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Cybernetic-Economic Systems' Sensitivity

The aim of this paper is to point out some peculiarities of the cybernetic-economic systems' sensitivity, and mainly the different time constants which must be taken into account in such systems.

SOME BASIC DEFINITIONS

Sensitivity. The sensitivity measures the relative change of a variable as effect of variation of a specific parameter. For instance, in feedback theory the sensitivity measures the effectiveness of the feedback in reducing the effects of variation of a specific parameter. This parameter, denoted by n, may be in electronic systems a time constant, an amplifier gain, a supply voltage or any parameter affecting system performance. In engineering n can be a moment of inertia, a friction coefficient, a speed and so on. In economic systems n can be the average age of a population, saving, consumption and investment or any other economic parameter.

If we denote by A the dependent variable, the absolute sensitivity of A, by rapport to n is

$$Sa_n^A \triangleq dA/dn.$$

The relative sensitivity is defined by the relation

$$S_n^A \triangleq (dA/A)/(dn/n) \triangleq d\log A/d\log n.$$

System Transfer Function. In system analysis we are ordinarily interested in system dynamics. A system can be described in terms of the system transfer function or transmittance $T(s)$, which is the overall transfer function relating system output to system input; s is the complex variable from the Laplace transform. The concept of transmittance is related to linear systems.

The transmittance $T(s)$ is the ratio of $Y(s)$, the Laplace transform of the output $y(t)$ to $U(s)$, the Laplace transform of the input $u(t)$. From the engineering point of view, s is the complex frequency variable.

For a single-loop system (Fig. 1), the transmittance is

$$T(s) = A(s)/(1 + A(s)B(s)). \tag{1}$$

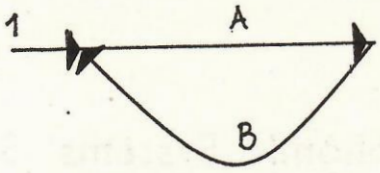

Fig. 1. Basic single-loop system

SENSITIVITY OF A SINGLE-LOOP SYSTEM

In a single-loop feedback system, we may calculate the sensitivity of T with respect to A and B by straightforward differentiation of the expression of T in Eq. (1).

It is easy to calculate:

$$Sa_A^T = (T/A)^2$$
$$Sa_B^T = -T^2$$
$$S_A^T = T/A$$
$$S_B^T = -BT.$$

MASON'S REDUCTION FORMULA

A cybernetic system can be represented by a graph, more precisely by an oriented graph. The transmittance of a graph is the sum of all different paths leading from a specified source to some chosen dependent node, where each path transmittance becomes weighted by factors arising from configuration of loop s. When there are no loops, the weighting factor for each path is unity.

Based upon this view of graph topology, Mason's reduction formula for graph transmittance takes the form of the summation

$$T_{i,j} = \Sigma P_k \Delta_k / \Delta$$

the summation being taken over all k different open paths from source i to dependent node j; Δ is the graph determinant. The path factor Δ_k is the determinant of the remaining graph after the k th path has been deleted from the graph.

TOPOLOGICAL INTERPRETATION OF SENSITIVITY

The evaluation of sensitivity of a given system is readily accomplished by carring out on Mason's formula for transmittance the steps of the formal derivative definition of sensitivity. We have

$$S_x^T = (x/T)(\partial M/\partial x)$$

$$M = \sum_k P_k \Delta_k / \Delta.$$

For convenience we introduce the symbol

$$\Sigma \triangleq \sum P_k \Delta_k.$$

Then

$$S_x^T = (x/\Sigma)(\partial \Sigma/\partial x) - (x/\Delta)(\partial \Delta/\partial x).$$

We introduce also the symbols

$$x(\partial \Delta/\partial x) \triangleq \Delta - \Delta_0$$

$$x(\partial \Sigma/\partial x) \triangleq \Sigma - \Sigma_0.$$

We arrive at the following result:

$$S_x^T = \Delta_0/\Delta - \Sigma_0/\Sigma.$$

LINEAR SYSTEMS' SENSITIVITY

Models in the main, and particularly economic models are of two types: structural and dynamic. Structural models reflect the internal elements of an economy at a given moment, and their relations. Dynamic models involve the body of parameters defining time-development of the system.

