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Economic-Mathematical Model of Determining the Optimal Development Plan for an Industrial Sector

It is presented a model of optimizing the development plan for an industrial sector by considering as a basic element the project variants concerning the new objectives as well as those ones referring to development, modernization and reorientation of old units.

The worked out model is a dynamic linear model with discrete variables with more objective functions including bivalent variables. In the final part of the paper, an algorithm for solving the model is given.

The optimization of investments is among the prioritary domains where the use of modern modelling and optimization methods and techniques is required. The approach to the investment problems by means of the methods of economic-mathematical modelling aims at the determination of the efficient solutions from the social, economic, technical and financial points of view so that a maximal social effect be obtained in the frame of the existing concrete conditions (internal demand, export charges, the volume and structure of the available resources and so on).

We have to outline the fact that the decisions in the domain of investment are extremely complex from the point of view of economic-mathematical modelling, since they refer to important human, material and financial resources and, at the same time, they have the properties of uniqueness and irreversibility. The complex character of these decisions is augumented due to the great number of connexions and interferences formed between different branches and sectors of the national economy, since every investment decision in one sector engenders multiple influences in many others.

The principles at the basis of the model that we present are the following:

a) the development programme of the sector refers to the time horizon $[0, T]$.

b) the charges concerning the production that the sector has to provide annually of every product are given exogenously, on the basis of prognoses concerning the consumer's demand and the exportation possibilities. These are minimal charges.

c) for every year, the sites where the new industrial units can be placed are given, exogenously. These are chosen by specialists, on the basis of economic criteria: closeness to the sources of raw materials, fuel and energy, to the centers of sale of finite products, the rational use of labour force resources, the involvement of the natural resources of all zones in the economic flow and so on.

At the same time, in the choice of the sites for the new industrial units one has in view important social-political criteria: the development of the backward zones of the country, employment of the labour force in the less developed zones, protection of the environment and polution control; the rational use of unreproducible resources (land, water, and so on).

From the aspects mentioned above and from the fact that the location decisions influence in the long run many domains of the economy and society, we conclude that these decisions have to play an important part in the model.

d) for every possible point of location one works out a number of variants of project which differ one from another in technical, economic, functional, construction characteristics, in the technological procedures adopted, in the technological equipment that they are supposed to use, and so on.

Every project will be characterized by the volume and the structure of the attainable production, the specific consumption of labour force, of raw materials, fuel, energy, by the consumption of water, by the land area occupied by the unit, the pollution it can generate and so on.

Obviously, the variants of projects will differ one from another as concerns also the specific investment (direct, related and collateral), the volume of equipment, machines, necessary licenses from import, the cost at which the production will be realized, the benefit and profitableness of the unit, the attainable level of the productivity of the labour force and so on.

e) for every stage of the considered time horizon one assigns the production units that can be developed, modernized or re-orientated. For these units variants of projects will be worked out concerning the way of development, modernization or reorientation. The variants will differ one from another in the technical, economic, technologic characteristics mentioned at point d).

f) The model is intended to answer the following problems:

— the sites where the new units will be located and the projects that will be adopted;

— the way of development, modernization or reorientation of the existing capacities.

The choice of the development variant of the industrial sector has to be done in such a way, that a maximal production be realized with a minimal consumption of labour, natural resources, raw materials, fuel and energy.

The solution that will be adopted has also to ensure as high as possible productivity, benefit and profitableness, and costs of production as low as possible.

At the same time, the development programme has to be realized with as low as possible investment expenditures, (including those concerning import of equipment, technologies, licenses).

Since, in general, the costs implied by the realization of development, modernization or reorientation are lower, due especially to lower collateral investments (water supply, sewerage, gas, air, steam, roads, garage lines and so on), the model has to put into evidence the most rational way of combining the projects for the building of new units with those of development, modernization and reorientation of the existing ones.

g) Taking into account the costs implied by the realization of every variant of project, the available designing capacity, as well as the augmentation of the difficulties in solving the model caused by the increase of the number of variants, a reasonable number of projects has to be considered in the model.

