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"Nowadays cybernetics has an increasing importance in the management of industrial and economical procsses, in reducing the human effort and in increasing the special productivity of man's work, in the overall progress of mankind".

NICOLAE CEAUȘESCU

from the Message addressed by the President of the Socialist Republic of Romania to the Third International Congress of Cybernetics and Systems.

The Labour Productivity System. Cybernetic-Economic Models of the System

The building up of the many-side developed socialist society and the advance to communism suppose to attain a high level of social labour productivity, to efficiently use the national assets of time and work—the most precious wealth of the nation. The Romanian Communist Party grants a particular importance to the continuous increase of labour productivity, to economic efficiency in all fields of activity within the general policy of country's economic-social development.

The Programme of the Romanian Communist Party — masterly scientific work of designing the trajectories of country's economic-social development, of leading Romania towards the general progress, substantiates the necessity of continously increasing the labour productivity by creating an advanced highly efficient economy.

The permanent increasing of labour productivity, the source of the national income and people's wealth magnifying, is according to Romanian Communist Party's concept, the decisive element for securing the continuous

increase of the national economy, for achieving the society's general progress and the ever increasing of people material and cultural living standard.

President Nicolae Ceauşescu, Secretary General of the party, pointing out the importance of qualitative factors of our economic increase shows the followings: "The more emphasized increase of labour productivity is also a primordial need for our economic development in the future years" [1].

The materialist-dialectic analysis of economic mechanism reveals the fact the permanent growth of labour productivity, the utilization with maximum efficiency of the time fund of the nation is a basic law of the economic development within the national economy space.

The objective law of social labour productivity increase mirrors the development as a whole of production forces, the improvement of production social relations.

1. SYSTEM TOPOLOGY AND CONNECTIONS

Taking into account the outstanding part which belongs to labour productivity in the framework of the economic mechanism, in securing the upward evolution on growth economic trajectories of all the national economy components, the *labour productivity system* has many tasks regarding the optimal functioning of the national economy general cybernetic system (Fig. 1).

The labour productivity system is strongly connected to the management, command, controlling and regulation system and goes through the whole economic body, giving an impulse to the rate of all the activities taking place within the national economy space.

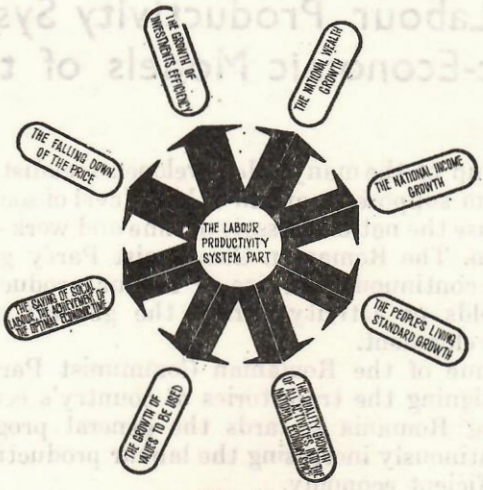


Fig. 1.

[1] Nicolae Ceauşescu: "Report to the National Conference of the Romanian Communist Party", December 7–9, 1977, Editura politică, Bucureşti, 1977, p. 19.

In the contemporary age, the economic growth as a whole, the permanent raising of the people's living standard and the drawing nearer of the evolution trajectories of the developing countries to those of the economically developed countries essentially depend upon the turning to good account of all the human and material resources, upon the level and working mode of the labour productivity system.

The possibility to multiply its economic potentiality, the national wealth is in any country, decisively determined by the labour force capacity to forge more goods in one time unit. By analysing the influence of labour productivity upon the increase of national wealth, Karl Marx shows: "The real wealth of society and the possibility to permanently extending its reproduction process do not depend on the length of superwork but on its productivity" [1].

The statement that the labour productivity represents an important "aparatus board" of the management, command, controlling regulation system can be made. It calculates in every moment the efficiency of using labour force in each cell of the economic body.

At the same time, by means of some specific mechanisms, the labour productivity system takes directly part in the implementation of the decisions adopted by the management system regarding the controlling and regulation of the processes that take place in the national economy space in order to direct them in the sense of a full efficiency use of the nation's work resources. In this way, it is emphasized the majour part played by labour productivity system in achieving the tasks foreseen in the single national plan of country's economic-social development.

The continuous growing up of the labour productivity within every unit of the national economy system represents a task of an outstanding importance in the economic growth programs fulfilment, a main direction for carrying out the general progress.

The national economy general cybernetic system has in every moment at its disposal a limited size of human resources; the more the labour productivity is higher, that means the working time spent for the production of an unit of good is smaller, the more we can obtain greater speeds in the systems development in the national economy space and especially in the goods production and services system, in the system of social product and national income.

The increasing of the labour productivity is equivalent to the "dillatation" of the working time fund at the nation's disposal, being the decisive element for the growing size of production, of the national income, of the national wealth and finally, of the general welfare. Therefore, the permanent raising of the labour productivity is one of the basic objective laws of the optimal economic growth within the economic space.

The labour productivity level, its directions of growth are determined as a vectorial issue of many interrelated economic-social agents that act according to specific laws. Firstly, the labour productivity level is in a tight connection with the "*Productive force of work*", about which Karl Marx shows

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[1] K. Marx, Fr. Engels: "Works", vol. 25. second part, Editura politică, București, 1973, p. 358.

that "it is determined by different circumstances as: the average level of workers skill, science development degree and its techonological applicability, the volume and efficiency of production means, as well as the natural conditions." [1]

The cybernetic analysis of the multitude of agents *acting in a strong connection* upon the labour productivity growth puts in bold relief the fact that these can be classified in the following three decisive groups for the production processes in the economic space: (Fig. 2).

a) agents regarding the labour force development: the ever improvement of the socio-professional structure of labour force; the ever raising of the training level, of the working people's qualification and skill; the optimal staff allocation on branches, subbranches, fields of activity and in territorial profile; the integration of training and practice.

