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**"Nowadays cybernetics has an increasing importance in the management of industrial and economical processes, in reducing the human effort and in increasing the special productivity of man's work, in the overall progress of mankind".**

NICOLAE CEAUŞESCU

**from the Message addressed by the President of Socialist Republic of Romania to the Third International Congress of Cybernetics and Systems**

# Economic Logistics and Quantum

The plan based, unitary management of the whole mechanism of the socialist economy represents a decisive objective condition for the optimal, proportional and balanced development of the whole national economy system. This objective need is a consequence of laws governing the economic organism, the alert development of production forces.

The contemporary economy is characterised by an ample process of multiplying the connections among its components as a result of activities diversity taking place into the national economy space, of deepening the social division of labour, of continuously stressing the process of concentration, specialisation and co-operation in production. At the same time, it is characterized by deep changes in the technologies field, as follows from the plenary turning to account the valences of science production binomial, of over faster application into production of the got results by the scientific research.

The scientific management of many-sided developed socialist society has as a fundamental base the deep knowledge of objective laws for developing the social-economic phenomena, the multiple connections and interdependences which are formed among the components of national economy's system into the complex process of economic growth.

The Programme of the Romanian Communist Party on building the many-sided developed socialist society and Romania's advancement towards communism strongly emphasises the objective need for a deep understand-

ing of laws governing the economic mechanism in order to use them for the perpetual improvement of the ways of managing and organising the whole economic-social life. This understanding is based upon the dialectic and historic materialism, upon the practical experience gained by Romanian Communist Party, by the working class, by the whole people during the Romanian society revolutionary conversion.

"The provision of a scientific management and planning — shows the Party's Programme — implies the solid knowledge of universally valid social laws, of economic and social laws proper to socialism, as well as their correct use under the circumstances of our country according to the Party's Programme". [1]

The contemporary era, characterised by high rhythms and profound changes in all fields of economic and social practice imposes more and multiple requirements upon the scientific knowledge and on this basis, upon the management of the phenomena and processes taking place in nature and society. The achievement of a comprehensive view over the mechanisms govering the development of social-economic processes, the laws of movement and dynamics, balanced development of the national economy is of great importance during the present stage.

The planned development in socialism of all national economy components represents the fundamental movement form of cybernetic-economic systems within the economic space. Based on this, the system spatial-temporal movement gains a dynamic balanced and proportional character, specific to the process of optimal planned economic growth.

Under these circumstances, the deep knowledge of the laws of growth of the cybernetic-economic systems, characterising the essence of the development process, is of capital significance for the planned economy, for the continuously raising of the national wealth and on this base of people's welfare, the main goal of the whole policy of Romanian Communist Party. This creates the premises of guiding, coordinating and synchronising in time and space, all elements of the economic growth process, of the operation with maximal efficiency of the component parts of the national economy system. At the same time, there are provided the scientific premises for founding the design of the trajectories of optimal, balanced and proportional growth, the convergence of the huge energies and potential of the nation material, technical-scientific and human resources, towards a unique direction — the optimal economic-social balanced, proportional development of the country.

The achievement of a firm unity, of a strong symbiosis between the knowledge of objective law and the practical action is a sine qua non condition for the efficient operation of the whole social-economic mechanism. In a foundation of problems set by this direction, Nicolae Ceaușescu shows: *In fact, the whole organism of the socialist society must operate as the most perfect electronic machine* — if one can say so, since the clock is obsolete nowadays — able to synthesise everything good and valuable in the practical experience,

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[1] The Programme of the Romanian Comunist Party for building up the many-sided developed socialist society and Romania's advance to communism. Editura Politică, Bucharest, 1975, pp. 106.

to elaborate new concepts and objectives, ensuring at the same time, that the established tasks are strictly achieved.[1]

The country's economic-social development, the continuous increase of national income, of national wealth and on this basis of people's welfare represents a dynamic very complex process, implying the permanent modernisation of the national economy technological-scientific and production structure, of the whole economic efficiency, the conversion of quantity into a new quality by raising the technological level, the product quality and the reliability, the perpetual reduction of raw materials, energy and fuel consumptions, the optimal use of all national wealth components, the more and more intense and effective participation to the international division of labour.

Under these conditions, *the substantiation of forecast strategies of country's social-economic development and designing on this base the trajectories of optimal, balanced and proportional economic growth into the national economy space*, the settlement of options and decisions to achieve the exceeding of the *adopted political objectives*, are problems of greatest importance facing the management science.

The adoption of social-economic development strategy, the decisions and options concerning the optimal utilisation of the national income for development and consumption, the distribution of *the national fund for development among the material production and cultural-social sectors*, among several branches, subbranches of the national economy, *groups of products* as well as *over the territory* — fundamental elements for ensuring the country's rapid progress, for increasing the people's welfare — have as a base the deep knowledge of optimal economic growth mechanism and the objective laws of social-economic development.

Economic cybernetics using the scientific arsenal of dialectic and hystoric materialism of scientific socialism and political economy studies the specific way by which the laws of movement and growth of the cybernetic-economic systems act and manifest themselves in time and space, and expresses them in a mathematical form. In this way, premises are created for quantifying the intensity of the feedback and forward connections that are formed among various cybernetic-economic systems during the process of their development — essential items for elaborating the economic-mathematical models for optimal economic growth. In this way, economic cybernetics by the methods, techniques and models of investigation which it works out, brings an ever greater contribution to knowing the complex mechanisms within the national economy space, to studying the connections among its several sides to printing out the optimal development laws of national economy's general system components.

The process of many-sided investigation of dynamic properties of prospective trajectories of optimal, balanced, proportional economic growth, of the conditions ensuring the settlement and maintenance of cybernetic-economic systems on the optimal growth trajectories, as well as the many-sided

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[1] Nicolae Ceaușescu: Speech at the opening session of the colloquim concerning the problems of society's management science, Editura Politică, Bucharest, 1972, pp. 30.

study of structural-topologic properties of national economy's space, have lead to developing two new chapters of economic cybernetics: ECONOMIC LOGISTICS and ECONOMIC QUANTUM.

## 1. ECONOMIC LOGISTICS

Starting from the dynamic character of phenomena occuring into the economic space, from its entropic and gravitational properties, *economic logistics has adopted a new vision concerning the notion of optimal*, substantiating the concept of *dynamic optimal.*

According to this interpretation the dynamic optimal of economic-cybernetic systems and subsystems aims at their maintenance on an *optimal trajectories range* which to ensure the attainment of the established goals under the conditions of most efficiently using all human and material resources. Unlike the mechanistic conception where the optimal evolution is conceived as an automatic coincidence of phenomena unfolding on only one pre-established trajectory, in *economic logistics* the notion of *optimal trajectories range* appears as an essential element.

At the same time, economic logistics substantiates the *notion of optimal from the viewpoint of general cybernetic system ensemble of national economy*, not of a single individual component, thus being achieved the symbiosis between the two factors of the *optimal microeconomic-optimal macroeconomic binominal.* By taking into account the direct and inverse connexions (feedbacks) occuring between the components of national economy's general cybernetic system, between their growth zones ( Fig. 1), optimal development of each system can be ensured only within the overall optimal evolution.

The specific mechanism ensuring the growth process of economic-cybernetic systems on an optimal balanced, proportional range according to the adopted political strategy, is synthesised in the *command-control-adjustment triad.* In the socialist economy the economic-cybernetic systems are launched on optimal trajectories of economic growth. These are designed according to the plan of economic-social development strating from the fundamental political objective of the whole strategy of building the many side developed socialist society — *the maximisation of national wealth and nation's welfare increase*, of continuous increase of life quality, of living, of material and cultural standard of population.

*The management system* carries out the permanent *control* of each component of national economy's general cybernetic system, their evolution way on the settled trajectories through the economic-social development programs.

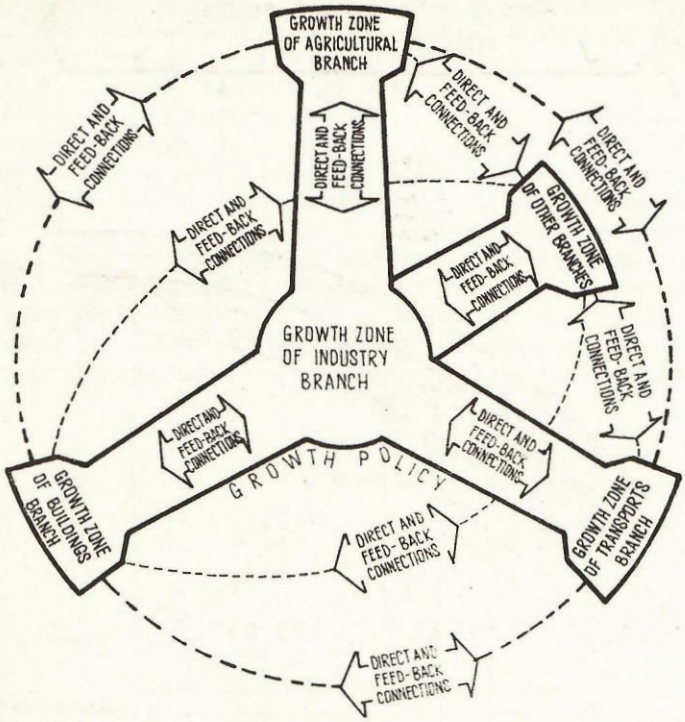



Fig. 1:

When deviations from the designed trajectories are noticed, *control decisions* are made with a view to *reducing* the respective component on the optimal trajectories range (Fig. 2.).

*Control mechanism* of growth trajectories of economic-cybernetic systems have a great importance as result of creative activity sustained by collectives of working people, in case when new elements allow the established parameters.

In this respect, it is of relevance the fact that based on the successes obtained by working people during the first two years of 1976—1980 five year plan, on the action initiated and directly controlled by President Nicolae Ceauşescu, General Secretary of the Romanian Communist Party, on the optimization of material production expenses and of investments, conditions have been ensured allowing to obtain a supplementary industrial production. This allows an important acceleration of national economy development, the achievement of new production objectives, the more rapid increase of national economy material production branches, the improvement of territorial distribution of production forces, the more efficient and complete use of country's material and work force resources.

The approval of the Party National Conference of the Supplementary Development Programme for years 1978—1980, allows a substantial *acceleration* of the development speed of national economy system, the stronger incre-

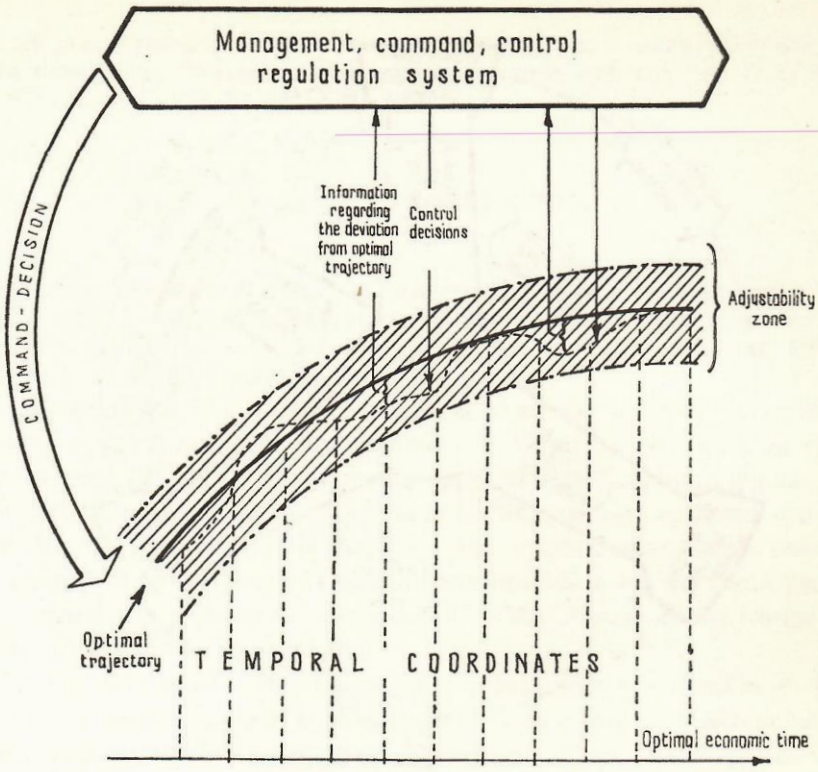



Fig. 2:

ase of national income, of nation's wealth of the people's material and cultural standard of living.

An important aspect of each nation's social-economic development is the intensification of participating in the world economic circuit, on mutual and advantage terms, the amplification of flows intensity taking place among the components of national economy space and among the components of world economy space. Under these circumstances, *control mechanisms* have the role of efficiently and rapidly turning to account the favourable conjuncture situations appearing within the world market and to attenuate the unfavourable influences which can be propagated from the world economy space, as a result of prices increase over the forecasted ones for certain products which are imported, of non-observing the settled time of delivery by certain external partners.

In tight connection to ensuring an optimal evolution of economic-cybernetic systems, to their management and control,the problem of stating the conditions of *dynamic stability* appears.

From the economic logistics point of view, the dynamic stability of an economic-cybernetic system refers to its property of derivating from the optimal growth trajectory only within certain feasible limits.

The study of *dynamic stability* has in this interpretation the role to outline the most efficient modalities for maintaining the economic-cybernetic systems

on the optimal growth trajectories range. This fact involves a knowledge of the dynamic properties which are specific to each system and subsystem, of its cybernetic behaviour, as well as of the evolution of its connexion intensities to other systems evoluating in the national economy space and of flows forming within the world economy space.

In the activity of working out the general strategies of social-economic development, of drawing up the programs and plans of optimal economic growth, a very important part is played by the precise knowledge of structural-topologic properties of national economy space of growth zone for each component of national economy space. To this end, economic logistics aims at creating an arsenal of methods and techniques which to allow the identification of topological properties *of observation*, *accesibility*, *growth and control zones* for all components of national economy system (Fig. 3).

To know the zones within the national economy space which characterize the growth potential of each system—from the fundamental unit till the

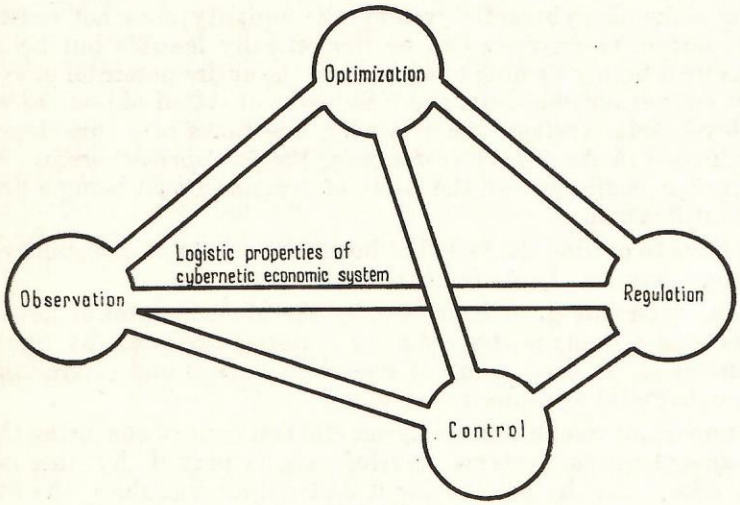



Fig. 3.

national economy ensemble—represents the fundamental elements in the activity of designing the forecast trajectories of social-economic development.