For every unit, one excludes from the beginning the "inefficient" variants, that is those variants for which there exists at least one other variant that allows to obtain the same increase of products, with lower expenditures.

h) The present model has to be included in an integrated system of models of territorial planning, planning at the level of a branch and inter-branch planning [1]. In this way, one will be able to determine more precisely a number of elements which, in a first phase, are included exogenously in the model.

Thus, for example, on the basis of a territorial planning model, one can determine the optimal way of distribution of the labour resources between different economic-social sectors, the volume of labour force which can be attracted in the considered sector, the optimal way of using the natural resources, and so on.

The way of stating the problem (see, especially, points c) and f)) shows that we are dealing with a model with several objective — functions (multicriterial model).

In the B section of the paper, some considerations concerning the generation of efficient (Pareto optimal) points and the way of choosing the "optimal" solution will be presented.

The present model is a multicriterial linear dynamic model with discrete variables, which contains also bivalent variables.

A. THE ECONOMIC-MATHEMATICAL MODEL

In order to describe the model, the following notations will be used:

J — the set of indices of the types of products that the sector has to realize;

P_t^j — the quantity of the j-th product that has to be provided in year t for the internal consumers (outside the sector) ($j \in J$; $t = 0 \div T$)

E_t^j — the minimal quantity of the j-th product, that has to be ensured in year t for export ($j \in J$; $t = 0 \div T$);

Q_t^0 — the set of indices that denote the industrial units existing in the sector at moment t;

Q_t^1 — the set of indices that denote the points where new capacities can be located at moment t or where unachieved units exist (at moment t)

Q_t^2 — the set of indices which denote the production units which can be or are in course of development, modernization or reorientation at moment t;

$H_t^{1,i}$ — the set of indices which denote the projects which can be realized in year t at the location point i ($i \in Q_t^1$);

$H_t^{2,i}$ — the set of indices which denote the projects of development, modernization or reorientation that can be realized in year t for the unit i, ($i \in Q_t^2$);

t_1 — the time necessary for the realization of a new unit;

t_2 — the time necessary for the realization of a development, modernization or reorientation project.

Obviously, one can also consider the case when t_1 and t_2 depend on the type of the adopted project, but this does not change the structure of the model.

One considers also the following sets of indices:

$$\hat{Q}_t = Q_t^0 \cup \left(\bigcup_{\tau=t_1}^{0} Q_{t-\tau}^1 \right)$$

$$H_t = \left(\bigcup_{i \in Q_t^1} H_t^{1,i} \right) \cup \left(\bigcup_{i \in Q_t^2} H_t^{2,i} \right)$$

$$t_3 = max\ (t_1,\ t_2)$$

$A_{t,\tau}^{i,h}$ — the production capacity in year t of the new unit i, whose construction begins in year τ according to the variant of project h ($i \in Q_\tau^1$, $h \in H_\tau^{i,1}$; $\tau \leqslant t \leqslant \tau + t_1$);

$B_{t,\tau}^{i,h}$ — the production capacity in year t of the production unit i whose development, modernization or reorientation begins in year τ according to the variant of project h ($i \in Q_\tau^2$, $h \in H_\tau^{i,2}$, $\tau \leqslant t \leqslant \tau + t_2$);

$I_{t,\tau}^{i,h}$; $V_{t,\tau}^{i,h}$ — the volume of investment and valutary funds, respectively, necessary in year t for the variant of project h, whose realization begins at the site i in year τ ($i \in Q_\tau^1 \cup Q_\tau^2$; $h \in H_\tau$; $\tau \leqslant t \leqslant \tau + t_3$);

R — the set of indices which denote the types of resources (labour force, raw materials, semi-products, natural resources, fuel, energy, water and so on) which are consumed in the sector for the realization of the production, as well as for the realization of investments.