At the same time, the deepening of the social labour division is a powerful agent in the magnifying of the labour productive force, in the labour productivity growth.

b) technical-material agents, in connection to the technical progress implementation: the technic and technological structure by branch, of the goods production and services system; the mechanization, automation, cybernetization and chemization degree of the production processes; the level of the labour technical endowment and the weight of the fixed assets with high technical level in all the fixed assets; the securing of the production technologic flow continuity; the implementation of new types of raw materials, substitues; the raising of the technical level of all goods, of the functioning viability of the equipments.

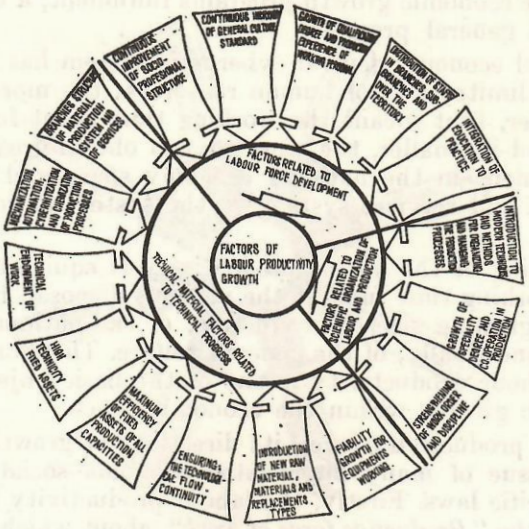


Fig. 2.

[1] K. Marx, Fr. Engels: "Works", vol. 23, Editura politică, Bucureşti, 1966, p. 54.

c) agents concerning the production and labour scientific organization: the strengthening of labour order and discipline; the implementation of modern methods and techniques in the organization and management of the production processes; the raising of the production specialization and cooperation degree.

The unitary classification of the productivity growth agents, the examination of their combined action mode, have an outstanding theoretical and practical importance in order to render evident the optimal ways and methods in the stirring up of the economic growth process.

The identification of the most efficient ways of increasing the labour productivity is a decisive factor in achieving our programs of economic growth, of building up the many-side developed socialist society.

At the National Conference of the Romanian Communist Party on December, 7—9, 1977, where it was adopted the "Programme concerning the supplementary methods of Romania's economic-social development till 1980", President Nicolae Ceaușescu, General Secretary of our party shows that for an emphasized growth of labour productivity: "it is necessary to faster pass to modernizing and automatizing both the production process and the management of units by firmly applying the programs of raising the professional qualification of the entire working personnel".[1]

The systemic approach of labour productivity, the analysis of the part this main system plays in the whole topology of the national economy space strongly underlines the combined action mode of the labour productivity growth agents. At the same time, the social feature of the processes that develop in the national economy space, the decisive importance of labour in the transformation of matter, are clearly revealled.

The labour force, intensified by its productivity, is the one that transforms the goods flows into the social product macroflow within goods production. At the same time, the systemic approach of labour productivity enables the equation-form presentation of the effects of all the activities that take place in the economic body cells, so as to ease the *quantification* of each productive agent part to the national income growth, to the nation's general wealth and finally to magnifying people's material and cultural living standard.

The labour productivity system has a great force of expression. On the base of the commands coming from the macro-economic decision body, it intensifies, conducts and regulates the performances of the general cybernetic system (Fig. 3), for a given level of the labour intensity. By means of many forward and feedback economic technical and social connections, the labour productivity system is connected to all the systems and subsystems developing in the national economy space. The analysis of these connections allows a better knowledge of the social labour growing productivity general economic law action mode, as well as of its effects upon the level of the activities developing in the framework of the general cybernetic system components.

There are many and strong connections and interferences between the labour productivity system and the population and labour force system, on one side, as well as between the human resources — a basic component

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[1] Nicolae Ceaușescu: "Report at the National Conference of the Romanian Communist Party", December, 7—9, 1977, Editura politică, București, 1977, p. 19.