Let us consider a model of economic growth described by the equations

$$x_i(t) = \sum a_{ij} x_j(t - \theta_{ij}) \quad i, j = \overline{1, n}$$

With the aid of the Laplace transform we get

$$s\hat{x}_i(s) - x_i(0) = \sum a_{ij} x_j(s) \, exp \, (-\theta_{ij} s).$$

We define

$$a'_{ij} \triangleq a_{ij} \exp(-\theta_{ij} s)$$

so we can write

$$(s_{ij} - a'_{ij}) \, \hat{x}_j(s) = x_i(0)$$

or in matrix form

$$A\hat{X} = X_0,$$

A being the matrix with the general term

$$(s\delta_{ij} - a'_{ij}).$$

The evolution of the system is given by the relation

$$X(t) = \mathfrak{L}^{-1}(A^{-1}X_0).$$

As an exemplification, let us consider the one sector model described by the equation

$$x(t) = ax(t - \theta).$$

We get

$$\hat{x}(s) = \frac{x_0}{s - a\exp(-\theta s)}.$$

If it is possible to develop the function exp (.), we have

$$\hat{x} = C\,(s - a')^{-1},$$

$$C \triangleq x_0/(1 + a\theta)$$

$$a' \triangleq a/(1 + a\theta).$$

Using the inverse Laplace transformation, we have

$$x(t) = C\exp(a't).$$

If the delay θ is small, it is possible to approximate

$$a\theta \ll 1 \quad x(t) = C' \exp(at);$$

if the delay θ is important

$$a\theta \gg 1 \quad x(t) = C'' \exp(t/\theta).$$

The evolution has a strong dependence on θ: if θ is small, the evolution is determined by a; if the delay is important, we have also an exponential growth, but with another coefficient, the time constant being exactly θ. The conclusion is a simple one: the longer the delay, the longer the moment the effect is visible.

It is possible to define the sensitivity of x by regard to θ, as the partial derivative

$$S^x_{a\theta} = \partial x/\partial\theta$$

and the relative sensitivity by the relation

$$S^x_\theta = (\partial x/\partial\theta)(\theta/x).$$

It is easy to compute that for θ small the sensitivity is zero, and for important θ the relative sensitivity is 1.

It is known that economic growth models are described also by other types of equations. So the short-term forecasting model for the economy of Belgium, or the Federal Reserve — MITE econometric models are also with time delayed argument, but the arguments are defined only on Z — the set of integers. In this case instead of the Laplace transformation £ it is indicated to utilise the $\mathfrak{Z}$ transformation.

CYBERNETICS AND EDUCATION

Every educational system is a cybernetic one, implying the two factors specified in the definition Wiener postulated for Cybernetics: information and control.

Undoubtedly, education was usually seen as affecting society by socializing individuals. The metaphor man -the computer suggest us to consider man as an amphybian, having a hardware system and a software system: education is influencing his software. Socializing means implementation of a software system in individuals. Recently the view that education is affecting society only by socializing individuals has been attacked with the argument that education is a system of allocation, conferring success on some and failure on others.

The traditional socialization model is based on three assumptions:

A_1 *(Socialization)*. Schooled persons are socialized to expanded levels of knowledge and competence and expanded levels of modern values or orientations;

A_2 *(Socialization and Adult Competence)*. Early socialization to higher levels of knowledge, competence, and modern values or orientations creates higher levels of adult status and competence;

A_3 *(Individual Competence and Social Progress)*. The expansion of the number of skilled adults expands the complexity and wealth of society and social institutions (Meyer, 1977).

We must add some more assumptions.

A_4 *(Education Allocation)*. In modern societies, adult success is assigned to persons on the basis of duration and type of education, holding constant what they have learned in school (Meyer, 1977).

A_5 *(Chartering)*. Students tend to adopt personal and social qualities appropriate to the positions to which their schools are chartered to assign them.

A_6 *(Lagged Socialization)*. Adults tend to adopt qualities appropriate to the roles and expectations to which their educational statuses have assigned them (Meyer, 1977).

It is easy to recognize a contrast between socialization and allocation ideas, i.e. education as a socializing process versus education as a status competition.

It is essential to think on education as on the process by which the work productive force is increased.

As academician professor Manea Mănescu pointed out, the central feedback connection over the space of the national economy is achieved through the flows of labour and of materials, as follow: "the population and labour force system→labour force→ goods production and services system → social product and national income system → national fund of economic social development → national wealth system → the increase of nation's welfare → nation's welfare system → the standard of living → population and labour force system".

"The analysis of the structure of the above-mentioned connections shows that an essential element of the increase of the national wealth and of the nation's welfare is the initial component, i.e. the labour force and its productivity" (Manea Mănescu, 1978).

But it is evident that a higher productivity means a higher training: no higher productivity without skilled workers, without good managers and so on. All this means investment in human capital.

It is by now widely recognized that investment decisions play a major role in the determination of the national wealth, as well as in the individual age-earning profiles (Blinder, Weiss, 1976).