$c_\tau^{i,j,r,h}$ — specific consumption of resource r for a unit of j-th product at the production site i, where the realization of project h begins in year τ ($i \in Q_\tau^1 \cup Q_\tau^2$,, $h \in H_\tau$, $j \in$ J, $r \in R$, $\tau = -t_3 \div T$);

M — the set of indices which denote the efficiency—profitableness indicators by means of which the activity of the sector is estimated;

$f_\tau^{i,j,m,h}$ — the value of the efficiency — profitableness indicator m per a unit of the j-th product at the production site i where the realization of project h begins in year τ ($i \in Q_\tau^1 \cup Q_\tau^2$; $h \in H_\tau$; $j \in J$, $m \in M$, $\tau = -t_3 \div T$);

$d_{l,\tau}^{i,h,r}$ — consumption of resource r in year t for the realization of project h begun at point i in year τ ($i \in Q_\tau^1 \cup Q_\tau^2$, $h \in H_\tau$, $r \in R$, $\tau \leqslant t \leqslant \tau + t_3$).

The unknowns of the model are the following:

a) The state variables of the system:

X_t^i — the production capacity of unit i in year t; ($i \in \hat{Q}_t$, $t = 0 \div T$)

$c_t^{i,j,r}$ — specific consumption of the resource r for the realization of a unit of product j at point i, in year t ($i \in \hat{Q}_t$, $j \in J$, $r \in R$, $t = 0 \div T$);

$f_t^{i,j,m}$ — the value of the efficiency — profitableness index m per unit of product j, realized at point i, in year t ($i \in \hat{Q}_t$, $m \in M$, $j \in J$, $t = 0 \div T$).

b) The decision variables of the system are the following:

$Y_t^{i,j}$ — the volume of production of product j, that will be obtained at point i, in year t ($i \in \hat{Q}_t$, $j \in J$, $t = 0 \div T$).

We introduce also the following bivalent decision variables:

$$\alpha_\tau^{i,h} = \begin{cases} 1, & \text{if in year } \tau \text{ begins the realizations of project } h \text{ at point } i; \\ 0, & \text{if in year } \tau \text{ the project } h \text{ for point } i \text{ is not adopted.} \end{cases}$$

$$(i \in Q_\tau^1 \cup Q_\tau^2,\ h \in H_\tau).$$

In order to make easier the description of the mathematical model, we make the following working hypotheses:

i) The set of projects $H_t^{2,i}$ contains the "identical" project, which corresponds to the situation when no development, modernization or reorientation activities begin at unit i in year t;

ii) During the first θ_1 years of their functioning, the new units are not submitted to development, modernization or reorientation;

iii) for every point of production, projects of development, modernization or reorientation can be considered only after a period of θ_2 years since the completion of the preceeding project.

Assuming that the index of the "identical" project of the set $H_t^{i,2}$ is $h = 1$, we have, according to hypothesis i)

$$B_{i,t}^{i,1} = X_{t-1}^i, \quad i \in Q_t^2, \quad t = 0 \div T. \tag{1}$$

$$X_t^i = X_{t-1}^i, \quad X_0^i - \text{given} \quad i \in Q_t^0 \setminus \left\{ \bigcup_{\tau=-t_2}^{0} Q_{t-\tau}^2 \right\}. \tag{2}$$

The above equations refer to the production capacities of the units which are not submitted to development, modernization or reorientation

$$X_t^i = \sum_{\tau=t-t_1}^{t} \sum_{h \in H^{1,i}} \alpha_\tau^{i,h} \cdot A_{t,\tau}^{i,h}; \quad i \in \bigcup_{\tau=t-t_1}^{t} Q_\tau^1 \tag{3}$$

where

$$\alpha_\tau^{i,h} \in \{0; 1\}; \sum_{\tau=-t_1}^{T} \sum_{h \in H^{1,i}} \alpha_\tau^{i,h} \leqslant 1 \tag{4}$$

If $$i \in Q_\tau^1 \text{ and } \sum_{h \in H_\tau^{1,i}} \alpha_\tau^{i,h} = 1 \tag{5}$$

then

$$i \in Q_{\tau+t_1+t}^0 \setminus Q_{\tau+t_1+t}^2; \; t = 1 \div \theta_1.$$

Relations (3)—(5) refer to the new production units; equations (3) refer to the dynamics of their production capacities; relation (4) shows that at most one unit may be built at location point i, and relations (5) show that, according to hypothesis ii) ,the new units will not be submitted to development, modernization or reorientation during the first θ_1 years of their functioning.