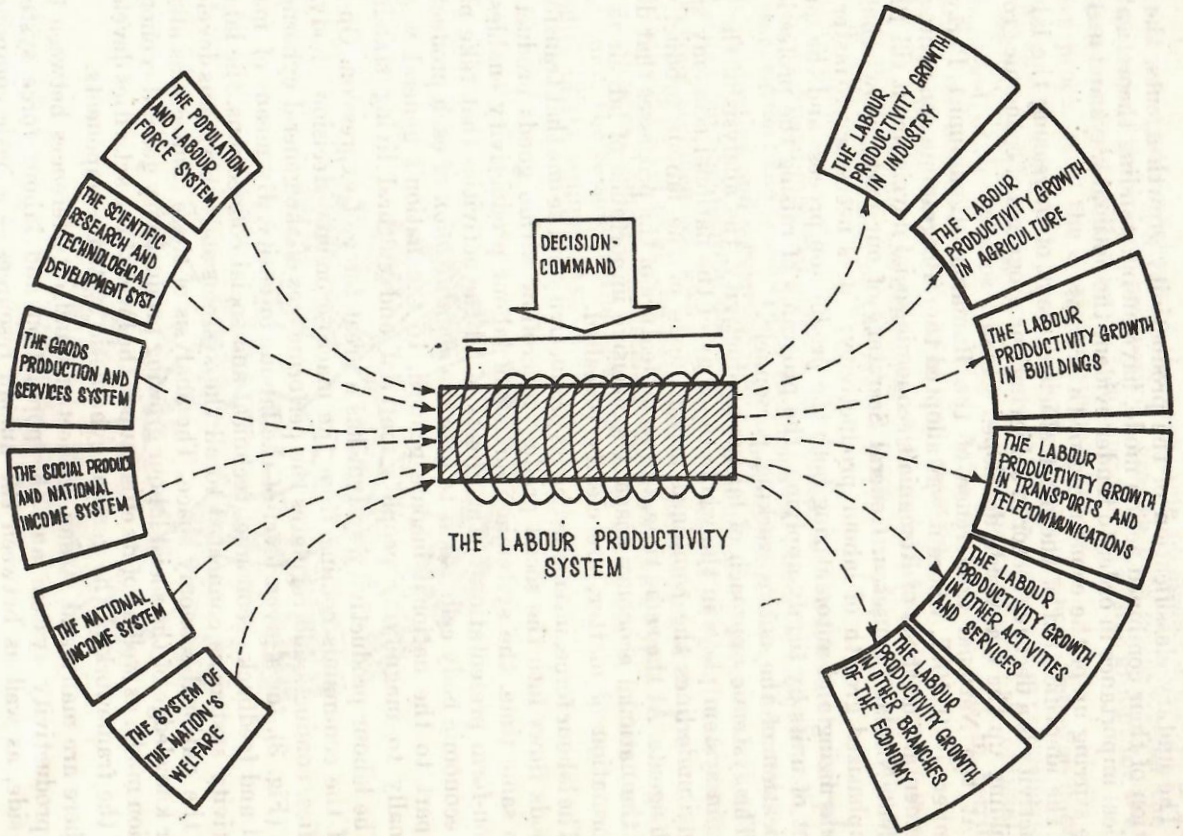

THE POPULATION AND LABOUR FORCE SYSTEM
THE SCIENTIFIC RESEARCH AND TECHNOLOGICAL DEVELOPMENT SYST.
THE GOODS PRODUCTION AND SERVICES SYSTEM
THE SOCIAL PRODUCT AND NATIONAL INCOME SYSTEM
THE NATIONAL INCOME SYSTEM
THE SYSTEM OF THE NATION'S WELFARE
DECISION-COMMAND
THE LABOUR PRODUCTIVITY SYSTEM
THE LABOUR PRODUCTIVITY GROWTH IN INDUSTRY
THE LABOUR PRODUCTIVITY GROWTH IN AGRICULTURE
THE LABOUR PRODUCTIVITY GROWTH IN BUILDINGS
THE LABOUR PRODUCTIVITY GROWTH IN TRANSPORTS AND TELECOMMUNICATIONS
THE LABOUR PRODUCTIVITY GROWTH IN OTHER ACTIVITIES AND SERVICES
THE LABOUR PRODUCTIVITY GROWTH IN OTHER BRANCHES OF THE ECONOMY

Fig. 3.

of the national wealth system, "the energetic tank" of the economic-social development, of the development in the economic space — and the public training subsystem — a nation's welfare system component, on the other side.

There is a very osmosis process between these four components of the national economy general cybernetic system (Fig. 4). The main part in the labour productivity growth belongs to labour force, to man endowed with modern production technique and by its means he puts to work the whole growth mechanism of the national economy general cybernetic system. The qualification degree and experience of labour force, its branch, professional, organizational structure, are basic elements that cause the labour productivity level.

By means of a better qualification, the labour force gets the possibility to use a higher volume of fixed assets, with a higher technical level, so as to produce a bigger volume of goods in the same time unit. The raising qualification of labour force enables an easier adaptation to the new manufacture technologies, to the scientific organization methods of production, thus facilitating the innovations and rationalizations that finally lead to the labour productivity growth.

The accomplishment of a harmonious structure of labour force on branches of the national economy and on territorial profile has an outstanding importance for the labour productivity growth. Any lack of balance in this field can lead to the unreasonable utilization of labour force, to losses in the *national fund of working time* and thus to the falling down of productivity. Therefore, in order to use with maximum of efficiency the labour force, the management, command, controlling, regulation system sets up for each stage, according to the labour productivity system, the optimal structure which; will have to be achieved in the population and labour force system.

By means of the labour productivity system, starting from the knowledge of the evolution laws of population and its structure, in connection

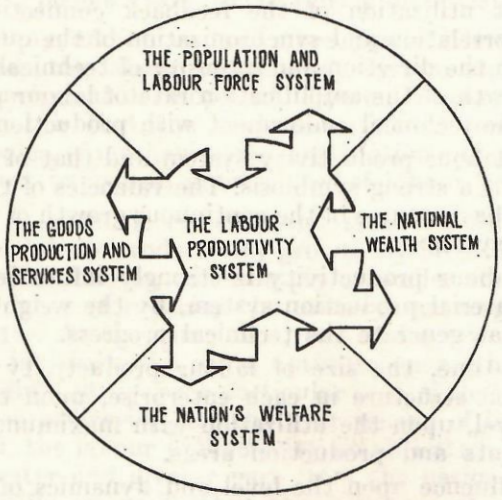


Fig. 4.

to the public training subsystem, the optimal trajectories for the training of new generation of workers, masters, technical experts and specialists with high qualification, according to the progress necessities of the national economy general cybernetic system components, are settled. All these are fulfilled according to the progress necessities of all the national economy general cybernetic system components.