Thus, an essential stage in the activity of designing the growth trajectories in the national economy space, is the multilateral analysis of the already attained development level, of existing financial, material and human resources, of structural tendencies taking shape within the economy. These elements allow the identification of system's "initial position" and "reference position" into the national economy space.

Using the got information from the studies concerning the evolution tendencies, as well as those concerning its initial position into the national economy space, the *accesibility zone* of the system is identified.

The accesibility zone is made up of the trajectories set which the economic-cybernetic system can evoluate on, taking into account the development level which the "reference position" starts from and its specific growth laws.




Fig. 4

On the base of the options resulted from country's social-economic growth strategy concerning system's *growth zone* is identified within the accesibility zone and the set of its optimal social-economic growth trajectories, respectively (Fig. 4).

It is obvious the fact that between the accesibility zone and the growth one of an economic-cybernetic system, the equality does not exist. Indeed, certain evolution trajectories can be theoretically feasible but because they do not ensure a higher turning to account of the entire potential of system growth, they are not optimal from the view point of stated objectives within the general developing strategy. That is why, one takes into consideration only the growth zone in *the activity of designing the development forecast trajectories of economic-cybernetic systems*, the study of accesibility set being a preliminary to its identification.

We have to outline the fact that both the concept of accesibility zone and growth zone have a *dynamic character.*

In the economic growth process by the identification of new resources and reserves of economies, by reducing material expenses, by finalizing the new technologies, a "dilatation" of accesibility zones and of growth ones for economic-cybernetic systems takes place.

An important role in organizing an efficient control concuring the process of economic-cybernetic systems development is played by the *observation zone* that allows, on the base of input and output variables, the evolution's permanent control.

*Control zone* identification into the national economy space has a special practical importance, because it allows the efficient establishment of measures ensuring the full turning to account of new growth reserves of economic-cybernetic systems, as well as the avoidance of those situations where the control of certain economic-cybernetic systems would require a great consumption of social energy.

Delimitation of this zone for each system is done in a tight interdependences to the study of other components of national economy's general-cybernetic system. Thus, in case of identifying the control zone for an engineering industry sector this involves the maintenance of proportionality and equilibrium both as regards the development of the entire branch and the related branches, firstly regarding the metal-working industry, its main dealer.

## 1.1. Logistic methods and techniques for designing the optimal economic growth trajectories of economic-cybernetic systems

A fundamental field of economic logistics, where the worked out methods prove to be very efficient, is the field of techniques and algorithms, for designing the growth optimal trajectories into the national economy space. To this end, the conditions which have to be met by the elements of the economic-cybernetic model of a system, will be pointed out, namely the state and command variables, in order to assure the optimal growth process into the national economy space.

The basic elements where the economic logistics starts from with a view *to design the optimal growth trajectories of an economic-cybernetic system* are as follows:

1) *vectorial equation describing the dynamics of system's state variables*:

$$x_{t+1} = F_t(x_t, x_{t-1}, ..., x_{t-\theta_1}; \quad u_t, u_{t-1}, ..., u_{t-\theta_2}; \quad w_t, w_{t-1}, ..., w_{t-\theta_3}; \quad b).$$

Notions are those used in the study "Economic Space and Economic Time-Systemic Approach"[1] for describing the overall cybernetic model of evolution into national economy space, according to space-time coordinates.

We denoted by $w$ the input-connecting vectors. As these vectors into the system are exogeneous elements to the model, they are not included in the incidence of optimizing the analysed economic-cybernetic system, but in the incidence of another systems. By utilizing a convenient variable change, the vectorial equation describing the growth process of economic-cybernetic system for which the designing of optimal growth trajectory is made, can be written as follows:

$$x_{t+1} = f_t(x_t, x_{t-1}, ..., x_{t-s}; \quad u_t, u_{t-1}, ..., u_{t-s}; \quad b)$$

State vectors $x_t$, command-decision vectors $u_t$ and $b$ belong to the following zone in the national economy space:

$$x_t \in G_1 \subseteq R^q; \quad u_t \in U \subseteq R^h; \quad b \in B \subseteq R^l.$$

We denoted by $s$ the maximal retardation period of model variables:

$$s = max \ (\theta_1, \theta_2)$$

2) *initial conditions* formed of three elements:

a) *initial orders*

$$u_{t_0-i} = \eta_{t_0-i}; \quad i = 0, 1, ..., s$$

b) *initial moment* $t_0$ considered for system optimization,

c) *initial states*

$$x_{t_0-i} = \alpha_{t_0-i}; \quad i = 1, ..., s.$$

Vector $\alpha_{t_0-i}$ quantizes the economic development level which the system starts from, system whose growth process will be optimized; vectors $\eta_{t_0-i}$ give information regarding the decisions adopted in the previous period

[1] Study published in: Economic Computation and Economic Cybernetics Studies and Research, no. 3, pp. 5–20, Bucharest, 1978.

and which will action also in the time interval which the optimization is done for. These vectors represent the links ensuring the *continuity* among decision-making periods. In order to illustrate, one may mention the case of decisions-making as concerns the investments. Indeed, certain investment objectives which have been began for a previous period are also continued for the next period, thus influencing the decision concerning the output, accumulation, a.s.o.

We have to point out that the initial moment $t_0$ may be fixed from the very beginning or presented as a parameter function $b$:

$$t_0 : B \rightarrow N$$

In this case, the initial moment $t_0$ is going to be determined under the concrete social-economic conditions under which the growth trajectory is designed.

Referring to system initial state, the following situation could appear in practice:

a) the points in the national economy space are known:

$$x_{t_0-i} = \alpha_{t_0-i} \in G_1; \quad i = 0, 1, 2, ..., s$$

which represent *system initial states.*

b) the surfaces in the economic space where the initial states will be placed, are known:

$$G_0^i \subseteq G_1; \quad i = 1, 2, ..., s.$$

"Launching" surfaces are represented parametrically by the following vectorial surfaces:

$$x_{t_0-i} = \alpha_{t_0-i}(b); \quad i = 0, 1, 2, ..., s$$
$$\alpha_{t_0-i} : B \rightarrow G_1; \quad i = 0, 1, 2, ..., s.$$

Case b) is met particularly in practice because the works of designing a new growth trajectory began, according to certain preliminary figures, by 2—3 years before the period which the respective trajectory refer to.

As generally the fulfilments record exceedings at numerous indicators, the optimizing works related to the new trajectory have to take into calculation these situations. Thus it is outlined the necessity of considering the "*launching*" *surfaces* as initial elements of the model, not simple points in the national economy space.

3) *the objective of optimal economic growth* which has to be attained by the system is quantized by the following vectorial equation:

$$x_T = \omega(b)$$
$$\omega : B \rightarrow G_1.$$

*The objective of optimal growth* — fundamental element in the logistics of designing developing forecast trajectories—quantizes the elements of maximal importance for country's social-economic development strategy. The components of vector $\omega(b)$ represent the economic levels which are going to be attained at the end of optimizing period on branches, subbranches, products and groups of products, and the increase of population's livelihood and cultural standard, utilizing structure of labour force, the territory economic development, a.s.o.

The objective of optimal economic growth shows the zone in the economic space where the system is desired to be directed during the plan period.

Regarding the final moment $T$ of time horizon which the system is analysed on, this can be fix or given by a function:

$$T : B \to N.$$

In certain cases, system objective is under the form of a surface into the economic space.

4) *growth zone of economic cybernetic system in the economic space*

The zones where the economic-cybernetic systems can evoluate in the national economy space, depending on the concrete conditions related to the control of these systems, are determined.

The constraints upon the evolution of economic-cybernetic systems quantize, under the equation form, the correlations and proportionalities which have to be kept in comparison with the evolution of other components of the overall cybernetic system. In fact, this quantizes the intensity of input-connecting variables into the system which the economic-social, optimal growth trajectory is designed for.

The general mathematical form, under which the constraints appear, constraints influencing system evolution with a view of *ensuring an optimal, balanced and proportional growth*, is the following:

$$I_\gamma(u, b) \leqslant 0; \ \gamma = 1, 2, ..., p'$$

$$I_\gamma(u, b) = 0; \ \gamma = p' + 1, p' + 2 ..., p.$$

Functions occuring within the constraints have the following general form:

$$I_\gamma(u, b) = g_\gamma(b) + \sum_{k=t_0}^{T-1} L_{\gamma,k}(x_k, ..., x_{k-s}; \ u_k, ..., u_{k-s}; \ b).$$

Based on information concerning the system under analysis, the following functions are determined:

$$g_\gamma : B \to R$$

$$L_{\gamma k} : G_1 \times U \times B \to R$$

$$\gamma = 1, 2, ..., p$$

$$k = 1, 2, ..., T.$$

Thus, evolution zone of cybernetic system in the national economy space coincides with the points set which are solutions for the next equations and inequations system:

$$\begin{cases} I_\gamma(u, b) \leqslant 0; & \gamma = 1, 2, ..., p' \\ I_\gamma(u, b) = 0; & \gamma = p' + 1, p' + 2 ..., p \\ x_T = \omega(b). \end{cases}$$

One assumed above that the *feasible orders field* $U$ is known, and $u_t \in U$.

5) *system performance* — decisive element in testing the growth trajectory optimality in the national economy space — is given by the following functional:

$$I_0(u, b) = g_0(b) + \sum_{k=t_0}^{T-1} L_{0k}(x_k, x_{k-1}, ..., x_{k-s}; \quad u_k, u_{k-1}, ..., u_{k-s}; \quad b).$$

The selection of system performance, of the optimal criterion respectively, in concordance to the general strategy of country's social-economic development, is a decisive stage in the process of designing the growth trajectories of economic-cybernetic systems, because a strong dependence exists between the stated optimal criterion and resulted trajectory.

Further on, the concepts of *command-feasible decision* and of *command-optimal decision* will be determined.

The command-decision $\{u_t, b\}_{t_0-s}^{T-1}$ is *feasible* for the analysed economic-cybernetic system if the following conditions are met:

$$\text{a)} \quad u_{t_0-i} = \eta_{t_0-i}; \quad i = 1, 2, ..., s;$$

$$\text{b)} \quad u_t \in U, \; t = 0, 1, ..., T-1.$$

Let $\{x_t\}_{t_0-s}^{T}$ be the corresponding solution of system's evolution equation

$$x_{\alpha-i} = t_{0-i}(b); \; i = 0,1, ..., s.$$

The triplet $\{u_t, b, x_t\}_{t_0-s}^{T-1}$ checks the conditions:

$$I_\gamma \leqslant 0; \; \gamma = 1, 2, ..., p'$$

$$I_\gamma = 0; \; \sigma = p' + 1, p' + 2, ..., p$$

the established economic goal being attained: $x_T = \omega(b)$.

The command — decision $\{\tilde{u}_t, \tilde{b}\}_{t_0-s}^{T-1}$ is an *optimal* if minimizes the functional $I_0$ in the class of all feasible commands.

*The Principle of Maximum* — a very important instrument for optimizing the evolution trajectories of dynamic systems — outlines the necessary conditions which have to be met by the parameters of optimal economic growth trajectories. In order to present this very important theorem, we shall denote by $\{\tilde{u}_t, \tilde{b}_t\}_{t_0-s}^{T-1}$ the *optimal command* and by $\{\tilde{x}_t\}_{t_0-s}^{T}$ *the optimal corresponding trajectory.*

*The Principle of Maximum* ensures the existence of multiplies $\lambda_0, \lambda_1, ..., \lambda_{p+q}$ which satisfy the following properties:

$$\text{(i)} \sum_{i=0}^{p+q} |\lambda_i| \neq 0$$

$$\text{(ii)} \quad \lambda_\gamma \geqslant 0; \; \gamma = 0, 1, ..., p'$$

$$\text{(iii)} \quad \lambda_\gamma I_\gamma(\tilde{u}, \tilde{b}) = 0; \; \gamma = 1, 2, ..., p'.$$

Further on, we shall denote by $\tilde{\psi}_t$ the solution of adjunct equations system, stated as follows:

$$\psi_{t-1} = \psi_t A_t + \sum_{j=0}^{s} \psi_{t+j+1} B_{t+j+1}^{j+1} - \sum_{\beta=0}^{p} \lambda_\beta \left[ C_{\beta,t} + \sum_{j=0}^{s} D_{\beta,t+j+1}^{j+1} \right], \quad \tilde{\psi}_{t-1}^{i} = -\lambda_{p+i},$$

$$\tilde{\psi}_t \equiv 0, \quad t \geqslant T.$$

*The system's Hamiltonian* is stated with the aid of the economic-mathematic model's elements of the analysed system:

$$\mathcal{H}_t(x_{t+s},\ x_{t+s-1},\ ...,\ x_{t-s};\quad u_{t+s},\ u_{t+s-1},\ ...,\ u_{t-s};\quad b) =$$

$$= \sum_{j=0}^{s} H_{t+j}(x_{t+j},\ x_{t+j-1},\ ...,\ x_{t+j-s};\quad u_{t+j},\ u_{t+j-1},\ ...,\ u_{t+j-s};\quad b)$$

where:

$$H_t(x_t,\ y_1,\ ...,\ y_s;\quad u_t,\ v_1,\ ...,\ v_s;\quad b) = \tilde{\psi}_t f_t(x_t,\ y_1,\ ...,\ y_s;\quad u_t,\ v_1,\ ...,\ v_s;\quad b) -$$

$$- \sum_{\beta=0}^{p} \lambda_\beta L_{\beta t}(x_t,\ y_1,\ ...,\ y_s;\quad u_t,\ v_1,\ ...,\ v_s,\ b).$$

If the growth trajectory $\{\tilde{x}_t\}$ is optimal, it has to necessarily satisfy the following conditions which are the "*Principle of Maximum*":

$$\alpha)\ \frac{\partial \mathcal{H}_t}{\partial u}(\tilde{x}_{t+s},\ ...,\ \tilde{x}_{t-s};\quad \tilde{u}_{t+s},\ ...,\ \tilde{u}_{t-s};\quad b)\,[u - \tilde{u}_t] \leqslant 0$$

for any $u \in U$ and $t = t_0,\ t_0 + 1,\ ...,\ T-1$.

$$\beta)\ \sum_{\rho=0}^{p} \lambda_\rho \frac{\partial g_\rho(b)}{\partial b^\sigma} + \psi_{T-1}\frac{\partial \omega(\tilde{b})}{\partial b^\sigma} - \tilde{\psi}_{t_0-1}\frac{\partial x_{t_0}(b)}{\partial b^\sigma} -$$

$$- \sum_{k=t_0}^{T-1} \frac{\partial H_k}{\partial b^\sigma}(\tilde{x}_k,\ ...,\ \tilde{x}_{k-s};\quad \tilde{u}_k,\ ...,\ \tilde{u}_{k-s};\quad \tilde{b}) = 0.$$

The first condition ensures the *Hamiltonian value maximization* and the second one represents the *cross-cutting conditions.*

The following matrix notations were used for presenting the *Principle of Maximum:*

$$A_t = \frac{\partial f_t}{\partial x}(\tilde{x}_t,\ \tilde{x}_{t-1},\ ...,\ \tilde{x}_{t-s};\quad \tilde{u}_t,\ \tilde{u}_{t-1},\ ...,\ \tilde{u}_{t-s};\quad \tilde{b})$$

$$B^j_{t+1} = \frac{\partial f_t}{\partial y_j}(\tilde{x}_t,\ \tilde{x}_{t-1},\ ...,\ \tilde{x}_{t-s};\quad \tilde{u}_t,\ \tilde{u}_{t-1},\ ...,\ \tilde{u}_{t-s};\quad \tilde{b})$$

$$C_{\gamma,t} = \frac{\partial L_{\gamma,t}}{\partial x}\tilde{x}_t,\ \tilde{x}_{t-1},\ ...,\ \tilde{x}_{t-s};\quad \tilde{u}_t,\ \tilde{u}_{t-1},\ ...,\ \tilde{u}_{t-s};\quad \tilde{b})$$

$$D^j_{\gamma,t+1} = \frac{\partial L_{\gamma,t}}{\partial y_j}(\tilde{x}_t,\ \tilde{x}_{t-1},\ ...,\ \tilde{x}_{t-s};\quad \tilde{u}_t,\ \tilde{u}_{t-1},\ ...,\ \tilde{u}_{t-s};\quad \tilde{b}).$$

The functions $f_t$ and $L_{\gamma,t}$ are stated in relation to variables $(x_t,\ y_1,\ ...,\ y_s;\ u_t,\ v_1,\ ...,\ v_s;\ b)$.