$$X_t^i = \sum_{\tau=t-t_2}^{t} \sum_{h \in H_\tau^{2,i}} \alpha_\tau^{i,h} \cdot B_{t,\tau}^{i,h}; \qquad i \in \bigcup_{\tau=t-t_2}^{t} Q_\tau^2 \tag{6}$$

where

$$\alpha_\tau^{i,h} \in \{0; 1\}, \sum_{\tau=t-t_2}^{t} \sum_{h \in H_\tau^{2,i}} \alpha_\tau^{i,h} = 1; \quad i \in \bigcup_{\tau=t-t_2}^{t} Q_\tau^2; \tag{7}$$

if $i \in Q_\tau^2$ and $\alpha_\tau^{i,1} = 0$, then

$$i \in Q_{\tau+t_2+t}^0 \setminus Q_{\tau+t_2+t}^2; \quad t = 1 \div \theta_2. \tag{8}$$

Relations (6)—(8) refer to the production units proposed for development, modernization or reorientation. Equations (6) refer to the dynamics of the production capacities. Since, for every $i \in Q_\tau^2$, the "identical" project is included in $H_\tau^{2,i}$, the sum of the variables $\alpha_\tau^{i,h}$, has to be equal to 1 (equation 7).

Relations (8) show that for the units for which a project h of development, modernization or reorientation was accepted at moment τ (hence $\alpha_\tau^{i,1} = 0$) no other project will be considered during the first θ_2 years after its completion (according to hypothesis iii).

The dynamics of specific consumptions and of the efficiency — profitableness indicators are given by the following equations:

$$c_t^{i,j,r} = c_{t-1}^{i,j,r} \qquad (9)$$

$$i \in Q_t^0 / \left\{ \bigcup_{\tau=t-t_2}^{t} Q_\tau^2 \right\}$$

$$f_t^{i,j,r} = f_{t-1}^{i,j,r} \qquad (10)$$

$$c_t^{i,j,r} = \sum_{\tau=t-t_1}^{t} \sum_{h \in H_\tau^{1,i}} \alpha_\tau^{i,h} \cdot c_\tau^{i,j,r,h} \qquad (11)$$

$$i \in \bigcup_{\tau=t-t_1}^{t} Q_\tau^1$$

$$f_t^{i,j,m} = \sum_{\tau=t-t_1}^{t} \sum_{h \in H^{1,i}} \alpha_\tau^{i,h} \cdot f_\tau^{i,j,m,h} \qquad (12)$$

$$c_t^{i,j,r} = \sum_{\tau=t-t_2}^{t} \sum_{h \in H_\tau^{2,i}} \alpha_\tau^{i,h} \cdot c_\tau^{i,j,r,h} \qquad (13)$$

$$i \in \bigcup_{\tau=t-t_2}^{t} Q_\tau^2$$

$$f_t^{i,j,m} = \sum_{\tau=t-t_2}^{t} \sum_{h \in H_\tau^{2,i}} \alpha_\tau^{i,h} \cdot f_\tau^{i,j,m,h}. \qquad (14)$$

Equations (9) and (10) refer to the production units which are not submitted to development, modernization or reorientation, equations (11)—(12) refer to the new units, and equations (13)—(14) to those submitted to development, modernization or reorientation, respectively.

For the "identical" project, the following relations are satisfied

$$c_t^{i,j,r,1} = c_{t-1}^{i,j,r}, \quad r \in R \qquad (15)$$

$$f_t^{i,j,m,1} = f_{t-1}^{i,j,m}, \quad m \in M. \qquad (16)$$

II. THE CONSTRAINTS OF THE MODEL

As concerns the constraints of the model, we make the following assumptions, which complete assumptions a) — h) given at point I):

j) For a subset R_1 of types of resources used in the production process, one gives upper limits for the annual consumption.

For the remainder of the types of resources used in the production process, their consumption is required to be "as low as possible", hence its level will result from the optimization process.

k) For a subset R_2 of types of resources used for investment, one gives upper limits for the annual consumption.