In the circumstances of alert changes that happen in the economic space, namely as a consequence of the technical-scientific revolution, both the labour productivity system and the public training subsystem have the task to find out the most reasonable completion and renewing forms for knowledge, for the ever improvement of labour force, for staff requalification and manysided qualification.

A strong *interference relationship* forms itself between the working people growing qualification and the efficient utilization of the action of technical progress — base agents for the growth of the labour productive force and productivity.

The raising of the labour force technical endowment, the introduction of technical progress in all the cells of the national economy system, based upon the complex mechanization, automation and cybernetization of the production processes, by means of the quick implementation of the most modern manufacture technologies, are decisive agents for the labour productivity growth.

At the same time, the utilization with maximum efficiency of the new technologies requires a highly qualified labour force, with many technical-professional knowledge. The rapid connection, at the settled parameters, of the new technologies to the economic circulation of the national economy space, is possible only in the circumstances of a full synchronization and correlation between the level of technical endowment and the qualification degree of the working people. Thus the action mode of the *feedback connection law* upon the growth agents of labour productivity is revealed.

The conscient utilization of the feedback connection objective law action mode, the correlation and synchronization of the qualification growth of labour force with the direction and intensity of technical progress, enables the more rapid growth of the amplification rate of labour productivity compared to that of the technical endowment with production funds.

Between the labour productivity system and that of goods production and services, there is a strong symbiosis. The valencies of the labour productivity system are a basic source of the continuous growth of the goods production and services system.

The size of labour productivity is strongly influenced by the branch structure of the material production system, by the weight detained by the "top" branches, that generate the technical progress.

At the same time, the size of labour productivity directly depends upon the production structure in each enterprise, upon the mechanization and automation level, upon the utilization with maximum efficiency of the machines, equipments and production areas.

A decisive influence upon the level and dynamics of labour productivity belongs to the size and structure of the economic-social development

national fund that directly causes the rate of puting into operation, the new productive fixed assets, with high technical level, decisive agents for the raising of the labour technical endowment degree, for the magnifying of the productive force of labour.

The organization mode of production and labour has a strong influence upon the labour productivity growth in all the cells of the goods production and services system.

The standardization and specialization of enterprises, their efficient cooperation, the settling of rational technological flows, in each productive unit, section or workshop, of optimal goods lot that enable the full utilization of the equipments and the removal of the narrow places in production, as well as the unification, standardization of blanks, subensembles and finished products are important elements that directly influence the labour productivity growth.

The labour scientific organization is an important agent in the productivity growth, that enables the efficient utilization of the staff working time, according to its qualification and skill.

The scientific organization of production and labour in the framework of the goods production and services system is a dynamic process that has to be in permanent harmony with the evolution of the new technologies, with the raising qualification level of labour force, a.o.

In the framework of the labour productivity system it is carried out the continuous analysis of the achieved level in the cells of the goods production and services system and there are established the ways and methods for its raising, for the falling down of the dispersion compared to the best achievements in the labour field.

At the same time, the level and intensity of the activities taking place in the goods production and services system directly contribute to the development of the working people's skill, to the raising qualification of labour force.

There are many and more and more powerful forward and feedback connections between the labour productivity system and that of scientific research and technological development. The main reason that make these coonections to exist is that the implementation of technical progress, of advanced technologies, of mechanization, automation and production cybernetization, is the most important growth way of labour productivity.

The system of scientific research and technological development has a direct part in the labour productivity growth by providing new and improved products and technologies, which imply a smaller amount of labour and by conceiving the ways of reducing the unitary consumption and the utilization of substitutes. At the same time, in the framework of the scientific research system it is conceived a set of optimization methods and techniques that enable the finding out, for each particular case, of the most efficient issues for the labour force utilization, for the ever raising of labour productivity. On the other hand, the labour productivity amplification makes possible the allocation of a greater and greater part from the national fund of working time for the scientific research. Thus the existence of strong forward and feedback connections between the two systems is underlined.

The productivity magnifying, the production of a bigger quantity of goods and services in the same economic time unit, is the decisive agent for the national income growth and consequently, for the national wealth and nation's welfare growth.

In its turn, the national wealth system strongly acts upon the labour productivity growth by means of all its components: human resources, the nation's creative capacity, natural resources, fixed assets, a.o. Between the labour productivity system and the nation's welfare system there are many interferences.

The systematic growth of labour productivity is the decisive element for the ever raising of the working people's retribution, a certain base for the people's general living standard growth. In its turn, the nation's welfare system has a direct influence upon the labour productivity development, by means of its components, the public training, public health, goods and services consumption, a.o. Thus, the public training subsystem, being the main growth agent of the labour force qualification, has a direct part in the labour productivity amplification. At the same time, the public health, by means of its contribution to the health stock growth and consequently, to the used time fund growth, has an important contribution to the labour productivity growth.

The former analysis reveals the complexity of the labour productivity system, its strong forward and feedback connections to all the national economy general cybernetic system components (Fig. 5).