In its simplest form, the investment hypothesis rely on investment in capital as a leading single hypothesis, which can be summarized by the equation

$$Y = F(K, \dot{K}),$$

where Y is observed earnings, K is potential earnings and $\dot{K}$ is its rate of change. This model is due to Rosen, 1973, but similar formulation can be found in Becker, 1964 and Ben-Porath, 1967, as Blinder and Weiss point out.

The proposed model highlights the formal similarity to investment problem of the firm in the presence of adjustment costs. But there are also some important differences. So, for instance the human capital is embodied in the individual and can not be sold, so that the finitness of life plays a central role in human investment. We mention also the fact that it is not possible, in general, to separate the consumption and investment problems for individuals, as done in the theory of the firm. From an epistemological point of view we note that neither human capital nor the investment therein are normally observed, and that the lack of data on human capital makes it hard to distinguish the effects of investment among other influences on actual earnings.

However, the Blinder-Weiss model generates many concrete conclusions, of interest in the capitalistic society.

An essential part in this model is the utility, i.e. the individual utility, which has three sources: the stream of lifetime real consumption, $c(t)$; the fraction of time devoted to leisure, $l(t)$; the bequest, or terminal value of real assets, $A(t)$. Here T is length of life which is assumed known and exogeneous. The lifetime utility is

$$u = \int_0^T U(c,1) \exp(-\rho t)dt + B(A(T))$$

where $U(c, 1)$ and $B(A(T))$ are twice-differenciable, strictly concave functions of their arguments, and ρ is the "rate of impatience" (Blinder, Weiss, 1976).

It is assumed that

$$\lim_{c \to \infty} U_c(c, 1) = \infty \; \forall \; 1$$

$$\lim_{1 \to 0} U_1(c, 1) = \infty, \; \forall \; c$$

$$U_1(c, 1) > 0, \; \forall \; c.$$

In a normal life cycle, we distinguish three phases: schooling, on-the-job training and work; we can add a forth one, retirement. One can think of a utility-maximizing individual free to allocate his daily time budget among work, leisure and education.

On a macroeconomic scale, it is possible to say that schools may teach people useful skills and values. Schools also allocate people to positions of higher social status.

However, there are some other aspects. We mention two of them. One is connected to the fact that education has an ever increasing role in a global competition. For every community is now clear that a major need is in qualified people. The second one is an informational limit to growth. With any amont of raw materials, it is impossible to put them in circulation, without trained people. And training means qualifications, schools and on-the-job training. So it appears that education has many cybernetical implications.

The last thing we would like to point out is the relative sensitivity of economic systems to changes in the educational system. We can compare the situation with a living body. An organism is very sensitive to changes in oxigen, less sensitive to changes in water changes, and still less sensitive to changes in other kind of food. In an economic system, the sensitivity is high for energy, less for raw-material and even less noticeable at short time intervals, for changes in the structure of the educational system.

From the system theory point of view, the sensitivity can be defined as follows:
the absolute sensitivity

$$S_a \triangleq \Delta x / \Delta n;$$

the relative sensitivity

$$S_r \triangleq (\Delta x / x)/(\Delta n / n) = d\log x / d\log n.$$

If we note the sudden relative changes in every economic system, the sensitivity is very low for changes in the educational system, but they can have a tremendous one at long time intervals.

Research on education systems from a cybernetic point of view must examine the effects of education considering effects of variables quite beyond the level of the classroom. One must think that in modern society education is one of the most important variables and that no significative economic progress can be made, in every society, without a proper educational policym i.e. without an educational system based on the most advanced scientific knowledge, among the cybernetics having an authoritary role.

REFERENCES

BLINDER, ALAN S. WEISS YORAM: *Human Capital and Labor Supply*, Journal of Political Economy, 84, 3, 1976, p. 449—472.
MANEA MĂNESCU: *Cybernetic-Economic Systems*, Bucharest, 1978, p. 35.
MEYER, JOHN W. *The Effects of Education as an Institution*, American Journal of Sociology, 83, 1, 1977, p. 55—77.
MALIȚA MIRCEA: *Aurul cenușiu*, (Grey Glod)vol. I, II, III, București.

SENSIBILITE DES SYSTEMES CYBERNETIQUES-ECONOMIQUES

Le but de cet ouvrage est celui de mettre en évidence certaines particularités concernant la sensibilité des systèmes cybernétiques économiques et en premier lieu, les constantes de temps différentes qui doivent être prises en considération dans un tel système.

ЧУВСТВИТЕЛЬНОСТЬ ЭКОНОМИКО-КИБЕРНЕТИЧЕСКИХ СИСТЕМ

Целью настоящей статьи является выявление характерных черт чувствительности экономико-кибиернетических систем относительно различных постоянных, которые должны быть учтены в таких системах.