The consumption of the remainder of the types of resources used for investment is required to be "as low as possible".

l) for the indicators of efficiency — profitableness indexed by indices $m \in M_1 \subseteq M$, minimal levels to be attained are given. As concerns the remainder of the indicators of efficiency — profitableness, they have to be "as good as possible".

m) the resources indexed by indices belonging to the set $L \subseteq R \cap J$ have to be provided inside the sector, e.g. by cooperation in production (semiproducts, a.s.o.).

In order to write the constraints of the model we introduce the following notations:

$\hat{C}_t^{1,r}$ — the maximum level that consumption of resource r for the production process of the sector may reach in year t; $(r \in R_1;\ t = 0 \div T)$;

$\hat{C}_t^{2,r}$ — the maximum level that consumption of resource r for investment in the sector may reach in year t $(r \in R_2;\ t = 0 \div T)$;

$\hat{I}_t$ — the maximum volume of investment funds that can be allotted in year t for the development of the sector;

$\hat{V}_t$ — the maximum volume of valutary fund that can be allotted in year t for investment necessary to the development of the sector;

$\hat{F}_t^{i,m}$ — lower limit for the level of the efficiency — profitableness indicator m at unit i in year t $(m \in M_1;\ i \in \hat{Q}_t;\ t = 0 \div T)$;

$Q_{t,j}$ — the set of indices that denote the production units which can realize the product j in year t $(j \in J;\ t = 0 \div T)$;

J_t^i — the set of indices which denote the products j that can be realized at production unit i, in year t $(i \in \hat{Q}_t;\ t = 0 \div T)$;

S_t^j — the set of indices of the production units which use resource j produced inside the sector $(j \in L \subseteq R \cap J,\ t = 0 \div T)$.

Further, we introduce the following decision parameters:

$Z_t^{i,j}$ — the volume of the resource j (semiproducts, a.s.o.) realized in the sector for production unit i, in year t $(i \in \hat{Q}_t;\ j \in L;\ t = 0 \div T)$.

With the above notations, the constraints of the model are the following:

$$\sum_{i \in Q_{t,j}} Y_t^{i,j} \geqslant P_t^j + E_t^j + \sum_{i \in S_t^j} Z_t^{i,j};\quad j \in J. \tag{17}$$

Relations (17) express the fact that the production of product j satisfies the necessities of the internal consumers, the minimal export planned, as well as the internal necessities of the sector.

$$\sum_{j \in J_t^i} c_t^{i,j,1} \cdot Y_t^{i,j} \leqslant X_t^i; \quad i \in \hat{Q}_t. \tag{18}$$

Relations (18) refer to the balance of the production capacity of every production unit. We mention that the production capacities are also considered as resources and we index them by $r = 1$.

$$\sum_{i \in \hat{Q}_t} \sum_{j \in J_t^i} c_t^{i,j,r} \cdot Y_t^{i,j} \leqslant \hat{C}_t^{1,r}; \quad r \in R_1 \tag{19}$$

$$\sum_{j \in J_t^i} f_t^{i,j,m} \cdot Y_t^{i,j} \geqslant \hat{F}_t^{i,m}; \quad i \in \hat{Q}_t, \ m \in M_1. \tag{20}$$

Constraints (19) express the requirement that consumption of resource r for the production process of the sector in year t should not exceed the given upper limit; constraints (20) express the requirement that every production unit should reach the imposed lower level for the efficiency — profitableness indicators.

The constraints concerning the balance of semi-products and other resources produced inside the sector are the following:

$$\sum_{j \in J_t^i} c_t^{i,j,r} \cdot Y_t^{i,j} \leqslant Z_t^{i,r}; \quad r \in L; \quad i \in S_t^r. \tag{21}$$

The following constraints refer to the activity of investment in the sector:

$$\sum_{\tau=t-t_3}^{t} \sum_{i \in Q_\tau^1 \cup Q_\tau^2} \sum_{h \in H_\tau^{1,i} \cup H^{2,i}} \alpha_\tau^{i,h} \cdot d_{t,\tau}^{i,h,r} \leqslant \hat{C}_t^{2,r}; \quad r \in R_2 \tag{22}$$

$$\sum_{\tau=t-t_3}^{t} \sum_{i \in Q_\tau^1 \cup Q_\tau^2} \sum_{h \in H_\tau^{1,i} \cup H_\tau^{2,i}} \alpha_\tau^{i,h} \cdot I_{t,\tau}^{i,h} \leqslant \hat{I}_t \tag{23}$$

$$\sum_{\tau=t-t_3}^{t} \sum_{i \in Q_\tau^2 \cup Q^2} \sum_{h \in H^{1,i} \cup H_\tau^{2,i}} \alpha_\tau^{i,h} \cdot V_{t,\tau}^{i,h} \leqslant \hat{V}_t. \tag{24}$$