The time propagation and ramification phenomena characterize the connections that form themselves between the labour productivity system and the other general system components. The flows generated in the labour productivity system ramify to the goods production and services system, to

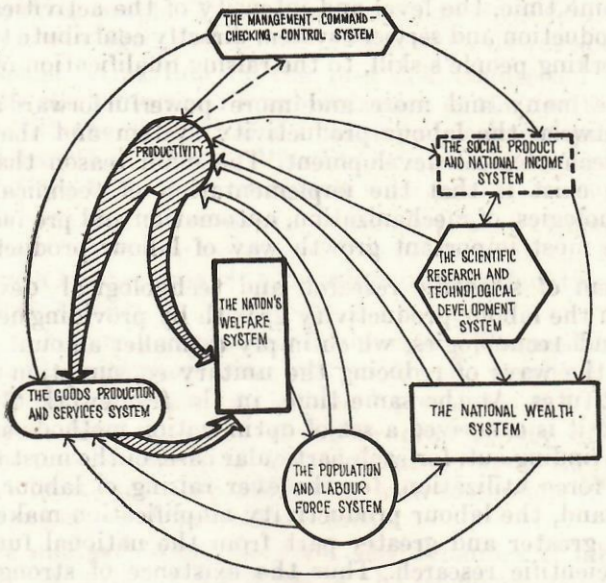


Fig. 5.

the scientific research and technical development system, to the national wealth system, to the nation's welfare system as well as to the other systems in the economic space, thus causing broad shiftings that are convergent to the alive and dead work sparing, to the labour productivity growth. All these are finally mirrored in the systematic growth of the people's material and cultural living standard, the basic target of the country economic-social development strategy.

The fact that the economic progress cordinates and those of social progress interfere in the framework of the labour productivity system has to be underlined.

Labour productivity — as it is shown by V.I. Lenin — is finally, *the most important factor of the victory of the new social order*. Capitalism has created a labour productivity not known by feudalism. Capitalism can be and will be definitely defeated by the fact that socialism creates a new labour productivity, a higher productivity.[1]

In this context, the international comparison of the labour productivity levels becomes a main criterion for the evaluation of the national economy general cybernetic system performances, an essential agent in the drawing nearer of the development level of the economically developed countries.

The systemic approach of labour productivity, the profound study of the labour productivity system complex problems, the analysis of its forward and feedback connections to all the components of the economy general cybernetic system, reveal the fact that the labour productivity continuous growth law is a general law of the development in the national economy space. The knowledge of its particular action mode contributes to the settling of the decisions that enable the better turning to account of the growth reserves of the labour productive force, upon which decisively depends the economic and social progress.

2. CYBERNETIC-ECONOMIC MODELS

The cybernetic analysis of the role and functionality of the labour productivity system in the framework of the economic mechanism has revealed its basic part in the whole process of economic growth, in assuming an upward development of the national economy general cybernetic system, on optimal growth trajectory.

In order to accomplish the various tasks it has regarding the fast and continuous growth of productivity in all the economic body cells, the utilization with maximum efficiency of the working time national fund, many types of cybernetic-economic models for analyses and optimization are used in the framework of the labour productivity system: *the labour productivity growth equation, the economic-mathematic model of the correlation labour productivity — the level of labour technical endowment with fixed assets, the input-output model in labour units, the economic-mathematic model of the correlation between the working people's qualification level and the labour productivity growth a.o.* These models ensure both the exact knowledge, at each moment,

[1] V. I. Lenin: Works, vol. 39, Editura politică, București, 1966, p. 22.

of the labour expenditure efficiency in every system and subsystem, and the finding out of the optimal growth ways of their efficiency, of the circumstances that enable the fast growth of labour productivity.

Thus, for instance the circumstances that assure the faster growth of the labour productivity rate compared to that of the technical endowment with fixed assets can be measured by means of the economic-mathematic model of the correlation labour productivity — the level of labour technical endowment with fixed assets.

The cybernetic-economic models of the labour productivity system ensure a strict comparison of the labour consumption in the framework of all the economic body components to the issue measured, finally, by the growth of the national income and of the nation's welfare.

The indicators worked out in the framework of the cybernetic-economic models of the labour productivity system have to mirror the productivity at the level of each national economy general system component and, at the same time, to render evident the elements which enable a full turning to account of the agents that lead to the increasing efficiency of all the activities in the national economy space.

2.1. The equation of labour productivity growth

As it was rendered evident in the former paragraphs, the labour productivity in the national economy space depends upon many agents: the qualification and training degree of the labour force, the technical endowment of labour with fixed assets and their technologic level, the proportion between the productive and auxiliar staff, the scientific organization of production and labour, the branch structure of the goods production system, the level of mechanization, automation and cybernetization of the production processes, a.o.

Denoting by W_t — the labour productivity level in the national economy space in the t year, its growth equation can be written, in a general form, as follows:

$$\frac{\dot{W}_t}{W_t} = \xi_t(\alpha_t, \beta_t, k_t, z_t, m_t, \gamma_t, h_t, q_t)$$

The following notations were used:

$\frac{\dot{W}_t}{W_t}$ — the growth rate of labour productivity;

α_t — a vector whose components quantify the staff socio-professional structure;

β_t — a vector whose components quantify the staff qualification degree and skill;

k_t — the technical endowment of labour with fixed assets;

z_t — a vector whose components quantify the branch structure of the goods production and services system, the industry weight in the whole system included;

m_t — a vector whose components mirror the fixed assets structure, on types of technologies, the new fixed assets weight in all the fixed assets included;

γ_t — a vector that quantifies the development levels of the activities belonging to the scientific research and technological engineering system;

h_t — a vector that quantifies the level of complex mechanization, automation and cybernetization of the production processes;

q_t — an element that quantifies the scientific organization of labour and production.