The application of economic logistics' methods and techniques, of computing algorithms worked out on this base, allow the definition of the coordinates of optimal economic growth trajectories of economic-cybernetics systems and sub-systems.

## 1.2. Logistic properties of growth trajectory in the national economy space. Trajectories of proportional, balanced, optimal economic growth

In order to outline logistic properties of economic-cybernetic systems' growth trajectories, one shall identify the *growth zone* of the system denoted by $\mathcal{C}$, within the national economy space.

Denoting by $x_t$ the state vector of the economic-cybernetic system, the sequence $x_0, x_1, ..., x_T$ is *its growth trajectory.*

The growth trajectory of the economic-cybernetic system on the time horizon $T$, will be denoted as follows:

$$\{x_t\}_{t=0}^{t=T} \text{ or } \{x_t\}_0^T.$$

For more exactly rendering the phenomena taking place within the economic space, it is required the very complete identification of topologic structural properties of economic-cybernetic systems' growth zones.

Generally, if the mathematic-economic model describing the growth process of the analysed cybernetic system is of the form:

$$x_{t+1} = f_t(x_t, u_t); \quad u_t \in U_t$$

then

$$x_{t+1} \in F(x_t).$$

The following notation was used above:

$$F_t(x_t) = f_t(x_t, u_t)$$

where $u_t$ is the command-decision vector.

A very great number of growth trajectories can be generated from each initial state where an economic-cybernetic system exists.

Denoting by $X(x_0, T)$ the set of all trajectories which the economic-cybernetic system can evoluate on for the time interval $[0, T]$ if it starts from state $x_0$, it is necessary to adopt an *efficiency criterion*, on whose base the analysis and the comparison of respective trajectories to be possible to be made.

Denoting by $X$ the states space, and by $\psi$ the outputs space, the efficiency of activities developed by the analysed system can be quantized with the aid of functions:

$$\varphi_i : X \to \psi, \qquad i = 1, 2, ..., q.$$

These correspond to the indicators of assessing system's economic efficiency. In this case, the comparison between the growth trajectories will be done on the base of *development level* which the system can attain for the considered time horizon, quantized with the aid of vectorial function $\varphi$.

According to this comparison criterion, the growth trajectory $\{x_t\}_0^T$ is *more efficient* than the trajectory $\{y_t\}_0^T$ if the development level attained by the system at the end of horizon $T$ is greater on the first trajectory than on the second one, therefore:

$$\varphi(x_T) \geqslant \varphi(y_T).$$

The above vectorial relation synthesizes the following scalar inequalities:

$$\varphi_i(x_T) \geqslant \varphi_i(y_T), \quad i = 1, 2, ..., q.$$

Thus, the growth trajectory $\{x_t\}_0^T$ will be considered more efficient than the trajectory $\{y_t\}_0^T$ if it ensures the fulfilment of higher values for all efficiency indicators taken into consideration.

In this way, a sequence relation is introduced into the outputs space. As one takes into consideration a *vectorial criterion* of evaluating the efficiency, the sequence structure introduced in the outputs space is not total, this fact generating another way of approaching the optimization problem than that one corresponding to the scalar case.

A special problem which has to be solved in this case, concerns the identification of all trajectories set generating maximal points in relation to the introduced sequence relation.

The growth trajectory $\{x_t\}_0^T$ will be called *efficient* if there is no other feasible trajectory which to ensure higher values for all efficiency indicators.

In this way, the trajectory of efficient growth at the national economy level for years 1980, 1985, or 1990 is that which ensures the achievement of the highest values of national income, work productivity, the highest standard of living.

The totality of all efficient trajectories generate the set of maximal points. Generally, there are more maximal points, this fact involving the adoption of a selection criterion on the base of a unique indicator which to outline the trajectory corresponding to the proposed strategic objectives in the best way.

With a view to identifying a unique criterion which to allow the comparison of all system's growth trajectories from the efficiency point of view, a series of structural-topologic properties of feasible trajectories set $X(x^0, T)$ will be analysed.

A special practical importance is devoted to the mathematical result according to which there exists a vector $v = (v^1, v^2, ..., v^2)$ on whose base the efficient trajectories can be obtained as maximal points in relation to a unique criterion which accumulates in a well-balanced way $q$ efficiency criteria $\varphi_i$. In other words, if trajectory $\{x_t\}_0^T$ is efficient, then it maximizes the performances

$$J_T = \sum_{i=1}^{q} v^i \cdot \varphi_i(x_T).$$

The optimization is made in relation to all trajectories assumed by the system. Components $v^i$ of vector $v$ quantizes the importance devoted to each efficiency criterion, achieving their hierarchization.

In case when the functions $\varphi_i$ are defined as follows:

$$\varphi_i(x_T) = x_T^i$$

the appreciation of trajectory's efficiency will be made on the base of absolute value attained in year $T$ by each component of state vector.

In this case, trajectory $\{x_t\}_0^T$ will be more efficient than $\{y_t\}_0^T$ if the relation $x_t \geqslant y_t$, $x_t^i \geqslant y_t^i$, $i = 1, 2, ..., n$ respectively, is met.

One assumed above that the states space is $n$-dimensional.

Vector $v$ will correspond to any trajectory $\{x_t\}_0^T$ that is efficient according to the mentioned criterion, in such a way as the following relation to be satisfied:

$$\sum_{i=1}^{n} v^i x_T^i \geqslant \sum_{i=1}^{n} v^i y_T^i$$

regardless the existence of any other trajectory:

$$\{y_t\}_0^T \in X(x^0, T).$$

The proof of the aforementioned properties is based on the hypothesis that the set $X(x^0, T)$ of permitted system growth trajectories is not empty, is compact and convex.

In order to identify the vectorial efficiency index the following set is considered:

$$M = \{y_t - x_t / \{y_t\}_0^T \in X(x^0, T)\}.$$

Since the set $X(x^0, T)$ of system growth trajectory is convex, it results that $M$ is also a convex set. Moreover, since $\{x_t\}_0^T$ is an efficient growth trajectory, one can write

$$M \cap R_+^n = \{0\}.$$

$R_+^n$ is the positive octant of the $n$ — dimensional space. $R_+^n$ and $M$ are convex sets and have a single common point. Thus there exists a hyperplane separating them. Let $v$ be the vector definying the separation hyperplane of the two sets. The following relation results:

$$vz \leqslant 0$$

which is satisfied for any $z \in M$. Since the vectors of set $M$ have the form $y_t - x_t$ one can write

$$vx_T \geqslant vy_T$$

which proves the above statement.

*The above property* emphasised the existence of a support hyperplane for growth trajectory set $X(x^0, T)$ passing through the vertex of an efficient trajectory and that group into the same half space of all permitted system growth trajectories.

In case when the weights $\tilde{q}^i$ *quantizing the role of each component within the growth process* have been settled on the base of economic analysis concerning the importance devoted to each component of state vector $x_T$, the existence of an efficient trajectory which *maximizes the next synthetic indicator* can be proved:

$$z_T = \sum_{i=1}^{n} \tilde{q}^i x_T^i, \; q^i \geqslant 0$$

in relation to any other growth trajectory of the system. We denoted by $\tilde{q}^i$ the coefficient quantizing the importance of component $x_T^i$ of state vector.

To prove the stated property, let's consider the next function

$$f : X(x^0,\ T) \to R_+$$

where

$$f(\{y_t\}_0^T) = \sum_{i=1}^{n} \tilde{q}^i y_T^i.$$

The linear function $f(.)$ defined over a compact set reaches its maximum. Let $Q$ denote the growth trajectory set achieving the maximum of the synthetic index $y_T$, thus

$$Q = \{\{y_t\}_0^T / \sum_{i=1}^{n} \tilde{q}^i y_T^i = max = m;\quad \{y_t\}_0^T \in X(x^0,\quad T)\}.$$

In what follows one will prove the existence inside set $Q$ of an efficient growth trajectory, ensuring the non-violation of the constant:

$$x_T \geqslant y_T$$

with respect to any other system growth trajectory.

To prove this, one shall consider a line vector $s$, having all components positive and the mathematical function stated as follows: $g(\{y_t\}_0^T) = sy_T$.

As the set $Q$ is compact, the function $g(.)$. reaches the maximum in the set; one shall denote by $\{x_t\}_0^T$ the trajectory performing the maximum.

One shall prove that the growth trajectory $\{x_t\}_0^T$ identified above is efficient; really, supposing that another trajectory $\{y_t\}_0^T$ feasible to the system would exist and that would be more efficient than trajectory $\{x_t\}_0^T$ this would imply the meeting of the following relations:

$$sy_T \geqslant sx_T$$

$$\tilde{q}y_T \geqslant \tilde{q}x_T.$$

From the above relation, it results:

$$m \geqslant \tilde{q}y_T \geqslant \tilde{q}x_T = m$$

which implies $\tilde{q}y_T = m$. The last relation proves the fact that the trajectory $\{y_t\}_0^T$ belongs to the set $Q$. But, in this case it is necessary to meet the relation:

$$sx_T \leqslant sy_T$$

this fact contradicts the inequality $sy_T > sx_T$ previously stated. The above proof proves that the growth trajectory $\{x_t\}_0^T$ is efficient and at the same time it maximizes the synthetic index $y_T$.

The analysis of logistic properties of *proportional*, *balanced*, *optimal growth trajectories* is a very important problem. To this end, we have to start from the fact that the development level attained for each moment is the reference point for the next trajectory of economic growth. This is why, the designed trajectory has to ensure the concordance among all sides of phe-

nomena and processes characteristic to the activities developed by the analysed cybernetic-economic system.

Growth trajectory $\{\hat{x}_t\}_0^T$ is *proportional, balanced and optimal* if ensures in a maximum rate the growth of all output vectors' components of economic-cybernetic system, of output on branches, subbranches, products and groups of products of labour productivity of national income and of population living standard, respectively, on the base of superior turning to account the human, technical-scientific and material resources.

Thus, the coordinates of trajectory $\{x_t\}_0^T$ quantizes an optimal balanced, proportional, economic growth process if the following mathematical properties are satisfied:

$$\text{a) } \{x_t\}_0^T \in X(x_0, T)$$

$$\text{b) } g(x_t) \in \theta_t$$

$$\text{c) } r(\{x_t\}_0^T) \geqslant r(\{y_t\}_0^T)$$

for any $\{y_t\}_0^T \in X(x_0, T)$.

Relation a) outlines the fact that trajectory $\{x_t\}_0^T$ is feasible for the system, namely it is in its growth zone. The second relation quantizes the constraints concerning the proportional, balanced growth process of the system. We denoted by $\theta_t$ the zone into the national economy space which assures a balanced proportional evolution to the analysed system.

Relation c) underlines the fact that a maximal growth rate for the output vector's components of the system is assured on the trajectory $\{x_t\}_0^T$. We denoted by $r(\{x_t\}_0^T)$ the value of system performance which underlines the options resulted from the general strategy of social-economic development.

The worked out analysis proves the complex subjects of studying the logistic properties of growth trajectories into the national economy space.

Identification of logistic properties of optimal economic growth trajectories is an important stage for the activity of forecasting and working out the economic growth programs.

### 1.3. Methods and techniques of analysing the logistic properties of economic-cybernetic systems

In order to substantiate the coordinates of proportional, balanced, optimal economic growth trajectories, an important part is played by precise sizing the zones within the national economy space, zones of accesibility respectively, and according to this base the identification of economic-cybernetic system's growth zones.

The knowledge of these zones is of particular importance both for working out the forecasts, programs of social-economic development and for the management activity of economic-cybernetic system on the trajectories designed with a view to attain the objectives of optimal economic growth.

Thus, the statement of accesibility and growth zones for the ensemble of the national economy general cybernetic system in year 1980 or 1990

will emphasise the growth of labour productivity and of potential of the social product and national income, of the industrial and agricultural production, of other branches of the national economy on the base of superior turning to account the human, technical-scientific, material resources, the possibilities of amplifying all components of national wealth system and of nation's welfare, of population's living standard, of life quality.

At macroeconomic level, for example in an industrial enterprise, the identification of accesibility and growth zones will emphasise the possibilities of increasing the production and labour productivity of developing and modernising the production capacities, of enlarging the product range, of increasing their technological level and quality, the ensurance of goods selling for the internal market and for export.

Determination of accesibility zone and of growth zone into the national economy space of an economic-cybernetic system is based on the cybernetical notion of *development level* which outlines the state attained by the analysed process in its growing process. Notion of development level gives a *space-time* image upon system evolution, upon its dynamics in the growing process.

By considering the economic-cybernetic system $\mathcal{S}$ characterized by state vector $x_t$, its evolution into the national economy space is given by its interior behaviour which under an equation form can be written:

$$x_t = \varphi_t\ (\tau,\ x_\tau;\ \gamma_{[\tau,t)}).$$

We denoted by $x_\tau$ system state in moment $\tau$, by $\gamma_{[\tau,t)}$ the ensemble of input flows into the system during time period $[\tau,\ t)$; function $\varphi_t(.,.,.)$ quantizes the interior behaviour of the system under an equation form.

The development level attained by economic-cybernetic system $\mathcal{S}$ at moment $t$ is outlined by:

$$(t,\ x_t) \in T \times X.$$

We denoted by $T$ the time period which the study of considered system is made for any by $X$ the states space.

With a view to identify all development states where the analysed system can reach during the forecasting a plan period, it is necessary to previously settle *the set of input flows* $\Phi_{\mathcal{S}}$ *feasible to the system.*

The development level $(t_1,\ x_1)$ *is feasible* to the economic-cybernetic system which is at the development level $(\tau,\ x_\tau)$ if input flows $\gamma_{[\tau,t_1)}$ exist from the set $\Phi_{\mathcal{S}}$ which to bring the system to state $x_1$ at moment $t$.

It results that the development level $(t_1,\ x_1)$ is feasible to the system if satisfies the following vectorial equation:

$$x_1 = \varphi_{t_1}\ (\tau,\ x_\tau,\ \gamma_{[\tau,t_1)}).$$

Further on, we shall denote by $M_t[(t_0,\ x_0),\ \Phi_{\mathcal{S}}]$ the set of all development levels where the economic-cybernetic system can reach at moment $t$, if it starts from the development level $(t_0,\ x_0)$ therefore:

$$M_t[(t_0,\ x_0),\ \Phi_{\mathcal{S}}] = \{x/x = \varphi_t(t_0,\ x_0,\ \gamma_{[t_0,t)});\ \gamma \in \Phi_{\mathcal{S}}\}.$$

The *entire accesibility zone* of economic-cybernetic system for time horizo T is determined as follows:

$$\widehat{M}_T = \bigcup_{t \in [t_0, T]} M_t \, [(t_0, \; x_0), \; \Phi_{\mathcal{S}}].$$

We mentioned above that the system starts from the developmer level $(t_0, \; x_0)$.