Constraints (22) (23) and (24) express the requirements that the material resources which are allotted annually for the realization of investment, the investment fund and the valutary fund which are allotted annually for the development of the sector respectively, should not be exceeded, by the corresponding expenditures.

The following constraint refers to the final conditions concerning the production capacity of the sector at the end of the considered planning horizon:

$$\sum_{i \in \hat{Q}_T} X_T^i \geqslant \beta_T. \tag{25}$$

We noted by β_T the lower limit imposed upon the production capacity of the sector in year T.

III. THE OPTIMALITY CRITERIA OF THE MODEL

The optimality criteria will refer to the following aspects:

a) maximization of the annual production of the sector. Assuming that the quantities P_t^j planned for the internal consumers satisfy entirely their demand, we shall aim at the maximization of the valutary contribution of the sector, i.e. of the production for export.

Correspondingly, we have the following alternative optimality criteria:

$$max\ \gamma^{1,1} = max \sum_{t=0}^{T} \sum_{i \in \hat{Q}_t} \sum_{j \in J_t^i} [Y_t^{i,j} - Z_t^{i,j}] \tag{26}$$

$$max\ \gamma^{1,2} = max \sum_{t=0}^{T} \sum_{i \in \hat{Q}_t} \sum_{j \in J_t^i} \pi_t^j [Y_t^{i,j} - Z_t^{i,j} - P_t^j]. \tag{26'}$$

In relations (26'), we noted by π_t^j the foreign trade exchange rate of product j in year t.

b) minimization of total consumption of every resource $r \in \bar{R}$; we noted by $\bar{R} \subseteq R$ the set of indices which refer to resources the sparing of which has the precedence.

$$min\ \gamma^{2,r} = min \sum_{t=0}^{T} \sum_{i \in \hat{Q}_t} \sum_{j \in J_t^i} c_t^{i,j,r}\ Y_t^{i,j}; \quad r \in \bar{R}. \tag{27}$$

c) maximization, at the level of the sector, of the magnitude of every efficiency — profitableness indicator $m \in \bar{M}$.

We noted by $\bar{M}$ the subset of M containing the indicators that we have in view for optimization.

$$max\ \gamma^{3,m} = max \sum_{t=0}^{T} \sum_{i \in \hat{Q}_t} \sum_{j \in J_t^i} f_t^{i,j,m} \cdot Y_t^{i,j}; \quad m \in \bar{M}. \tag{28}$$

d) minimization of the total fund of investment and of the valutary fund that will be used for the development of the sector during the whole planning horizon

$$min\ \gamma^4 = min \sum_{t=0}^{T} \sum_{\tau=t-t_3}^{t} \sum_{i \in Q_\tau^1 \cup Q^{2,i}} \sum_{h \in H_\tau^{1,i} \cup H_\tau^{2,i}} \alpha_\tau^{i,h} I_{t,\tau}^{i,h} \tag{29}$$

$$min\ \gamma^4 = min \sum_{t=0}^{T} \sum_{\tau=t-t_3}^{t} \sum_{i \in Q_\tau^1 \cup Q_\tau^2} \sum_{h \in H_\tau^{1,i} \cup H_\tau^{2,i}} \alpha_\tau^{i,h} V_{t,\tau}^{i,h}. \tag{30}$$

Therefore, the development of the sector should be realized in such a way that a number of $3 +$ card $\bar{R} +$ card $\bar{M}$ indicators be optimal.