The knowledge of the function $\xi_t(\cdot)$ and of the vectors that interfere in the equation of labour productivity growth enables the quantification of each element contribution to the complex process of labour productivity magnifying, as well as the evident rendering of the combined mode of their action.

2.2. The model of the correlation between the labour productivity and the level of labour technical endowment with fixed assets

Between the productivity level, on one side, and the size of the endowment with fixed assets on an engaged person in the productive activity, on the other side, there is an extremely complex correlation (Figs. 6. a and 6. b).

This correlation can be quantified, in a synthetic form by means of the production function, both at macroeconomic and branch level, on enterprise level or on territorial profile.

The main problems which appear in the framework of the cybernetic-economic models, concerning the production functions utilization, are the following:

a) The statement of a scientifically substantiated methodology for the computing of the indicators used in the production functions theory.

b) The statement of the criteria for the choice of the concrete mathematical forms of the production functions that modelate most correctly the production activities developed in the frame of the systems and subsystems in the national economy space.

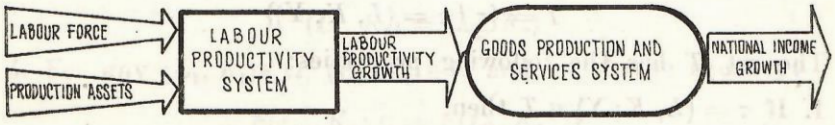


Fig. 6a

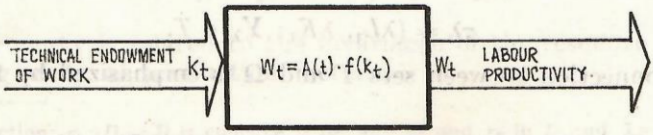


Fig. 6 b.

c) The most correct mirroring and quantification, by means of the production functions of the phenomena connected to technical progress.

d) The utilization of those mathematical techniques that enable the most correct estimation of the parameters that interfere in the production functions (the method of the maximal verisimilitude, the least squares method in more stages, a.o.).

e) The finding out of the economic-mathematical circumstances and of the computing techniques that enable the correct aggregation of the elements we work with (the labour force with various qualifications, the fixed assets of different types and technologies, a.s.o.), and the aggregation of some production functions computed on enterprises, branches and on territorial profile.

The *production function* shapes the functional dependence that forms itself between the following elements:

L_t — the size of the labour force that exists in the modelled system or subsystem;

K_t — the fixed assets volume;

Y_t — the output of the production activity of the system (the created social product and national income of the whole national economy system, the gross product and the net product on the branches, industrial corporation).

The mathematic model of the production function is the following:

$$F : \Omega \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$$

where the set Ω wherein the (L,K) variables have values, has to be described on the base of the specificity of the production processes performed by the modelled system or subsystem.

In order to characterize Ω set, it will be used the notion of activity, analysed in chapter 3:

$$\tau = (L, K; Y)$$

We shall denote by T the set of all production activities which can be developed by the analysed system and subsystem:

$$T = \{\tau / \tau = (L, K; Y)\}$$

The set T has the following properties:

1. If $\tau = (L, K; Y) \in T$ then

$$L \geqslant 0, K \geqslant 0 \text{ and } Y \geqslant 0$$

2. If $\tau = (L_1, K_1; Y_1) \in T$ then a production volume Y_λ corresponds to each real number $\lambda > 0$ such that

$$\tau\lambda = (\lambda L_1, \lambda K_1; Y_\lambda) \in T.$$

The connection between sets T and Ω is emphasized by the following relation:

$$\Omega = \{(L, K) / \exists\, Y \text{ such that } (L,K; Y) \in T\}$$

namely Ω is the set of possible combinations of production factors which are technologically feasible to the analysed system.

From the concrete character of production processes developed within the system and subsystems of the national economy space, it follows that the production functions:

$$F: \Omega \times \mathbf{R}_+ \to \mathbf{R}_+$$

must satisfy the following properties:

1. For any pair $(L, K) \in \Omega$

$$\lim_{\lambda \to 0} F(\lambda L, \lambda K, t) = 0$$

2. If $(L_1\ K_1) \in \Omega$, $(L_2, K_2) \in \Omega$ and $(L_1, K_1) \leqslant (L_2, K_2)$

then:

$$F(L_1, K_1, t) \leqslant F(L_2, K_2, t)$$

namely, the production function is increasing to both arguments.

3. For any $(L, K) \in \Omega$ and $\lambda > 0$ the function $\varphi(\lambda) = F(\lambda L, \lambda K)$ is concave like function of λ[1].

For certain cybernetic-economic systems, the concavity property may be replaced by the quasi-concavity one.

3′. For any $(L_1, K_1) \in \Omega$ and $(L_2, K_2) \in \Omega$ and for any $y > 0$ and $0 < \lambda < 1$

if:

$$F(L_1, K_1; t) \geqslant y \text{ and } F(L_2, K_2; t) \geqslant y$$

then:

$$F(\lambda L_1 + (1 - \lambda)L_2, \lambda K_1 + (1 - \lambda)K_2; t) \geqslant y$$

being equal only for the case where:

$$(L_1, K_1) = (L_2, K_2)$$

4. For any $(L_1, K_1) \in \Omega$, $(L_2, K_2) \in \Omega$ and $\lambda > 0$ the equality:

$$F(L_1, K_1; t) = F(L_2, K_2; t)$$

is satisfied if and only if:

$$F(\lambda L_1, \lambda K_1; t) = F(\lambda L_2, \lambda K_2; t)$$

This property is related to the invariance of the result to the changes of measuring units of production factors.