To exemplify, one shall suppose that the analysed system is at momer $t = 0$ in state $x_0 = a$.

The set of all development levels which can be reached succesively i each year, are the following:

$$M_1 [(0, \; a), \; \Phi_{\mathcal{S}}] = \{x/x = \varphi_0(0, \; a, \; \gamma_{[0,1)}); \quad \gamma \in \Phi_{\mathcal{S}}\}$$

$$M_2[(0, \; a), \; \Phi_{\mathcal{S}}] = \{x/x = \varphi_1(1, \; z, \; \gamma_{[1,2)}); \quad \gamma \in \Phi_{\mathcal{S}}; \quad z \in M_1[(0, \; a), \; \Phi_{\mathcal{S}}]\}$$

.....

$$M_t[(0, \; a) \; \Phi_{\mathcal{S}}] = \{x/x = \varphi_{t-1}(t_{-1}, \; z, \; \gamma_{[t-1,t)}); \quad \gamma \in \Phi_{\mathcal{S}}; \quad z \in M_{t-1}[(0, \; a), \; \Phi_{\mathcal{S}}]\}.$$

It results that $M_t[(0, \; a), \; \Phi_{\mathcal{S}}]$ represents the zone within the nationa economy's space where the cybernetic-economic system can reach after period of $t$ years.

The entire accesibility set forms a cone into the (time-state) develop ment levels which has the vertex in point $(t_0, \; x_0)$ (Fig. 5.).

On the base of general strategy of social-economic development of esta blished goal, within the region into the national economy space that is feasibl to the system, its economic growth zone is sized:

$$\mathcal{C}_t \subseteq M_t[(0, \; a), \; \Phi_{\mathcal{S}}].$$

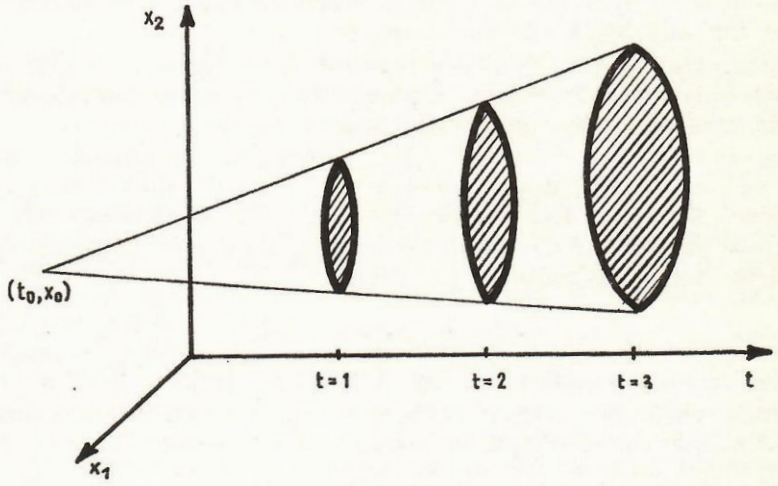



Fig. 5

Further on, we shall analyse the problem of identifying the accesibility zone and the growth zone for he class of economic-cybernetic systems whose interior behaviour is described by the following vectorial equation:

$$x_{t+1} = A_t x_t + B_t \gamma_t.$$

The systems whose growing process is described by the above vectorial equation is an important class, they being met both at macroeconomic level and at microeconomic level. We denoted by $x_t$ the state vector, by $\gamma_t$ the command-decision vector and by $A_t$ and $B_t$ the structure matrix of the system, therefore:

$$x_t = \|x_t^i\|_{n\times 1}; \quad \gamma_t = \|\gamma_t^i\|_{n\times 1}$$

$$A_t = \|a_{ij}^t\|_{n\times n}; \quad B_t = \|b_{ij}^t\|_{n\times n}.$$

General solution of evolution equation of system state giving the mathematical expression of vector $x_t$ depending on the initial state $x_0$ and on the inputs sequence $\gamma_0, \gamma_1, \ldots, \gamma_{t-1}$ is the following:

$$x_t = \prod_{i=t-1}^{0} A_i x_0 + \sum_{j=0}^{t-2} \prod_{i=t-1}^{j+1} A_i B_j \gamma_j + B_{t-1} \cdot \gamma_{t-1}.$$

In case when one studies the system evolution considering that the initial level of development where from the system starts is $(\tau, x_\tau)$, then the level $(t, x_t)$ is yielded by the following expression:

$$x_t = \prod_{i=t-1}^{\tau} A_i x_\tau + \sum_{j=\tau}^{t-2} \prod_{i=t-1}^{j+1} A_i B_j \gamma_j + B_{t-1} \gamma_{t-1}.$$

The properties of accesibility zone and of growth zone of economic-cybernetic system are determined by the mathematical structure of its evolution equation, as well as by the structure of set $\Phi_\mathcal{O}$ of the input vectors admitted by the system.

Thus, in case when the set $\Phi_\mathcal{O}$ is convex, then the set of all development levels feasible to the system at moment t is a convex set too.

Indeed, one will assume that $(t, x_t^1)$ and $(t, x_t^2)$ are two accesible development levels, therefore:

$$x_t^1, \; x_t^2 \in M_t[(0, \; x_0), \; \Phi_\mathcal{O}]$$

and $\gamma_j^1$, $\gamma_j^2$ $(j = 0, 1, \ldots, t-1)$ are the corresponding input vectors.

In this case, the following relations are satisfied:

$$x_t^1 = \prod_{i=t-1}^{0} A_i x_0 + \sum_{j=0}^{t-2} \prod_{i=t-1}^{j+1} A_i B_j \gamma_j^1 + B_{t-1} \gamma_{t-1}^1$$

$$x_t^2 = \prod_{i=t-1}^{0} A_i x_0 + \sum_{j=0}^{t+1} \prod_{i=t-1}^{j+1} A_i B_j \gamma_j^2 + B_{t-1} \gamma_{t-1}^2.$$

By making a convex combination of vectors $x_t^1$ and $x_t^2$ one obtains:

$$x_t^3 = \alpha x_t^1 + (1-\alpha)\, x_t^2 = \alpha \prod_{i=t-1}^{0} A_i x_0 + (1-\alpha) \prod_{i=t-1}^{0} A_i x_0 +$$

$$+ \sum_{j=0}^{t-2} \prod_{i=t-1}^{j+1} A_i B_j [\alpha \gamma_j^1 + (1-\alpha)\gamma_j^2] + B_{t-1}[\alpha \gamma_{t-1}^1 + (1-\alpha)\gamma_{t-1}^2].$$

Since $\Phi_{\mathcal{S}}$ is a convex set, it results that:

$$\alpha \gamma_j^1 + (1-\alpha)\gamma_j^2 = \gamma_j^3 \in \Phi_{\mathcal{S}}$$

and therefore $x_t^3 \in M_t[(0,\ x_0),\ \Phi_{\mathcal{S}}]$, thus, the set of the accessible system development levels is convex.

The management of economic-cybernetic systems, the designing of their optimal evolution trajectories into the national economy space according to the society's general development strategy, requires the precise knowledge of development levels which have to be attained. This is why, we shall further present a logistic criterion on whose base the accesibility of each development level is tested. We shall suppose that by an exchange of variable, the initial state $x_\tau$ is into the origin of coordinate axis, therefore $x_\tau = 0$.

The accesibility criterion further presented is based on the properties of the following linear operator:

$$W_{\tau,t} : X \to X$$

yielded by

$$W_{\tau,t} = \sum_{j=\tau}^{t-2} \left( \prod_{i=t-1}^{j+1} A_i \right) B_j B_j' \left( \prod_{i=t-1}^{j+1} A_i \right)' + B_{t-1} \cdot B_{t-1}'.$$

Nucleus of operator $W_{\tau,t}$ is:

$$Ker\ W_{\tau,t} = \{x / x \in X,\ W_{\tau,t} x = 0\}$$

and the image of the operator is the following one:

$$Im\ W_{\tau,t} = \{x / x \in X,\ (\exists\ Z_x \in X) \text{ such that } x = W_{\tau,t}\ Z_x\}.$$

If the states space $X$ is a $n$-dimensional vectorial space, then both the nucleus and the image of the operator are its subspaces. These subspaces disjoint and their direct sum [1] is equal to the entire vectorial space $X$ therefore:

$$Ker\ W_{\tau,t} \bigcap Im\ W_{\tau,t} = \Phi$$

$$Ker\ W_{\tau,t} \oplus Im\ W_{\tau,t} = X.$$

One proves on the base of operator $W_{\tau,t}$ the following *accesibility criterion:* the necessary and sufficient condition so that the development level $(t,\ x)$ to be feasible to the system, if it starts from the development level $(\tau,\ x_\tau)$ is that vector $\hat{x}$ to belong to the image of operator $W_{\tau,t}$.

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[1] The direct sum of two vectorial sub-spaces is defined as follows: $Y \oplus Z = \{x / x = y + z,\ y = Y,\ z = Z,$ and the representation is unique.

Further one we shall prove that the affilliation of state $x$ to the image of operator $W_{\tau,t}$ implies the fact that the development level $(t, \hat{x})$ can be reached by the analysed system.

The proof is based on emphasizing the way of effective selection of the orders guiding the system in the stated state $x$.

As $\hat{x} \in Im W_{\tau,t}$ it results that it exists a vector $z$ such *as*:

$$\hat{x} = W_{\tau,t} \cdot \hat{z}$$

for

$$\hat{x} = \left[\sum_{j=\tau}^{t-1}\left(\prod_{i=t-1}^{j+1} A_i\right) B_j B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' + B_{t-1} \cdot B_{t-1}'\right] \hat{z}.$$

The selection of input vectors, of orders respectively, is done according to the following rule:

$$\gamma_j = B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' \hat{z}; \quad j = \tau, \ \tau + 1, \ ..., \ t-2$$

$$\gamma_{t-1} = B_{t-1}' \hat{z}.$$

One will prove that the above relations ensure the *steering of the cybernetic-economic system* in order to reach the development level $(t, x)$.

Indeed, the general solution of the system growth equation yields:

$$x_t = \sum_{j=\tau}^{t-2} \prod_{i=t-1}^{j+2} A_i B_j \gamma_j + B_{t-1} \gamma_{t-1} =$$

$$= \sum_{j=\tau}^{t-2}\left(\prod_{i=t-1}^{j+1} A_i\right) B_j B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' \hat{z} + B_{t-1} \cdot B_{t-1}' \hat{z} =$$

$$= \left[\sum_{j=\tau}^{t-2}\left(\prod_{i=t-1}^{j+1} A_i\right) B_j B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' + B_{t-1} B_{t-1}'\right] \hat{z} = W_{\tau,t} \hat{z} = \hat{x}.$$

Therefore at time $t$, the system reaches state $\hat{x}$, which means that the development level $(t, \hat{x})$ can be attained.

In what follows, one will prove that when vector $x$ does not belong to the image of operator $W_{\tau,t}$ that state cannot be reached at time $t$. Thus the mention criterion is a necessary condition of accesibility.

The proof will use the following decomposition of vector $\hat{x}$:

$$\hat{x} = x^1 + x^2 \text{ with } x^1 \in Im W_{\tau,t} \text{ and } x^2 \in Ker\ W_{\tau,t}.$$

From the relation $(x^2)'\ W_{\tau,t} = 0$ one obtains:

$$(x^2)' W_{\tau,t}\, x^2 = (x^2)' \left[\sum_{j=\tau}^{t-1}\left(\prod_{i=t-1}^{j+1} A_i\right) B_j B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' +\right.$$

$$\left. + B_{t-1} B_{t-1}'\right] x^2 = \sum_{j=\tau}^{t-2}\left(B_j' \prod_{i=j+1}^{t-1} A_i' x^2\right)' \cdot \left(B_j' \prod_{i=j+1}^{t-1} A_i' x^2\right) +$$

$$+ (B_{t-1}' x^2)' \cdot B_{t-1}' x^2 = \sum_{j=\tau}^{t-2} \left\| B_j' \left(\prod_{i=t-1}^{j+1} A_i\right)' x^2 \right\|^2 + \left\| B_{t-1}' x^2 \right\|^2 = 0.$$

This implies the fact that the following vectors are zero:

$$B_j'\left(\prod_{i=t-1}^{j+1} A_i\right)' x^2 = 0,\ j = 0,\ 1,\ ...,\ t-1$$

$$B_{t-1}'x^2 = 0,\ i = t-1,\ ...,\ j+1.$$

*If one assumes that although* $\hat{x} \notin Im W_{\tau,t}$ *thus* $x^2 \neq 0$ *the development level* $(t,\ x)$ *is accesible, the following relations are obtained:*

$$\hat{x} = x^1 + x^2 = \sum_{j=\tau}^{t-2} \prod_{i=t-1}^{j+1} A_i B_j \gamma_j + B_{t-1}\gamma_{t-1}$$

$$(x^2)'\hat{x} = (x^2)' \cdot x^1 + (x^2)' \cdot x^2 = \sum_{j=\tau}^{t-2}\left[(x^2)' \prod_{i=t-1}^{j+1} A_i B_j\right]\gamma_j + (x^2)'B_{t-1}\gamma_{t-1}.$$

Since any vector in the image of the symmetric operator $W_{\tau,t}$ is orthogonal to all vectors in its kernel, it results that:

$$(x^2)'x^1 = 0.$$

Taking into account that the following relations are also satisfied:

$$(x^2)' \prod_{i=t-1}^{j+1} A_i B_j = 0 \text{ and } (x^2)'\ B_{t-1} = 0$$

it results that

$$(x^2)'\ x^2 = \| x^2 \|^2 = 0.$$

The above relation contradicts the hypothesis that vector $x$ does not belong to the image of operator $W_{\tau,t}$ hence the conclusion that within these hypotheses the level $(t,\ x)$ is not accesible.

The economic-cybernetic system whose growth is described by the equation:

$$x_{t-1} = A_t x_t + B_t \gamma_t$$

is *completely feasible* if the operator:

$$W_{\tau,t} : X \to X$$

is a bijective in other words its rank is maximum and its nucleus contain only the null vector.

When matrices $A_t$ and $B_t$ are invariant in time the condition concerning completely the system accesibility is as the matrix rank:

$$D = (B,\ AB,\ A^2B,\ ...,\ A^{n-1}B)$$

to be maximum.

Further on, we shall analyse the problems concerning the control and adjustment of economic-cybernetic systems. Thus, we shall assume that according to general strategy of social-economic development, the analysed system has to be controlled in such a way as to attain the established development

level in concordance to the general tasks of the national economy's cybernetic system.

So, we shall assume that the system has to be controlled in such a way as to attain the development level:

$$(t,\ \hat{x}) \in T \times X$$

which is the system "objective". State vector $\hat{x}$ is stated in concordance to the general tasks resulting from the growth programs of national economy's cybernetic system.

The development level $(\tau,\ x_\tau)$ of the economic-cybernetic system $\mathcal{S}$ can be directed to state $\hat{x}$ if and only if there exists a moment $t \geqslant \tau$ and input vectors (command-decision) $\gamma_\tau,\ \gamma_{\tau+1},\ \ldots,\ \gamma_{t-1}$ such that: $x_t = \hat{x}$.