B. SOME CONSIDERATIONS CONCERNING THE SOLUTION OF THE MODEL. COMPUTATIONAL ALGORITHM

We note by ξ_t **the vector of the state variables** of the system, the components of which are X_t^i, $C_t^{i,j,r}$ and $f_t^{i,j,m}$, and by w_t — **the vector of the decision** (control) variables with components $\alpha_t^{i,h}$, $Y_t^{i,j}$, $Z_t^{i,j}$.
The sequence

$$w = (w_{-t_3},\ w_{-t_3+1},\ w_{-t_3+2},\ \ldots,\ w_T)$$

is called **admissible control** if the following conditions are satisfied:

(i) components $\alpha_t^{i,h}$ satisfy constraints (4) and (7);

(ii) $\alpha_{-t}^{i,h} = \eta_{-t}^{i,h}$; $t = t_3 \div 1$
where $\eta_{-t}^{i,h}$ are given;

(iii) w, along with the corresponding states ξ obtained from relations (2), (3), (6), (9) — (14) satisfy constraints (17) — (25).

We note by Ω the **set of all admissible controls**; we shall assume this set nonvoid.

The admissible **decision $\bar{w}$ is called efficient** (Pareto optimal) if there exists no other decision $w \in \Omega$ such that:

$$\begin{aligned}
&\gamma^{1,k}(w, \xi) \geqslant \gamma^{1,k}(\bar{w}, \bar{\xi}); \quad && k = 1, 2 \\
&\gamma^{2,r}(w, \xi) \leqslant \gamma^{2,r}(\bar{w}, \bar{\xi}); \quad && r \in \bar{R} \\
&\gamma^{3,m}(w, \xi) \geqslant \gamma^{3,m}(\bar{w}, \bar{\xi}); \quad && m \in \bar{M} \\
&\gamma^{i}(w, \xi(\leqslant \gamma^{i}(\bar{w}, \bar{\xi}); \quad && i = 4, 5,
\end{aligned} \tag{31}$$

with strict inequality in at least one of the above relations.

In general, the set of the Pareto — optimal points is rich; this fact raises the problem of finding criteria for choosing an efficient decision which should correspond as well as possible to the given problem. Further, we shall consider as the **prioritary criterion** that concerning the maximization of production and of the export, respectively.

The admissible control w is called $\gamma^{k,l}$ — **optimal,** if it gives the best performance of the indicator $\gamma^{k,l}$ with respect to the set of all admissible controls:

$$\begin{aligned}
\tilde{\gamma}^{1,k} &= \gamma^{1,k}(\tilde{w}^{1,k}, \tilde{\xi}^{1,k}) = \sup_{w \in \Omega} \gamma^{1,k}(w, \xi); \quad k = 1, 2 \\
\tilde{\gamma}^{2,r} &= \gamma^{2,r}(\tilde{w}^{2,r}, \tilde{\xi}^{2,r}) = \inf_{w \in \Omega} \gamma^{2,r}(w, \xi); \quad r \in \bar{R} \\
\tilde{\gamma}^{3,m} &= \gamma^{3,m}(\tilde{w}^{3,m}, \tilde{\xi}^{3,m}) = \sup_{w \in \Omega} \gamma^{3,m}(w, \xi); \quad m \in \bar{M} \\
\tilde{\gamma}^{i} &= \gamma^{i}(\tilde{w}^{i}, \tilde{\xi}^{i}) = \inf_{w \in \Omega} \gamma^{i}(w, \xi); \quad i = 4, 5.
\end{aligned} \tag{32}$$

In general, it is not possible to reach the performance

$$\tilde{\Gamma} = (\tilde{\gamma}^1, \tilde{\gamma}^{2,r}, \tilde{\gamma}^{3,m}, \tilde{\gamma}^4, \tilde{\gamma}^5)_{(r \in \bar{R},\ m \in \bar{M})}$$

by means of admissible controls $w \in \Omega$.