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[1] A function: $\varphi : D \to \mathbf{R}$ is concave if for any x_1 and x_2 in D and $\lambda \in [0, 1]$, $\varphi(\lambda x_1 + (1 - \lambda)x_2) \geqslant \lambda\varphi(x_1) + (1 - \lambda)\varphi(x_2)$.

The following property is characteristic only for the class of homothetic fuctions.

5. For any $\lambda > 0$ it is satisfied the equality:

$$F(\lambda L, \lambda K; t) = \lambda F(L, K; t)$$

Property 5) shows the fact that in the case when each production factor is multiplied by λ times, then the result of production is also multiplied by λ times (for instance if each production factor is doubled, then the production volume must at least be doubled). We have to mention that for numerous systems it is charcteristic the fact that the result of production increases higherly than the increase pf production factors.

When function:

$$F:\Omega \to \mathbf{R}_+$$

is of $C^2(\Omega)$ class, then the property 2 is equivalent to

$$2'. \frac{\partial F}{\partial L}(L, K) \geqslant 0; \quad \frac{\partial F}{\partial K}(L, K) \geqslant 0$$

and property 3 is equivalent to:

$$3'. \text{ Matrix } \begin{Vmatrix} \frac{\partial^2 F}{\partial L^2}(L, K) & \frac{\partial^2 F}{\partial L \partial K}(L, K) \\ \frac{\partial^2 F}{\partial K \partial L}(L, K) & \frac{\partial^2 F}{\partial K^2}(L, K) \end{Vmatrix}$$

is negatively semi-defined.

The first problem which is set, knowing the production function properties is to identify the functional equation that points out the concrete type of the production function characterizing the modelled system or subsystem. A consistent help in solving this main problem is brought about by the indicators that characterize the production function of the system.

By supposing that the production function is homogeneous of one degree and by using the notations:

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1; t\right) = f\left(\frac{K}{L}; t\right)$$

$$\frac{Y}{K} = F\left(1, \frac{L}{K}; t\right) = \varphi\left(\frac{L}{K}; t\right)$$

the characteristic indicators of production function are the followings:

— average labour productivity:

$$y(L, K; t) = f\left(\frac{K}{L}; t\right)$$

— average efficiency of fixed funds:

$$z(L, K; t) = \varphi\left(\frac{L}{K}; t\right)$$

— technical endowment of labour:

$$k = \frac{K}{L}$$

— differential productivity of work:

$$n_L(L, K; t) = f\left(\frac{K}{L}; t\right) - \frac{K}{L} \cdot f'_{K/L}\left(\frac{K}{L}; t\right)$$

or:

$$\eta_L(L, K; t) = \varphi'_{L/K}\left(\frac{L}{K}; t\right)$$

— differential efficiency of fixed assets:

$$\eta_K(L, K; t) = f'_{K/L}\left(\frac{K}{L}; t\right)$$

or:

$$\eta_K(L, K; t) = \varphi\left(\frac{L}{K}; t\right) - \frac{L}{K} \cdot \varphi'_{L/K}\left(\frac{L}{K}; t\right)$$

— *production elasticity in relation to labour force:*

$$E_L(L, K; t) = 1 - \frac{K}{L} \cdot \frac{f'_{K/L}\left(\frac{K}{L}; t\right)}{f\left(\frac{K}{L}; t\right)}$$

or:

$$E_L(L, K; t) = \frac{L}{K} \cdot \frac{\varphi'_{L/K}\left(\frac{L}{K}; t\right)}{\varphi\left(\frac{L}{K}; t\right)}$$

— production elasticity in relation to fixed assets:

$$E_K(L, K; t) = \frac{L}{K} \cdot \frac{f'_{K/L}\left(\frac{K}{L}; t\right)}{f\left(\frac{K}{L}; t\right)}$$

or:

$$E_K(L, K; t) = 1 - \frac{L}{K} \cdot \frac{\varphi'_{L/K}\left(\frac{L}{K}; t\right)}{\varphi\left(\frac{L}{K}; t\right)}$$

— replacement degree of factors

$$r(L, K; t) = \frac{f\left(\frac{K}{L}; t\right)}{f'_{K/L}\left(\frac{K}{L}; t\right)} - \frac{K}{L}$$

or:

$$r(L, K; t) = \frac{\varphi'_{L/K}\left(\frac{L}{K}; t\right)}{\varphi\left(\frac{L}{K}; t\right) - \frac{L}{K} \cdot \varphi'_{L/K}\left(\frac{L}{K}; t\right)}$$

— elasticity coefficient of replacing the production factors:

$$\sigma(L, K; t) = \frac{K}{L} \cdot \frac{f\left(\frac{K}{L}; t\right) \cdot f''_{K/L,\ K/L}\left(\frac{K}{L}; t\right)}{f'_{K/L}\left(\frac{K}{L}; t\right)\left[\frac{K}{L} \cdot f'_{K/L}\left(\frac{K}{L}; t\right) - f\left(\frac{K}{L}; t\right)\right]}$$

or:

$$\sigma(L, K; t) = \frac{L}{K} \cdot \frac{\varphi\left(\frac{L}{K}; t\right) \cdot \varphi''_{L/K,\ L/K}\left(\frac{L}{K}; t\right)}{\varphi'_{L/K}\left(\frac{L}{K}; t\right) \cdot \left[\frac{L}{K} \cdot \varphi'_{L/K}\left(\frac{L}{K}; t\right) - \varphi\left(\frac{L}{K}; t\right)\right]}$$

It is noticed that each of these indicators depend on the technical endowement of work $\frac{K}{L}$ and on the temporal variable.