It results from the above definition the strong connection existing between the concept of control and the accesibility one. Indeed, the economic-cybernetic system at the development level $(\tau,\ x_\tau)$ can be controlled in such a way as to attain the development level $(t,\ \hat{x})$ if and only if $(t,\ x)$ is a feasible level to the system.

For the economic-cybernetic system whose growing process is described by the vectorial equation

$$x_{t+1} = A_t x_t + B_t u_t$$

the development level $(\tau,\ x_\tau)$ is controllable if and only if there exists a moment $s \geqslant \tau$ in such a way as $x_\tau$ to belong to the image of operator:

$$W_{\tau,s} = \sum_{j=\tau}^{s-2} \left( \prod_{i=s-1}^{j+1} A_i \right) B_j B_j' \left( \prod_{i=s-1}^{j+1} A_i \right)' + B_{s-1} \cdot B_{s-1}'.$$

We mentioned above that by a change of variable, the state which the system has to attained is $\hat{x} = 0$.

An economic-cybernetic system starting from the development level $(\tau,\ x_\tau)$ is completely controllable and adjustable if it can be a finite time in any state $x \in X$.

For the class of economic-cybernetic systems whose evolution is described by the vectorial equation:

$$x_{t+1} = A x_t + B u_t$$

the control condition and the complete adjusting one return to the fact that matrix:

$$D = (B,\ AB,\ A^2B,\ \ldots,\ A^{n-1}B)$$

has to have a maximum rank, namely rank $D = n$.

In order to illustrate the concrete way of unfolding the logistic properties analysis of an economic-cybernetic system, we shall take the case of a dynamic model of inter-branch connections type

$$X_t = A_t X_t + B_t[X_{t+1} - X_t] + C_t.$$

We denoted by $X_t$ the output vector in year $t$, by $A_t$ the matrix of direct expense coefficients, by $B_t$ the matrix of specific investments in year $t$ and by $C_t$ — vector of net final consumption.

The output vector represents, for the analysed exemple system state vector, it outlining the development level, output potential of considered branches. The vector of final net consumption $C_t$ represents the decision-command vector.

When matrix $B_t$ is invertible, the model can also be written as follows:

$$X_{t+1} = B_t^{-1}[E - A_t + B_t]\, X_t - B_t^{-1} \cdot C_t.$$

The above equation describes the output vector evolution of considered branches, depending on the adopted orders concerning the dynamic of net final consumption vector $C_t$.

Evolution equation of output vector can be written too as follows:

$$X_{t+1} = G_t X_t + F_t C_t.$$

Matrices $G_t$ and $F_t$ are yielded by the following relations:

$$G_t = B_t^{-1}\,(E - A_t + B_t); \quad F_t = -B_t^{-1}.$$

We shall assume the output vector in the initial year is $X_0$ and the evolution of final consumption vector is the following:

$$C_0,\ C_1,\ \ldots,\ C_{t-1}.$$

In this case, output vector for year $t$ is yielded by the following formulae representing the solution of dynamics equation:

$$X_t = \prod_{i=t-1}^{0} G_i X_0 + \sum_{j=0}^{t-2} \prod_{i=t-1}^{j+1} G_i F_j C_j + F_{t-1} \cdot C_{t-1}.$$

When matrices $G_t$ and $F_t$ are invertible for a certain time period, then the above formulae is changed as follows:

$$X_t = G^t X_0 + \sum_{j=0}^{t-2} G^{t-j-1} \cdot FC_j + FC_{t-1}.$$

In order to make concrete the application, one shall consider the case of a model including three branches: $R_1$, $R_2$, $R_3$.

The matrix of the coefficients of direct expenses is presented in Table 1.

The elements of matrix $A$ express the direct relations of interdependence which are formed among the three branches within the production process. The columns of matrix $A$ outline the structure of material expenses within each branch.

**Table 1**

| Producing branch \ Benefiting branch | $R_1$ | $R_2$ | $R_3$ |
|---|---|---|---|
| $R_1$ | 0.198 | 0.188 | 0.063 |
| $R_2$ | 0.165 | 0.141 | 0.043 |
| $R_3$ | 0.087 | 0.088 | 0.303 |

In this way, the material expenses for a product of 1000 lei for branch $R_1$ have the following structure: 198 lei represent the value of raw materials, etc., supplied by branch $R_1$ (self-consumption); 165 lei represent the value of inputs from branch $R_2$ and 87 lei — the value of inputs from branch $R_3$.

In a similar manner one can analyse the structure of material expenses for other branches on the basis of data contained by matrix $A$.

Table 2 shows the specific investments coefficients matrix:

**Table 2**

| Producing branch \ Benefiting branch | $R_1$ | $R_2$ | $R_3$ |
|---|---|---|---|
| $R_1$ | 0.590 | 0.442 | 0.475 |
| $R_2$ | 0.220 | 0.183 | 0.268 |
| $R_3$ | 0.221 | 0.393 | 0.277 |

The elements of matrix $B$ reflect interdependence relations formed during the investments activity. In order to achieve an increase by one unit of the production potential of branch $R_1$ it must deliver 0.590 units of its final product to be invested into this branch and in order to achieve an increase by one unit of the production potential of branch $R_2$ it is necessary for branch $R_1$ to deliver to it 0.442 units of its final product.

The analysis concerning the specific investments structure within each branch can continue in a similar manner, based on data contained in Table 2.

In order to obtain the dynamic vectorial equation calculations is required to calculate the elements of matrices $G$ and $F$ should be performed.

The following results are obtained:

$$G = B^{-1}(E - A + B) = \begin{Vmatrix} 6.132 & -5.631 & -2.035 \\ 0.576 & -3.963 & 3.573 \\ -5.221 & 11.217 & 0.007 \end{Vmatrix}$$

$$F = -B^{-1} = \begin{Vmatrix} -4.908 & 5.771 & 2.832 \\ -0.154 & 5.251 & -4.817 \\ 4.134 & -12.054 & 0.964 \end{Vmatrix}$$

Based on the above matrices one can write growth equations for each of the branches considered within the model:

— growth equation of branch $R_1$:

$$X^1_{t+1} = 6.132\, X^1_t - 5.631\, X^2_t - 2.035\, X^3_t. - 4\,908\, C^1_t + \\ + 5.771\, C^2_t + 2.832\, C^3_t;$$

— growth equation of branch $R_2$

$$X^2_{t+1} = 0 \cdot 576\, X^1_t - 3.963\, X^2_t + 35.73\, X^3_t - 0.154\, C^1_t + \\ + 5.251\, C^2_t - 4.817\, C^3_t;$$

— growth equation of branch $R_3$

$$X^3_{t+1} = -5 \cdot 221\, X^1_t + 11 \cdot 217\, X^2_t + 0 \cdot 007\, X^3_t + 4 \cdot 134\, C^1_t - \\ - 12 \cdot 054\, C^2_t + 0 \cdot 964\, C^3_t.$$

In order to study the logistic properties concerning the accesibility, the steering and the controllability of the system described by the analysed economic-mathematical model it is necessary to calculate the rank of the matrix:

$$D = (F,\ GF,\ G^2F).$$

It is obvious that this matrix has the rank 3, this fact meaning that the system is completely steering and adjusting.

For the case of economic-cybernetic systems whose growth is described by a vectorial equation under the form:

$$x_{t+1} = Ax_t + B\gamma_t$$

the size of operator image $W_{\tau,t}$ which supply information concerning the properties of system control and adjustment is equal to the rank of matrix $D$, where:

$$D = (B,\ AB,\ A^2B), ..., A^{n-1}B).$$

At the same time, we have to mention the states set of such a cybernetic-economic system where it can be controlled or adjusted form, is made up of vectorial sub-space $A$-invariant of minimum size, covering all columns of matrix $B$.

Economic logistics by its arsenal of worked out methods, techniques and algorithms, has become a very useful instrument in the activity of designing the economic growth trajectories for management, control and adjustment of the economic-cybernetic systems into the national economy space.

## 2. THE ECONOMIC QUANTUM

The scientific management of economy has to base upon the in-depth knowledge of the objective laws underlying the development of the economic and social phenomena, the multiple connexions and interdependences that arise among the parts of system of the national economy within the complex process of economic growth.

Unlike the systems of movement laws governing from mechanics or physics, the laws governing the evolution within the scope of the national economy are much more complex. Their degree of complexity gets incremented with the dynamic and fast character of the economic processes, with the way of the laws specific for the economic scope get manifested, with the requirements in order to ensure the optimum economic time and space, both at macro- and microeconomic levels, with the action of the phenomena of spreading (propagation) as well as branching out caused by feedback law.

The ever growing impact of the development of science and technology on the overall ec-onomic growth, the continuous increase of the forms of international cooperation and strengthening the flows of linking the scope of the national economy with that of the world economy as a result of growing the international division of labour are also elements that contribute to a larger extent to the increase in the complexity of the movement and development law of the economic and cybernetic system and consequently the act of leading the economic processes.

The laws of movement at the core of the economic and cybernetic systems show the essential and general connections that link different processes and phenomena that occur in the space of the national economy. They have a historical character generally related with a certain level of social development. By pointing out the time-space sequence of phenomena and economic processes, the dynamic laws of the economic and cybernetic systems displays the causality relationships that they produce.

Hence, the accurate knowledge, in terms of quality and quantity, of the concrete way of how the laws of space-time dynamics of the national economic system act has a majour significance in projecting the trajectories of economic and social growth.

The study on multiple parts of the economic and cybernetic systems dynamics, the pattern-like features of the national economic space has led to the development of a new branch of the economic cybernetics — the economic quantum.

The basic problems studied by the economic quantum concern the laws of *dynamic equilibrium* within the space of the national economy, the analysis of *the entropic* and *gravitational features*, the emphasis of the specific way of action of the phenomenon of *propagation* and that of *training*, as well as the mechanism of producing and displaying the *conditioned reflexes* from economy.

Making use of the results obtained from the economic quantum contributes to the knowledge of the control on the mechanisms that determines the process of economic growth. So, the prospective role of the economic quantum gets emphasized, its methods and techniques allow a better substantiation while forecasting the evolutionary trends of the phenomena and processes from the space of the national economy.

The economic quantum by taking ones and using some methods and techniques from the overall stock of the economic cybernetics, provides at the same time, its various fields, the economic logistics, particularly, with especially useful elements at proving scientifically the decisions as to guidance and adjustment of the economic and cybernetic systems. So, in the stage when the trajectories of economic and social development are projected, the economic quantum by the methods and techniques that have been worked out, provides information concerning the prospective trends from the space of the national economy, conditions that have to be fulfilled with a view to ensuring *a dynamic and optimized equilibrium.*

When working out some programmes regarding the development of specific branches and sub-branches of the economy, the knowing of the concrete way of action as to the gravitational forces from the space of the national economy, of the intensity of the training process and that of propagation

allows the pointing out of the connected parts of the system of the national economy which have to be developed in order to ensure the dynamic equilibrium for the entire economy.

The study of the *conditioned reflexes of the economy* contributes to the most exact acquaintance of the adjustment and self-adjustment mechanisms of the economic and cybernetic systems.

So there evinces the strong interferrence that occur amcng the economic quantum and the economic logistics, their contribution to the development of the overall stock of concepts, methods, techniques, algorithms and models of the economic cybernetics that are necessary to prove scientifically the economic decisions, both in the stage of projecting the paths of optimum economic growth and in the stage of execution in order to pursue the way how the targets trajectories have been achieved.

On the way of accomplishing the growth programs, the methods and techniques of the economic logistics and the economic quantum contribute in finding new solutions that ensure, by an efficient and versatile process of decision making, order, adjustment and self-adjustment, keeping all systems that develop within the space of the national economy on an area of trajectories of optimum balanced and proportional economic growth.

## 2.1. The dynamic equilibrium in the space of the national economy

A fundamental problem for the knowledge of movement laws acting in the space of the national economy is the identification of those conditions that ensure the *dynamic equilibrium* within the process for all the parts of the general cybernetic system of the national economy. *The dynamic equilibrium* mirrors the dialectical essence of the development of the cybernetic and economic systems on the space of the national economy. It points out the lively, evolutionary, accelerated character of systems development.

The concept of dynamic economic equilibrium presents itself under a multitude of aspects being the synthetic expression of the status of agreement and synchronization in time and space of all interdependent parts of the activities carried out in the national economy. It supposes fulfilment of some *optimum proportions* concerning the distribution of national income, the development funds, i. e the investments, the consumption fund, the material production, the imports and exports, by branches, sub-branches, products and groups of products, as well as by countries. At the same time, it implies the mostly efficient use of all human and material ressources of the nation, the providing of a high growth rates of the social labour productivity, the increase of the technological level, the quality and reliability of the products, the steady dwindling of material costs share in the social product a.s.o. The status of overall economic equilibrium requires, as an objective need the achievement of the material, financial, monetary and currency ballance in the development of the national economy.

A defining feature for the equilibrium of the space of the national economy is its lively, evolutionary character. The coordinates that determinates the equilibrium status at a given moment are, in their term, under the occurence of the permanent and fast changes that characterizes the entire

development process within the economic space. Consequently, within the economic space the equilibrium status appears as a general trend of permanent adapting to the wide variety of conditions of influence that occur by continuously surpassing the unappropriate states from the national economy and from the interference with the world economy. The dialectical understanding of the trinomial, *equilibrium-non-equilibrium-equilibrium* points out the dynamic character of evolution — the spring of development within the scope of the national economy.

The economic quantum, by studying the movement of cybernetic and economic systems on the path of dynamic equilibrium, points out, at the same time, the conditions that ensure *the proportionate growth* of the entire system of the national economy, the proportionate development of sector I and sector II of the social output, industry and agriculture and within the industrial subsystem the industry of productive means and consumer goods, the mining and processing industry a.s.o. Within the agricultural subsystem there are analysed the proportions between animal and vegetal production as well as those that arise among their various sectors.

The provision through the unitary national plan of the proportionality in the development of all systems evolving within the national economy space, of the proportions by branches, subbranches, products and groups of products, as well as in territorial profile, represents a decisive condition for the evolution of the whole national economy system along optimal dynamic balance trajectories.

It is characteristic for the cybernetic and economic systems that they have an extremely complex structure, being made up of a great number of parts integrated by a lot of direct links and feedbacks. The path of growth in a cybernetic and economic system is, therefore, *the vectorial rezultant of the paths bundle of their component elements.* Also, the status of dynamic equilibrium of a cybernetic and economic system corresponds to some dynamic inner equilibriums achieved among its multiple components.

There comes to the fore the need that when analysing the processes that occur within the scope of the national economy one should always take into consideration the fact that the laws of evolution for the cybernetic and economic systems provide information regarding the average sizes of the indicators that mark the process of growth. These make a synthesis of the large number of phenomena which are especially complex. So, each variable that identifies the status of one system within the scope of the national economy is as a matter of fact, a weighted average of those parameters corresponding to its component parts; in their turn, the state variables of the component parts swing between the average overall values of the system.

The economic quantum when studying the laws of evolution from the economic space, it starts from the fact that direction, speed and acceleration of a separated component in movement are not sufficient to characterize the overall development of a cybernetic and economic system. Only the dialectical outlook that allows for all the relations and links that form within the system, the inner logics of its laws of development, permits to acquire some useful information for the practical management of the economic and social processes and phenomena. As example, at the level of a national economy branch, the data concerning a single enterprise can not provide sufficient information

for the management of the whole branch. Also, nor the branch average indices can be considered sufficient for the knowledge of all specific processes taking place within each enterprise.

The achievement of complete information useful in the management activity of the whole system, requires the correlated analysis of branch average indices and those characterising the specific evolution of each component enterprise, vector, products or groups of products.