We note by Ω^δ the set of all decisions which satisfy the following constraints:

$$\Omega^\delta: \begin{cases} w \in \Omega \\ \dfrac{\gamma^{2,r}(w, \xi)}{\gamma^1(w, \xi)} \leqslant \dfrac{\tilde{\gamma}^{2,r}}{\gamma^1(\tilde{w}^{2,r}, \tilde{\xi}^{2,r})} + \delta_2^r; \quad r \in \bar{R} \\ \dfrac{\gamma^{3,m}(w, \xi)}{\gamma^1(w, \xi)} \geqslant \dfrac{\tilde{\gamma}^{3,m}}{\gamma^1(\tilde{w}^{3,m}, \tilde{\xi}^{3,m})} - \delta_m^3; \quad m \in \bar{M} \\ \dfrac{\gamma^i(w, \xi)}{\gamma^1(w, \xi)} \leqslant \dfrac{\tilde{\gamma}^i}{\gamma^1(\tilde{w}^i, \tilde{\xi}^i)} + \delta_i; \quad i = 4, 5 \end{cases} \tag{33}$$

where γ^1 is $\gamma^{1,1}$ or $\gamma^{1,2}$, and δ is the vector with components: δ_2^r, δ_3^m, δ_4, δ_5, where $r \in \bar{R}$ and $m \in \bar{M}$.

The algorithm suggested for the determination of an efficient solution "as good as possible" consists in the following:

0. We take $\delta = 0$ and pass to step 1.

1. We check whether $\Omega^\delta \neq \Phi$ or not. If $\Omega^\delta \neq \Phi$, we pass to 2, if not, we pass to step 3.

2. We determine a control $\hat{w}^\delta$ such that

$$\gamma^1(\hat{w}^\delta, \hat{\xi}^\delta) = \sup_{w \in \Omega^\delta} \gamma^1(w, \xi). \tag{34}$$

Let us consider:

$$\Delta^\delta = \gamma^1(\tilde{w}^1, \tilde{\xi}^1) - \gamma^1(\hat{w}^\delta, \hat{\xi}^\delta).$$

We note by ε the acceptable difference between the maximum volume of production $\tilde{\gamma}^1$, corresponding to the γ^1 optimal solution, and that corresponding to the decision $\hat{w}^\delta$.

If $\Delta^\delta \leqslant \varepsilon$, then decision $\hat{w}^\delta$ is satisfactory. If not, we pass to step 3.

3. We enlarge ("relax") the set Ω^δ assigning suitable values to the components of vector δ and we return to step 1.

One can show that the procedure converges.

The solution determined by the above algorithm gives the maximum level of production, whereas specific investment (including that from import), specific consumptions and the level of efficiency — profitableness indices are in a suitable neighbourhood of the values obtained by the γ — optimal solutions.

REMARKS

1. In the case when the prioritary criterion is other than that of maximization of the production, e.g. minimization of the total volume of investment, the algorithm has to be modified accordingly.

2. We can consider, in the model, the possibility of decisions of giving up some projects whose realization has begun in the period $[-t_3, -1]$ but which no longer correspond. In this case, we have also to consider the losses caused by giving up some of these projects.

3. We can also consider the case in which $\eta_{-t}^{t,h}$ are decision parameters, as well as the case in which the investment expeditures are summed over the period $[-t_3, \ T + t_3]$.

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MODELE ECONOMIQUE MATHEMATIQUE POUR DETERMINER LE PLAN OPTIMAL DE DEVELOPPEMENT D'UN SECTEUR INDUSTRIEL

On présente un modèle d'optimisation du plan de développement d'un secteur industriel prenant pour élément de base les variantes de projet concernant les nouveaux objectifs, ainsi que les variantes pour le développement, la modernisation ou le reprofilage des anciennes unités.

Le modèle élaboré est un modèle dynamique linéaire à variables discrètes ayant plusieurs fonctions objectif et contenant des variables bivalentes. La dernière partie de l'article présente un algorithme de résolution du modèle.

ЭКОНОМИКО-МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПРЕДЕЛЕНИЯ ОПТИМАЛЬНОГО ПЛАНА РАЗВИТИЯ ПРОМЫШЛЕННОГО СЕКТОРА

В работе приведена модель оптимизации плана развития отдельного промышленного сектора, учитывая в качестве основного элемента варианты проекта новых промышленных объектов, а также варианты развития, модернизации или перепрофилирования старых предприятий.

Разработанная модель является линейной динамической моделью с дискретными переменными и несколькими целевыми функциями, содержающими переменные с двумя возможными значениями. В последней части статьи приведён алгоритм разрешения модели.

15. XII. 1978