overline{CP}_j} = \beta_i, \; i = 1, ..., m, \; j = 1, ..., n \tag{2.66}$$

then the mean profit of capital j will be

$$\overline{pr}_j = \bar{p}' \; \overline{CP}_j \sum_i \frac{\beta_i}{1+\bar{k}_i} = \overline{pr}'_j \; \overline{CP}_j, \quad j = 1, ..., n \tag{2.67}$$

where $\overline{pr}'_j$ is the mean rate of the profit of this capital:

$$\overline{pr}'_j = \frac{\overline{pr}_j}{\overline{CP}_j} = \bar{p}' \sum_i \frac{\beta_i}{1+\bar{k}_i}, \; j = 1, ..., n. \tag{2.68}$$

In this case, the mean rate of the profit for the whole economy will be:

$$\overline{pr}'_M = \frac{\left(\bar{p}' \sum_i \frac{\beta_i}{1+\bar{k}_i}\right) \sum_j \overline{CP}_j}{\sum_j \overline{CP}_j} = \bar{p}' \sum_i \frac{\beta_i}{1+\bar{k}_i} = \overline{pr}'_j, \tag{2.69}$$

hence the profits'rates of capitals $\overline{CP}_j$ are equal between them and equal to the mean profit's rate in the whole economy (which is consequently a general profit's rate), the profits of the capitals $\overline{CP}_j$ being proportional to the respective capitals'volumes.

In the same conditions (2.64) — (2.66), the mean organic composition of capitals CP_j

$$\bar{k}_j = \frac{\sum_i \overline{CP}_{ij}^c}{\sum_i \overline{CP}_{ij}^v} = \frac{\sum_i \frac{\bar{k}_{ij}}{1+\bar{k}_{ij}} \overline{CP}_{ij}}{\sum_i \frac{1}{1+\bar{k}_{ij}} \overline{CP}_{ij}} = \frac{\sum_i \frac{\bar{k}_{ij}}{1+\bar{k}_{ij}} \beta_{ij}}{\sum_i \frac{1}{1+\bar{k}_{ij}} \beta_{ij}} \tag{2.70}$$

are equal between them and equal with the mean organic composition $\bar{k}_M$ of the total social capital.

The mean rate $\overline{pr'_j}$ of the profit of capital CP_j increases, as against the mean rate of the profit on the whole economy, if one of these quantities $\bar{p}'_{ij}$, β_{ij} increase s,or $\bar{k}_{ij}$ decreases as against $\bar{p}'$ or β_i and k_i, respectively, the other ones remaining constant.

We succeeded hence to prove the formation of the mean rates of the profits of the individual capitals and to find out the sufficient conditions for the formation of a general profit's rate. The proof was based upon the following essential suppositions: the investment decisions of the capitalists are made only in relation to the last state of the process; the homogeneity of capitals „behaviour"; the constance in time of the probabilities of selecting the investments structure (relations (2.33)), which leads to the stationarity of the process; the oscillation of the market prices round the momentary market values, and together with them round the values defined by (2.51) — (2.52″) and (2.54) — (2.56); finally the communicability of all states and the non-periodicity of the evolution, which lead to the ergodicity of the process (the independence of the limiting state probabilities as against the initial distribution). We supposed also the non-existence of the technical progress. This implies the constancy of the value of commodities over an horizon, no matter how distant, hence the non-existence of the moral wear.

All these hypotheses were, of course, restrictive enough, but even so, the results are significant. The concurrence between the capitals within the same sphere and between different spheres leads, in long periods, to a regulation of the value by social needs, hence by the social use value of the goods. "This is the law of value such sa it shows itself, not in relation with certain commodities and articles, but each time in relation with the totality of the products of the various spheres of the social production", Marx writes. And further on: "It is the social need, i.e. the use value on a social scale, which determines here the share of the total social working time devolved upon the various production spheres. But here the same law is acting, which acts in relation with each commodity taken separately, viz. the use value of a commodity is the premise of its exchange value, and hence of its value" ([1], part II, pp. 177—178).

The described process acts like a regulation cybernetic system, of adequation of the supply to the demand, but this adequation of the production to the social need is never perfect and it occurs through permanent inadequations and more or less acute contradictions.

Moreover, we could prove that the formation process of the general profit's rate is not in contradiction with the law of value, but is based directly upon its acheving made. At least three exposition of the law of value are to be found in "The Capital", which seem, at first sight, difficult to harmonize with one another: the first, at the beginning of volume I, wherein the value of a merchandise is given through the working time, socially necessary for its production, i.e. "the working time required to produce any use value whatever, in the existing production conditions, normal from the social viewpoint and with the mean social degree of skill and labour intensity", the second one, in volume III, with the opportunity if the description of the general profit's rate formation (where the individual values and the market prices appear) and which was exposed in the first part of the present work; finally, the third one was called by us above, through two short quotations.

In the model given by us, these three points of view are equally used and justified: to market values of the commodities produced in each state are means of the individual values, hence they are determined by the mean production conditions in the respective branches; the market prices corresponding to each state are expressed in terms of volume and distribution of the individual values of supply (production) and demand, consequently they are expressed also in working times, oscillating round the market value; finally, through the evolution in time of the process, the proper social value of commodities and capitals was defined as the mean, in whose vicinity the momentary market values and the market prices oscillate, hence as working time, socially necessary, not only from the point of view of the normal (mean) production conditions, but also of the regulation, of the value, thus defined, by the social use value of the produced commodities, a process through which the mean profit's rate is also concomitantly formed.

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UN MODELE STOCHASTIQUE MARKOV DE LA FORMATION DU TAUX GENERAL DU PROFIT

Utilisant la théorie des processus stochastiques Markov, l'auteur cherche à mettre en modèle la formation du taux général du profit par la transformation du taux moyen du taux individuel de profit, telle que cette formation fut présentée par Marx dans la seconde partie du troisième volume du "Capital" (v. [1], I^e partie, pag. 145—213).

L'ouvrage comprend tant l'exposé des idées de Marx mises en formules algébriques plus simples, que le modèle stochastique proprement dit.

МАРКОВСКАЯ СТОХАСТИЧЕСКАЯ МОДЕЛЬ ОБРАЗОВАНИЯ СРЕДНЕЙ НОРМЫ ПРИБЫЛИ

В работе автор пытается моделировать, используя теорию марковских стохастических процессов, образование общей нормы прибыли посредством преобразования частной нормы прибыли в среднюю норму в таком виде, как это изложено Марксом во второй части третьего тома «Капитала» (см. [1], стр. 145—213).

Первая часть настоящей работы содержит изложение идей Маркса в более простой алгебрайческой формулировке, а вторая часть — модель, как токавую.

7. XII. 1976.