The analysis of how specific is the process economic development renders evident the fact that the paths of evolution $\{x_t\}_o^T$ of the economic and cybernetic systems represent the vectorial resultant of three separate component parts that mark the movement within the economic space:

— the trend or the long-term tendency $g_t$ (figure 6);

— undulatory motion $\varphi_t$;

— the random component $\varepsilon_t$;

There results:

$$x_t = g_t + \varphi_t + \varepsilon_t$$

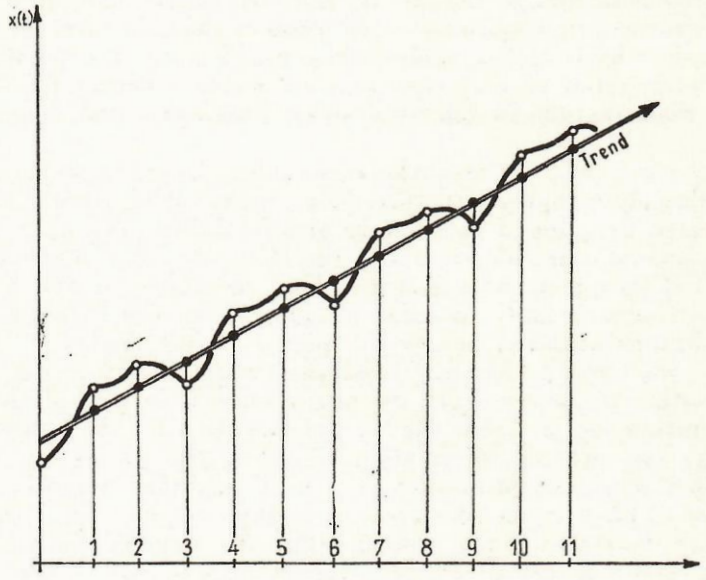



Fig. 6

*The first component*, respectively *the trend*, is identified by means of the criterion of system and the whole macrosystem stability. The evolution of the cybernetic and economic systems, their motion and development, is a continuous process of passing from a concrete state to another one, higher in terms of quality and quantity. Each status of a system, defined by the vector of the variable of state, is a stage in the process of growth marked by a large number of correlations determined by specific laws of motion underlain by trend, the long-term tendency, any future status, of the cybernetic and economic system gets shape in terms of course and direction of the previous states.

There results that the trend is the way of concrete display of the essential tendency in the system development. It is determined by the inner nature of that system, i.e. by the pattern of its links and its specific behaviour. The trend determines the main direction of the cybernetic and economic systems, of their development. It subordinates or cancel some other nonessential features or directions that occur on the way within process of system growing.

There have to be underlined that the concrete evolution of an economic and cybernetic system is determined as a *vectorial resultant of interaction* among its own laws of motion, mirrored by the trend with the conscious interference of the decision makers, substantiated on the knowledge of the objective laws.

The identification of the trend by a cybernetic and economic system is performed by using the specific methods of the economic quantum starting from the knowledge of its past evolution and the tendencies that are foreshadowed within the pattern of the economic process that characterizes the respective system. It bares a special significance for forecasting in identifying the tendencies of growth with the cybernetic and economic systems.

In order to identify the trend one has to begin from the synthetic indicators proper to the cybernetic and economic system, resulted from the analysis of its essence, the logic links, the connections and interferrences that are shaped within it. These indicators are mirrored, in terms of quantity by the dynamic series:

$$\{x_t\}_{t=t_0}^{t=t_p}$$

that characterizes the evolution of the system from the original moment $t_0$ till the moment $t_p$.

The identification and the quantification of the long-term tendency concerning the evolution of the cybernetic and economic system lies in finding, within a class of mathematical functions $\mathcal{F}$ the one which reflects best the laws of growth for the system. That is not to identify the function $\tilde{f}_t \in \mathcal{F}$ that gets most closely to the given dynamic series, i.e.:

$$\hat{\delta}(\tilde{f}_t,\ x_t) = \min_{f_t \in \mathcal{F}} \hat{\delta}(f_t,\ x_t).$$

The concrete way of computing the distance between the actual path and the computed one could be performed by adopting, according with the needs of the analysis, various metrics, specific to the finite spaces. Therefore, one can consider, for instance, the following types of metrics:

$$\hat{\delta}(f_t,\ x_t) = \sqrt{\sum_{t=t_0}^{t_p} (f_t - x_t)^2}$$

$$\hat{\delta}(f_t,\ x_t) = \sum_{t=t_0}^{t_p} |f_t - x_t|$$

$$\hat{\delta}(f_t,\ x_t) = \max_{t_0 \leqslant t \leqslant t_p} |f_t - x_t|.$$

As to the choice of the class of mathematical functions that shoul describe under equation form the growth tendency of the studied cyberneti and economic system, it is performed on the basis of a *process phenomenologica analysis*. It bases itself, on the main, on the speed study on the evolutio of the analysed economic process.

For example, when the concrete analysis of the process points to a evo lution speed proportionate to the level that has already been achieved, i.e a steady growth rate, then its trend has to be looked for in the class of expo nential functions.

Indeed from the equation with finite differences:

$$x_{t+1} - x_t = \alpha x_t$$

by solving it we get:

$$x_t = (1 + \alpha)^t \cdot x_0; \quad t = 1, 2, \ldots$$

It should be chosen the class of functions with the form:

$$f_t = (1 + \alpha)^t$$

and the parameter $\alpha$ should be identified based on the dynamic series.

In the other situations, the concrete analysis of the cybernetic and economic system modelled is able to emphasise the existence of a certain cei- ling level, and the speed of evolution of the phenomenon should be, at each moment $t$ proportionate with the difference between the value of the ceiling level $x^*$ and the level that have already been achieved.

The equation is as follows:

$$x_{t+1} - x_t = \beta(x^* - x_t).$$

The solution of the above equation is the following:

$$x_t = x^* - (1 - \beta)^t (x^* - x_0)$$

and its graphic is at figure 7.

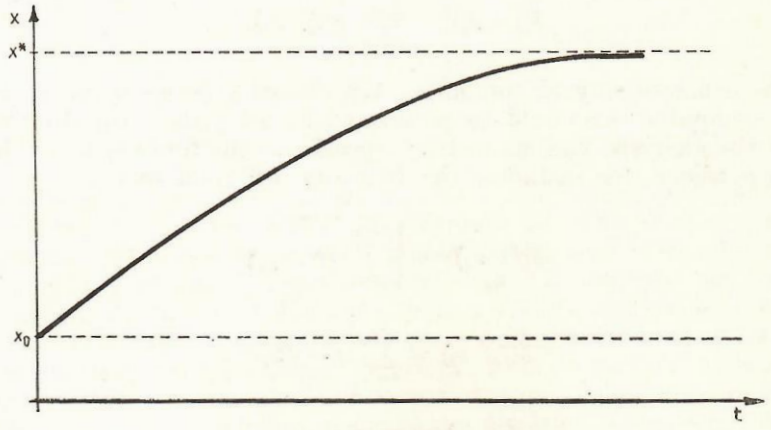



Fig. 7

In practice, the tendency of evolution for several systems could be described by the following function, defined as logistic function:

$$x_t = \frac{x^*}{1 + be^{-ct}}$$

the logistic curve (fig. 8).

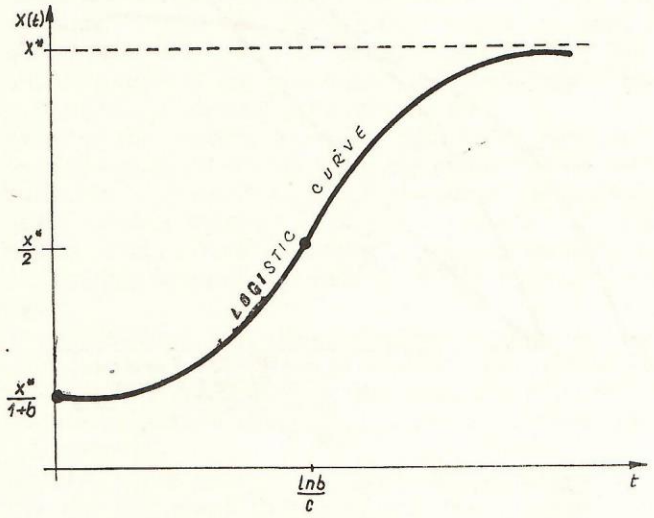



Fig. 8

This model represents, under equation form, the tendency of evolution of the system if the varying speed of it is proportionate both with the level that have already been attained and the distance against a ceiling-level $x^*$.

Such situations are likely to appear when, after attaining a certain level, the increases are practically insignificant. This is the case with the per capita consumption of some food-producers, of the amount of chemical fertilizers by hectar, etc.

In figure 9 there are shown the three curves of Törnquist used to determine the trend concerning the demand of the population, and in figure 10 the logarithmic curve are shown.

*The second component* item that marks the dynamics of a cybernetic and economic system, i.e. the *undulatory motion*, has got an extremely complex interpretation. It is in a close interrelationship with the specific nature of the laws of evolution within the scope of the national economy, depending on the intensity and rate of business that are carried out in each system, the need to ensure the process of optimum balanced and proportionate economic growth of the entire cybernetic system.

The lively dynamic character of the economic processes, the continuous shift of the concrete conditions where they are developing, make that the laws of evolution within the economic space have not a *linear character*. This fact was emphasised by Karl Marx when he studied the processes taking place in economy.

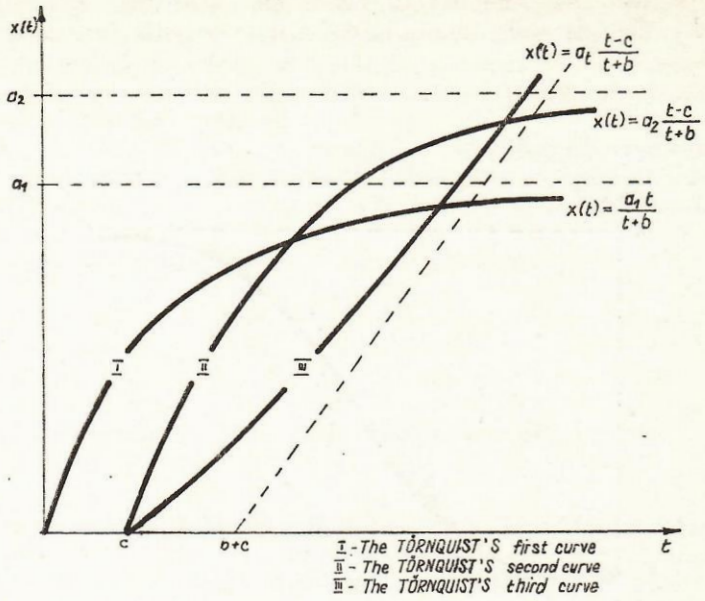



Fig. 9

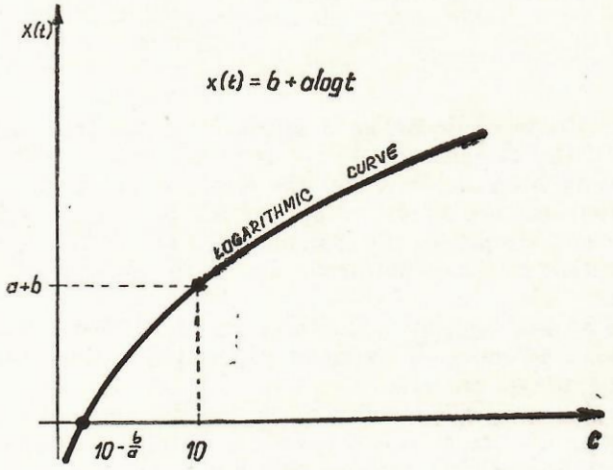



Fig. 10

In the analysis of the concrete way of how, the growth laws for the cybernetic and economic system manifest within the scope of the national economy it should be taken into account that they act lay a close interrelationship, and not isolated.

The growth trajectories of a cybernetic and economic system are strongly influenced by the joint action of its laws of evolution as well as by a series of factors originating from other systems of the space of the national economy or that of the world economy. The managing system, by the decision made

as to order control, self-adjustment or adjustment it adopts, guarantee the everlasting character of the systems development along the trajectories of economic growth that have been already set, as well as keeping the required ratios. The vectorial resultant of these extremely complex forces can render the systems, at different periods of time, an undulating motion around the direction of dynamic equilibrium whose amplitude is caused by concrete conditions.

One must point out that within the socialist economy, the permanent and objective tendency of ensuring *the dynamic, optimal balance* of the whole economic mechanism *is determinant*; the oscillatory component that might arise in the evolution of temporal balance state *represents only a moment in the dialectic of the dynamic process of optimal growth.*

The analysis of the specific forms by which the wave motion reveals itself within the scope of the motional economy prints out the nature of those factors contributing to its generation. Thus, the growth paths of the branches from the national economy may be affected by a range of factors connected with the emergence of some new technologies, the increase of the domestic as well as world market demand, the rate by which the new fixed assets are commissioned etc.

Also, some differences that would appear among the sectors of the national economy concerning the level of labour productivity, the consumption of raw materials, materials energy and fuels could temporarily produce wave motion whose amplitude dies off as a result of feedback law acting, the training phenomena.

By the impetus given to all components of the system of the national economy that is the alignment to the growth trajectories typical of those branches with the most dynamic and alert rates there could provisionally be generated certain kinds of wave motion. There have to be emphasized the positive role assigned in this case to wave motion as a factor of speeding the rates of growth, for all branches of the economy, with the view to their becoming in line with the most dynamic branches.

The strong interpenetration between the national and world economics accounts for another factor that could generate for a short time wave motion within the space of the national economy. In this respect, a series of floating prices on the world market for raw materials and materials, power resources, equipment and installations could affect the evolutionary paths within the scope of the national economy. Thus, if export prices for some delivered goods shaw an up-ward tendency on the world market, the economy will enjoy additional hard currency resources, that will allow speeding up of some branches and sectors from the national economy. On the contrary, if the import prices grow, there might momentary occur, some disturbance that must be immediately annihilated, lest waves of the evolutionary trajectories from some systems and sub-systems should be produced, in the context of balanced, proportional, economic growth.

Also, out of various reasons the failing to fulfil in due time the programmes in the fields of output, investment, goods quality, transportation, storage a.s.o., bring about disturbances that could propagate modulatorily within the scope of the national economy. There have to be mentioned that some undulatory phenomena under the form of disturbances could be produced

by natural disasters, for instance floads, the influence of the weather phenomena on agrarian output, a.s.o.

In socialism, the political decision body has available by means required to prevent and attenuate the effects of such disturbances by adopting the control steps which ensure the evolution of all systems and subsystems along planned optimal growth trajectories.

The economic quantum has the role to study the motion of systems, *its undulating component parts* the way the propagation and driving effects act with the view to working out some efficient algorithms of guiding and adjusting, *cuantification of the conditions that ensure the permanency of the dynamic equilibrium of the growth trajectories from the scope of the national economy.* If some negative waves appear, the algorithms that have been worked out have to allow to as quick as possible down the cybernetic and economic system along the optimum paths of growth, by a minimum consumption of social energy.

In order to represent the wave motion of a cybernetic and economic system under equation form, one should start from the fact that any function:

$$\varphi : [t_0, T] \to R$$

of class $C^1$ which describes its evolution in time could be decomposed into a series of mathematical functions of the form:

$$\varphi(t) = \sum_{\gamma} \left(a_\gamma \cos \frac{2\pi}{\delta} \gamma t + b_\gamma \sin \frac{2\pi}{\delta} \gamma t\right)$$

where $\delta = b - a$.

Coefficients $a_\gamma$ and $b_\gamma$ are obtained by the following relationships

$$a_\gamma = \frac{2}{\delta} \int_a^b \varphi(t) \cos \frac{2\pi}{\delta} \gamma t \, dt$$

$$\gamma = 0, 1, 2, \ldots$$

$$b_\gamma = \frac{2}{\delta} \int_a^b \varphi(t) \sin \frac{2\pi}{\delta} \gamma t \, dt.$$

If we note with

$$\rho_\gamma^2 = a_\gamma^2 + b_\gamma^2; \quad \theta_\gamma = \operatorname{arc\,tg} \frac{b_\gamma}{a_\gamma}; \quad \lambda_\gamma = \frac{\delta}{\gamma}$$

then the expression of function $\varphi(t)$ can be written in the following one:

$$\varphi(t) = \sum_{\gamma} \rho_\gamma \cos\left(\frac{2\pi}{\lambda_\gamma} - \theta_\gamma\right).$$

Taking into consideration that functions: $\sin \frac{2\pi}{\delta} \gamma t$ and $\cos \frac{2\pi}{\delta} \gamma t$ respectively $\cos\left(\frac{2\pi}{\lambda\gamma} - \theta_\gamma\right)$ are periodical functions the above equation points out that a patb of growth:

$$\{x_t\}_{t_0}^T = \varphi(t)\,|_{t_0}^T$$

along the period of time $[t_0, T]$ can be viewed as the vectorial resultant of a set of paths of some items $\gamma$ having an undulatory motion of amplitude $\rho_\gamma$.

Presentation under equation form of the two main component parts that determine the motion of the cybernetic and economic systems, the trend and the undulatory component, respectively is to be done based on the model that quantifies *the impact the rate of commissioning the investment objectives has on the trajectory of fixed assets from the system of material production and services.*

The economic-mathematical model of fixed funds increase which will be analysed, quantifies in a mathematical form the way in which the fixed funds evolution trajectory depends on the following basic elements:

— the national income volume;
— the magnitude of accumulation rate within the national income;
— the volume of national fund of economic-social development;
— the weight of investments for goods production and services system within the national fund for development;
— the rhythms of commisioning the investments.

*The equation showing the growth of fixed assets* is the following one:

$$K_{t+1} = K_t + F_t.$$

The notations are the following:

$K_t$ — the amount of fixed assets during year $t$

$F_t$ — the amount of the new fixed funds commisioned during year $t$.

The correlation between investment and the amount of fixed funds commissioned during year $t$ is pointed out by the equation for investment:

$$F_t = \sum_{\tau=0}^{\theta} \lambda_{t-\tau} I_{t-\tau} + \varepsilon_t.$$

We noted with $\lambda_{t-\tau}$ the share of investment allocated during year $t-\tau$ that are to be commissioned, that is materialized in fixed assets during year t, and with $\varepsilon_t$ a random variable. With regard to the random variable, $\varepsilon_t$ it is assumed they are of Gaussian type, independent with zero mean and the same dispersion. It was noted with $\theta$ the number of years where in the investments alloted in a certain period are fully commissioned. Generally the value of $\theta$ differs from branch to branch, and within the same branch it differs according with the type of work carried out within that sector.

The values of the coefficients $\lambda_{t-\tau}$ that occur in the equation for investments could be determined as solutions of the following problems of *square programming:*

$$\sum_{t=1}^{N} \varepsilon_t^2 = \sum_{t=1}^{N} \left( F_t - \sum_{\tau=0}^{\theta} \lambda_{t-\tau} I_{t-\tau} \right)^2 = minim,$$

with constraints

$$\sum_{\tau=0}^{\theta} \lambda_{t-\tau} = 1, \qquad t = 1, 2, \ldots N,$$

$$\lambda_{t-\tau} \geqslant 0, \qquad \tau = 0, 1, 2, \ldots \theta.$$

The dependency on equation that is to be seen between the share of investment designed for the productive fixed assets, the rate of accumulation the size of national income and the weight of investment allocated to productive fixed funds to the total national fund for development is represented by the following relationship:

$$I_t = \beta_t[v_t \cdot Y_t + a_t K_t]$$

where:

$I_t$ — the share of investment designed for the system of material output and services at year $t$;

$Y_t$ — the volume of national income during year $t$;

$v_t$ — the accumulation rate at year $t$;

$\beta_t$ — the share of investment designed to the system of material output and services to the total national fund for development;

$a_t K_t$ — the pay-off fund at year $t$.

In order to work out the model that is to be presented, it is assumed that the dependence between the size of national income and the amount of fixed assets from the system of material output and services takes the form:

$$Y_t = e_t K_t.$$

With $e_t$ it was noted the coefficient showing the net efficiency of fixed assets (the national income produced to the unit of fixed assets).

From the last two equations, there results:

$$I_t = \beta_t(v_t e_t + a_t) K_t$$

and, thus, the equation for investment could be written in the following way:

$$F_t = \sum_{\tau=0}^{\theta} \lambda_{t-\tau}\beta_{t-\tau}(v_{t-\tau}e_{t-\tau} + a_{t-\tau}) K_{t-\tau} + \varepsilon_t.$$

From the analysis, that have been elaborated, there results that *the equation of the fixed assets growth* from the system of material output and services, could be synthetically written as:

$$K_{t+1} = K_t + \sum_{\tau=0}^{\theta} \lambda_{t-\tau}\beta_{t-\tau}(v_{t-\tau}e_{t-\tau} + a_{t-\tau}) K_{t-\tau} + \varepsilon_t.$$

On the data concerning Romania's economy, there have been inferred the following concrete form of the equation:

$$K_{t+1} = 1{,}014\ K_t + 0{,}056\ K_{t-1} + 0{,}026\ K_{t-2}.$$

Above there have been made the following assumption:

$$v_t = v; \quad e_t = e; \quad a_t = a; \quad \beta_t = \beta.$$

By performing the required computation, there results the following general form for the solution of the equation for the increase of fixed assets:

$$K_t = 714{,}406\ (1{,}084)^t + (0{,}139)^t \left(217{,}768 \sin \frac{17}{12}\pi t - 46{,}406 \cos \frac{17}{12}\pi t\right).$$

The analysis of the solution for the equation of fixed assets growth points out that within the work hypotheses made the evolution law is determined by the two majour component parts:

— the trend that is the long-term outlook

$$g_t = 714{,}406\ (1{,}084)^t;$$

— the undulatory factor, produced by the rate of implementing the investment not yet completed:

$$\varphi(t) = (0{,}139)^t \left[217{,}768 \sin \frac{17}{12}\pi \cdot t - 46{,}406 \cos \frac{17}{12}\pi \cdot t\right].$$

There have to be emphasized that within the example considered, the trend element is the dominating growth factor of the fixed assets while the undulatory factor exports a small influence showing quite a tendency towards diminishing.

In the investigation of the properties of oscillatory component of the movement, the economic quantum is based upon the thesis of scientific socialism of dialectic materialism according to which all connections and interferences in the economic space take place within the frame of the *general law concerning the unity and fight of contraries.*

The socialist economy, characterised by a strong ascensive evolution by an optimal, balanced and proportional development is not free of some contradictions, but these have a temporary and non-antagonistic character. The provision of the process of planned growth, the overcoming of non-antagonistic contradictions which might appear represents the conscious work of the working class of the whole people lead by Romanian Communist Party.

In certain development stages of the general cybernetic system for the national economy, the undulatory component part could become amplified by some contradiction that rise within the economy.

In the countries with planned economy, by the national plan of economic and social development of the country there are drawn up optimum growth and if some disturbances occur, the contradictions that might arise can be solved by exerting a permanent control over the evolution of all systems and subsystems and by taking the proper decisions in due time of guidance and adjustment in due time. Thus, it is pointed out the need for the scientific studying of contradictions that may occur in the economy in order to take all measures to prevent and diminishing their effects, clearly emphasised in the documents of our party, in the words of its general secretary. In the address to the Colloquim regarding the problems of society management, President Nicolae Ceauşescu shows: "A scientific management requires the permanent knowledge of the contradictions acting in society and also appropriate steps in order to solve them — according to the social development laws, to the party's general orientations, so as to prevent the damages to become deeper, to produce conflicts.

What is demanded is not to turn the back to realities by declaring that in socialism contradictions can not arise.

On the basis of thoroughly knowing the contradictions, the cirumstances that generate and determine them, the consequences they would have it is

possible to be taken measures able to ensure the permanent increase of economic efficiency, the well ballanced, dynamic development of the overall economic and social life.

The deep-joing analysis of the laws of motion within the cybernetic and economic systems, the concrete manner of their manifestation and their representation by equation form are tasks of utmost topicality for economic quantum, for the economic cybernetics as a whole.

As Nicolae Ceauşescu, General Secretary of the Romanian Communist Party shows: "the thorough study of the social development objective laws, of the realities in permanent change and as result, the elaboration of solutions and steps best fitted to the requirements of these laws", represents a task of paramount importance in order to ensure a strongly ascending evolution of our socialist society.

### 2.2. Entropic and gravitational features in the economic space

The evolution of the cybernetic and economic systems within the scope of the national economy can occur on a wide cluster of growth trajectories that offer the decision makers the possibility to choose among various alternatives. The decision making body, by knowing the concrete possibilities, sets the growth trajectories for each cybernetic and economic system in tune with the requirements of optimum well-ballanced and proportionate of the whole system of the national economy.

When approaching the range of the growth trajectories in the cybernetic and economic systems, the motion laws from the economic scope, a special attention should be paid to the research of *random factors* produced by the dynamic shifts from science, by the emergence of the new technology, the identification of new sources of growth the increase of world economic and technological exchanges a.s.o.

As opposed to the mechanicist determinism which considers the system evolution as uniquely determined the scientific socialism, the materialist dialectics emphasises the *unity between necessary and random*. In this view a majour role is played by the *concept of probability*, the evolutionary tendency of cybernetic and economic systems, i.e. the trend, it being accomplished through a series of cases, each with a certain probability of incidence.

Including the random factor, the approach to the ranges of trajectories along with systems may develop, expands the scope of the concept on optimum growth, makes it acquire much more valences, closer to the practical reality from economy. With this approach, the optimum trajectory of economic growth is seen from the point of view ot the possibilities to constantly *exceed* the programmes on the basis of finding out some new inner reserves within the system by a better organization of work by rellying all human and material factors, by introducing quickly in the productive process the new scientifical and technological findings a.s.o.

Laying the stress on the random component of matrix and development within the cybernetic and economic systems plays a particular part in the activity of forecasting, of identifying, the likely "future period", as well as the optimum economic time and space. Setting out from the development level

on which the space of the national economy is located, the forecast has to identify the range area of the likely trajectories along which it can evolve as well as the likelihood of each possibility to turn into reality. This is a basic element at substantiating those decisions concerning the location of the systems on the trajectories of well ballanced and proportionate economic growth.

There exists a solid osmosis between the concepts of economic time and space and random factors; the economic time and space corresponding to the fulfilment of each activity carried out within the cybernetic and economic system have a certain probability to be diminished that is minimized, hence an ever larger amount of activities are fulfilled in the unit of the calender year.

Considering a cybernetic and economic system $\mathcal{S}$ which at moment $t$ is in $x_t$ state, by analyzing its laws of dynamics, one can identify the states that are likely to be attained at the next moment $t+1$, denoted with $\{x_{t+1}^i\}_{i=1}^{q_t}$ as well as the probability to attain these denoted with $p_i$ these are pointed out in the following scheme likelihood:

$$\begin{pmatrix} x_{t+1}^1, x_{t+1}^2, \ldots, x_{t+1}^{q_t} \\ p_t', \quad p_t^2, \ldots, p_t^{q_t} \end{pmatrix}.$$

With the probabilities $p_t^i$ shifting from state $x_t^i$ to state $x_{t+1}^i$ at moment $t$ there are made the following hypotheses:

$$p_t^i \geqslant 0; \quad i = 1, 2, \ldots, q_t$$

$$\sum_{i=1}^{q_t} p_t^i = 1 \cdot$$

When a certain probability $p_t^i$ is zero, it follows that the system cannot shift to state $x_{t+1}^i$ at moment $t$ on the contrary, if to a certain state there corresponds probability $p_t^i = 1$ it follows that the system at moment $t$ has a unique evolution.

The triplet $\{t, x_t, \mathcal{S}_{x_t}\}$ where $\mathcal{S}_{x_t}$ defined the probabilistic schetch from above is the likely *space of growth for the cybernetic and economic system* $\mathcal{S}$ *at moment t.*

There have to be noticed that between the concept of probabilistic space for the growth of the cybernetic and economic system and that of access area there exists a strong connection.

The concept of probabilistic space of growth is more general, being able to point out both the accessible states and the probabilities to attain it.

Of particular importance for forecasting and planning activities is to know the pattern of the probabilistic space of growth for the cybernetic and economic systems, the diversity of the likely trajectories of growth as well as the probabilities to locate the system on each of these ones.

In order to present under the form of equation the pattern of the probabilistic space of growth for a cybernetic and economic system, the identification of its set of growth trajectories the dynamism of its development, there should be defined *the concept of entropy for the probabilistic space of growth at moment t:*

$$H_t(\mathcal{S}_{x_t}) = -\sum_{i=1}^{q_t} p_t^i \log p_t^i;$$

where generally, the logarithm is in the second base.

In this way, *the entropy of the probabilistic space of growth for the cybernetic and economic system S measures as a matter of fact the dynamics of the system* within the complex process of economic development.

In figure 11 it is presented, for providing an example, the variation of entropy for a two-states system.

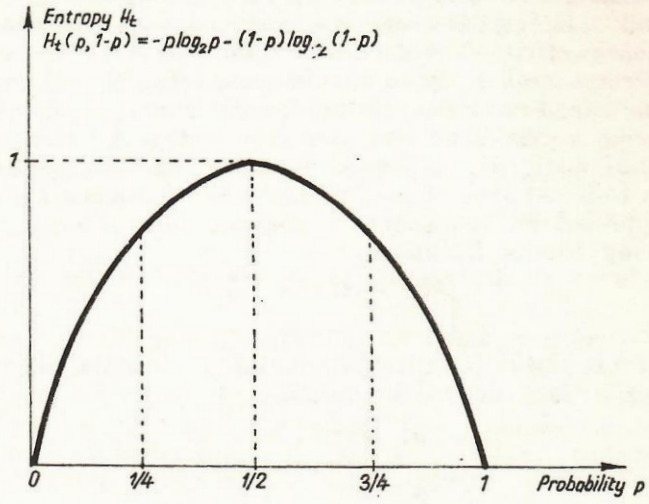



Fig. 11

It is obvious if one of the states accesible to the cybernetic and economic system that have been analysed has a probability equal with one, then the entropy of its space of growth has value zero, that is:

$$H_t(\mathcal{S}_{x_t}) = -\sum_{i=1}^{q_t} p_t^i \log p_t^i = 0.$$

The entropy of the probabilistic space of growth for a cybernetic and economic system has the *maximum value* when all probabilities $p_t^i$ are equal:

$$p_t^i = \frac{1}{q_t}; \quad i = 1, 2, ..., q_t.$$

Attaining the maximum entropic value of the probabilistic space of growth for the cybernetic and economic system illustrates its strong dynamics. It is the role of the macroeconomic decision making body to establish, according with the general strategy of economic and social development of the country the trajectory along which the system would develop.

In order to provide an example, we should consider two cybernetic and economic systems that at moment $t$ are being in states $x_t$ and $y_t$. It should be assumed that each of the two systems could develop into one of the possible five states; for the first system all shifting probabilities are supposed to be equal, being of value 0.2; for the second system, the probability to shift into the first of the five possible states, it is assumed to be 0.6, and the probabilities

for the other states are 0.1. In this situation, the entropy of the probabilistic space of growth for the first system has the value 2.32193, while that of the second one is only 1.77095 showing a somewhat less flexibility within the evolutionary process.

The properties of entropy underlines that it can quantify the pattern of trajectories range along which the cybernetic and economic system is able to evolve within the process of economic growth Also, the concept of entropy could be used in order to represent under the form of equation the *gravitational phenomena* (figure 12) that occur within the space of national economy.

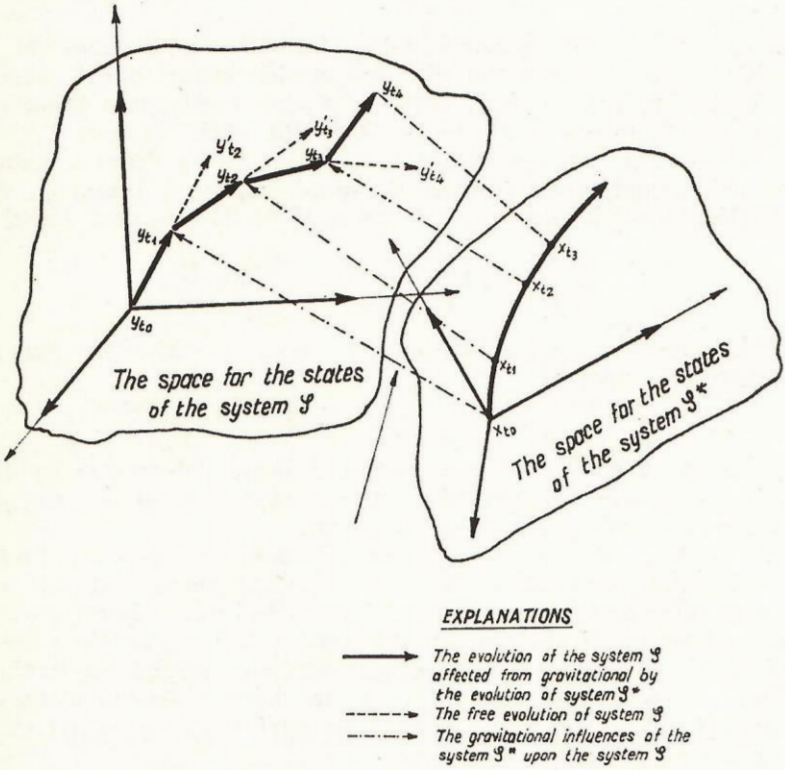



Fig. 12

Within the general cybernetic system of national economy some component parts i.e. branches, sectors or enterprises have a driving involving and speeding up role in the entire process of economic growth, of stirring the development of all systems that evolve in the economic space. Thus, industry by its high, dynamic growth accelerates the process of development in all branches of the economy. Also, within industry, some majour sectors, like machine building, chemistry, a.s.o. have a defining role in the overall development of the general cybernetic system of the national economy.

Knowing the effect if involvement, of forming actual gravitational economic fields round some cybernetic and economic systems or sub-systems that

represent propelling factors for the whole process of optimum economic growth plays a particular role in the activities of planning and economic guidance, of designing the trajectories for development.

So the conditions are created for the priority development of those branches that draw along and stirr the overall growth of the entire economic organism. In this respect, the industrialization policy applied consistently by our party, has had as effect ample and profound changes in the structure by branches, subbranches, groups of products and products, as well as in territorial profile, of the whole system of national economy in accelerating the increase of production forces, of the profound changes in the socialist relations in our country.

In order to present under equation form the phenomena of drawing along, that is the gravitational phenomena that occur in the space of the national economy, it should be considered the cybernetic and economic system $\mathcal{S}^*$ whose growth draws along the development of the system $\mathcal{S}$.

It is assumed that system $\mathcal{S}^*$ that propagates a drawing along force, is at moment $t$ and under state $x_t$ its space of growth being: $\{t,\ x_t,\ \mathcal{S}_{x_t}\}$. The probabilities of the states are shown in the following probabilistic chart:

$$\mathcal{S}_{x_t} = \begin{pmatrix} x_{t+1}^1, & x_{t+1}^2, & \ldots, & x_{t+1}^{n_t} \\ p_t^1, & p_t^2, & \ldots, & p_t^{n_t} \end{pmatrix}$$

With regard to the second system, that is $\mathcal{S}$, the states they may develop are the following ones:

$$y_{t+1}^1,\ y_{t+2}^2,\ \ldots,\ y_{t+1}^l$$

The growing process of the system $\mathcal{S}$ being determined by the way of evolution of system $\mathcal{S}^*$, the probabilities of its states of growth are conditioned by the way in which system $\mathcal{S}^*$ develops.

In order to provide an example, one can assumme the case of a machine building plant, its divisions being viewed as cybernetic-economic systems. It should be assummed that section $\mathcal{S}^*$ manufactures subcomponents for a product, and section $\mathcal{S}$ their mounting. With other words, the probabilities of the states the cybernetic and economic system corresponding to the mounting system will be at the moment $t + 1$, are directly dependent on the way the section of subcomponents will develop, that is the basic system.

Further on, we shall note with:

$$\pi_t^{ik} = p_t(y_{t+1}^i \mid x_{t+1}^k)$$

the probability that the system $\mathcal{S}$ is at moment $t + 1$ under the state $y_{t+1}^i$. With other words, $\pi_t^{ik}$ represents the probability through which system $\mathcal{S}$ will be at the moment $t + 1$ under state $y_{t+1}^i$, conditioned by the fact that the defining system $\mathcal{S}^*$ will be under the state $x_{t+1}^k$.

If, for instance the basic system is at moment $t + 1$ under state $x_{t+1}^1$, then the probabilities of the system states are given by the following probabilistic chart:

$$\begin{pmatrix} y_{t+1}^1, & y_{t+1}^2, & \ldots, & y_{t+1}^{l_t} \\ \pi_t^{11}, & \pi_t^{21}, & \ldots, & \pi_t^{l_1} \end{pmatrix}.$$

The entropy of the probabilistic space of growth for the cybernetic-economic system $\mathcal{S}$ if the main system $\mathcal{S}^*$ is at moment $t+1$ in $x_{t+1}^i$ under the state, is the following one:

$$H_t^1(\mathcal{S}_{y_t}) = -\sum_{i=1}^{l_t} \pi_t^{i_1} \log \pi_t^{i_1}.$$

Since the main system $\mathcal{S}^*$ that draws along the growth of the system $\mathcal{S}$, too may be at the moment $t+1$ with the probavility $p_t^i$ in anyone of its accessible states $x_{t+1}^1$, it is imposed that the overall probabilities should be taken into consideration.

In order to quantify the influence of the growth of the main system $\mathcal{S}^*$ on the development of system $\mathcal{S}$, it may be introduced the concept of *entropy of the growth probabilistic space for system $\mathcal{S}$ involved by the main system $\mathcal{S}^*$*, designated in the following way (figures 13 and 14):

$$H_t(\mathcal{S}_{y_t} | \mathcal{S}_{x_t^*}) = \sum_{j=1}^{n_t} p_t^i H_t^j (\mathcal{S}_{y_t}) = \sum_{j=1}^{n_t} \sum_{i=1}^{l_t} \pi_t^{ij} p_t^i \log \pi_t^{ij}.$$

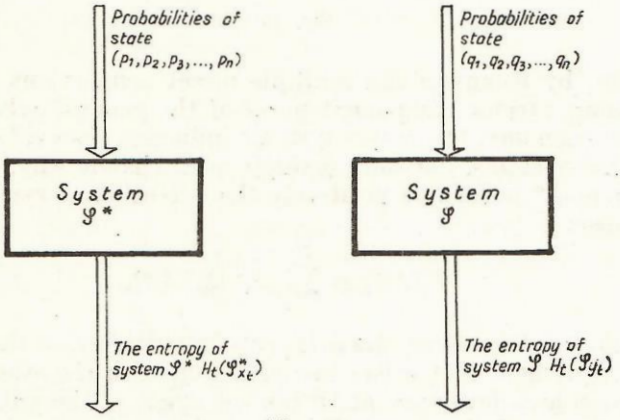



Fig. 13

Between the entropy of the growth probabilistic space belonging to an economic and cybernetic system and that which shows the phenomenon of involvement there is the following mathematic relationship:

$$H_t(\mathcal{S}_{y_t} | \mathcal{S}_{x_t^*}) \leqslant H_t(\mathcal{S}_{y_t}).$$

The equality between the both mathematical expressions that have been presented above occurs only if the increase of the cybernetic and economic system $\mathcal{S}^*$ does not affect the evolution of the system $\mathcal{S}$.

We shall assume that the difference:

$$\Delta_t(\mathcal{S}_{y_t} | \mathcal{S}_{x_t^*}) = H_t(\mathcal{S}_{y_t}) - H_t(\mathcal{S}_{y_t} | \mathcal{S}_{x_t^*})$$

quantifies the *strength of the involving effect* for the growth of system $\mathcal{S}$ caused by the development of the main system $\mathcal{S}^*$.

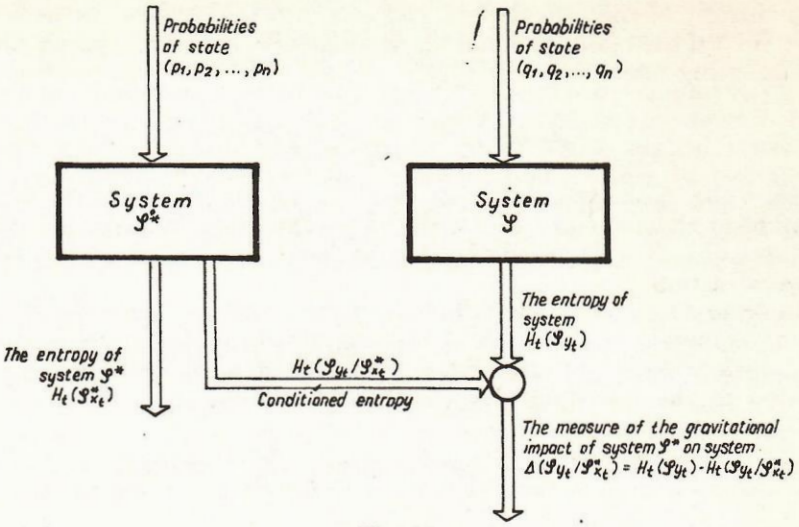



Fig. 14

Generally, by means of the multiple direct connections and feedbacks that arise among various component parts of the general cybernetic system of the national economy, the major systems influence the evolution of a large number of other systems, not only a single one. That is why, assuming that the main system $\mathcal{S}^*$ influences positively the evolution of systems $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$ the indicator:

$$F_t(\mathcal{S}_{x_t^*}) = \sum_{k=1}^{N} \Delta_t(\mathcal{S}^k_{y_t} / \mathcal{S}_{x_t^*}),$$

will quantify *the involving force, that is the gravitational force* of the main system.

A special problem that arises in connection with the study on the properties of motion and development within the space of the national economy regards the entropy of the set of trajectaries for their growth.

Supposing the cybernetic and economic system $\mathcal{S}$ at state $x_t$, its probabilistic space of growth at moment $t$ will be $\{t, x_t, \mathcal{S}_{y_t}\}$ whose entropy is $H_t(\mathcal{S}_{x_t})$. At moment $t+2$ the system could be under the states $x^1_{t+2}, x^2_{t+2}, \ldots, x^{q_2}_{t+2}$ whose probabilities of implementation are given in the following probabilistic scheme:

$$\begin{pmatrix} x^1_{t+2}, & x^2_{t+2}, & \ldots, & x^{q_2}_{t+2} \\ p^1_{1,t+1}, & p^1_{2,t+1}, & \ldots, & p^1_{q_2,t+1} \end{pmatrix}.$$

The probabilities shown above depend on the state attained by the system at $t+1$.

Thus, the necessity is pointed out to take into consideration probabilities of the form:

$$P_{t+1}(x_{t+2} = x^i_{t+2} / x_{t+1} = x^j_{t+1})$$

that quantify the probability that the system should be at $t+2$ under state $x^i_{t+2}$ if the moment $t+1$ is under state $x^j_{t+1}$.

*The entropy of the probabilistic space of growth of the cybernetic and economic system at moment* $t + 2$ *conditioned by its state at moment* $t + 1$, will be defined in the following way:

$$H_{t+1}(\mathcal{S}_{x_{t+2}} | \mathcal{S}_{x_{t+1}}) = -\sum_{k=1}^{q_2} p_{t+1}^k H_{t+1}^k(\mathcal{S}_{x_{t+2}}).$$

Above, $H_{t+1}^k(\mathcal{S}_{x_{t+2}})$ stood the entropy of the probabilistic space of growth of the cybernetic and economic system at moment $t + 2$ if at moment $t + 1$, it is under state $x_{t+1}^k$, hence:

$$H_{t+1}^k(\mathcal{S}_{x_{t+2}}) = -\sum_{j=1}^{q_2} \sigma_{t+1}^{jk} \log \sigma_{t+1}^{jk}.$$

With $\sigma_{t+1}^{jk}$ we defined the following conditioned probability:

$$\sigma_{t+1}^{jk} = p(x_{t+2} = x_{t+2}^j | x_{t+1} = x_{t+1}^k).$$

There follows that the entropy of the probabilistic space of growth for the cybernetic and economic system at moment $t + 2$ conditioned by its state at moment $t + 1$, could be also written:

$$H_{t+1}(\mathcal{S}_{x_{t+2}} | \mathcal{S}_{x_{t+1}}) = \sum_{k=1}^{q} \sum_{j=1}^{q_2} \sigma_{t+1}^{jk} p_{t+1}^k \log \sigma_{t+1}^{jk}.$$

Going on with the study on the properties inherent in the motion and development laws of the cybernetic and economic system, there could be similarly introduced the concept of entropy of the probabilistic space of growth of the system at moment $t$, conditioned by its evolution at moments $t - 1$, $t - 2$, ..., $t_0$. where $t_0$ is the original moment:

$$H_t(\mathcal{S}_{x_t} | \mathcal{S}_{x_{t-1}}, \mathcal{S}_{x_{t-2}}, \ldots, \mathcal{S}_{x_{t_0}}).$$

The identification of the probabilistic space of growth of the cybernetic and economic systems as well as its entropy is peculiarly useful for the actual forecasting, planning and directing the system within the space of the national economy. The analysis of the evolutionary trajectories of the cybernetic and economic systems on the basis of the philosophy of the stochastic processes allows to make a broad use of the methods and techniques proper to the economic cybernetics as well as those for probabilities calculus and stochastic brocesses. Thus the premises for a better mirroring within the cybernetic and economic models the complex realities from the economic space.

The economic quantum, by the methods and techniques it works out contributes to te extension of the means of searching the phenomena and the processes from the space of the national economy, *to the knowledge of the quantitative and qualitative dynamic and optimized balance* the entropic and gravitational properties of the cybernetic and economic systems. In this way, economic quantum alongside economic logistics, as well as the other cybernetic and economic fields contribute to steadily improving the methods for designing the trajectories of optimum, well balanced and proportional economic growth.

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