The following two indicators can be used in order to characterize the technical progress action:

$$T(L(t), K(t); t) = \frac{\frac{\partial F}{\partial t}(L(t), K(t); t)}{F(L(t), K(t); t)}$$

and

$$D(L(t), K(t); t) = \frac{\frac{\partial r}{\partial t}(L(t), K(t); t)}{r(L(t), K(t); t)}$$

where $L(\cdot)$ and $K(\cdot)$ are time-derivable functions. It was denoted by $r(\cdot, \cdot; \cdot)$ the substitution rate of the production factors.

The $T(\cdot, \cdot; \cdot)$ indicator measures the technical progress rate $D(\cdot, \cdot; \cdot)$ characterizes the orientation of the technical progress evolution.

Truly, the second indicator characterizes the change rate in time of the substitution coefficient of the alive work with the dead one, one of the most important fields the technical progress expresses itself in.

The evolution of those indicators that characterize the cybernetic-economic systems can be quantified by means of the exposed indicators:

— *the growth equation of production:*

$$\frac{\dot{y}(t)}{y(t)} = T(L(t), K(t); t) + E_L(L(t), K(t); t) \cdot \frac{\dot{L}(t)}{L(t)} +$$

$$+ E_K(L(t), K(t); t) \cdot \frac{\dot{K}(t)}{K(t)}$$

The above equation renders evident the fact that the production growth rates is equal to the sum of the growth rates of the two factors of production (the labour force and the fixed assets) weighted by the corresponding elasticities, enlarged with the technical progress rate. Above, there were denoted by E_L, the elasticity indicator of production compared to labour force, and by E_K the one by comparison to the fixed assets.

— *the equation of the work productivity growth*

$$\frac{\dot{\eta}_L(t), K(t); t)}{\eta(L(t), K(t); t)} = E_K(L(t), K(t); t) \cdot \sigma(L(t), K(t); t) \cdot$$

$$\cdot \left[\frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)}\right] - E_K(L(t), K(t); t) \cdot D(L(t), K(t); t) + T(L(t), K(t); t)$$

— *the growth equation of technical progress*

$$T(L, K; t) = E_L(L, K; t) \cdot \frac{\dot{\eta}_L(L, K; t)}{\eta(L, K; t)} + E_K(L, K; t) \cdot \frac{\dot{\eta}_K(L, K_j t)}{\eta_K(L, K; t)}$$

The above equation renders evident the fact that the growth rate of technical progress is equal to the weighted sum of the growth rate of labour productivity and the fixed assets efficiency, the elasticities $E_L(\cdot, \cdot; \cdot)$ and $E_K(\cdot, \cdot; \cdot)$ being the weights.

In order to expose the economic-mathematic model of the correlation between labour productivity and the technical endowment of labour with

fixed assets, the assumptions that the production function is homogenous of first degree and the technical progress acts by means of a multiplier $A(t)$ are made; the following can be written:

$$W_t = A(t) \cdot f(k_t)$$

wherein it was denoted by:

W_t — the labour productivity size in the t year;

k_t — the endowment with fixed assets on one engaged person;

$A(t)$ — technical progress element.

The correlation between the growth rate of labour productivity, on one side, and that of the endowment with fixed assets on an engaged person and the technical progress, on the other side, are rendered evident in the following mathematical relation:

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{A}(t)}{A(t)} + \dot{k}_t \cdot \frac{f'(k_t)}{f(k_t)}$$

which can be also written:

$$\frac{\dot{W}_t}{W_t} - \frac{\dot{A}(t)}{A(t)} + \frac{\dot{k}(t)}{k(t)} \cdot E_k f(k_t)$$

The economic meaning of the used above notations is the following:

$\frac{\dot{W}_t}{W_t}$ — the growth rate of labour productivity;

$\frac{\dot{A}_t}{A_t}$ — the growth rate of technical progress;

$\frac{\dot{k}_t}{k_t}$ — the growth rate of the endowment with fixed assets on one engaged person in the productive activity;

$E_K f(k_t) = \frac{f'(k_t)}{\frac{f(k_t)}{k_t}}$ — the elasticity indicator of labour productivity compared to the growth of the endowment with fixed assets.

The above equational description allows the quantification of the technical progress contribution and that of the endowment with fixed assets upon the labour productivity growth. It renders evident that each percentage of growth of the technical progress rate, for a given type of production function, leads to the raising of one percentage in the growth rate of labour productivity. At the same time, there is a proportionality between the growth rhythm of the endowment with fixed assets on one engaged person and that of the productivity, the proportionality coefficient being namely the elasticity indicator. In case the cybernetic-economic model is shaped by another type of production function, then the proportion 1:1 is consequently altered.

The above presented model will be shown for the case of industry — main component of material production system.

Identification of production function shape will be made by using the data concerning the dynamics of industrial global production, of fixed assets volume and of the number of working personnel within the production.

Industry of the Socialist Republic of Romania

Years	Global production (billion lei)	Fixed assets (billion lei)	Working personnel (tens thousand)	Technical endowement of labour (thousand lei /man) 2: 3	Labour productivity (thousand lei /man) 1: 3
0	1	2	3	4	5
1965	180.0	151.7	186.29	81.432	96.623
1966	205.2	166.7	193.14	86.310	106.244
1967	233.4	189.1	197.14	95.921	118.393
1968	260.1	212.5	203.14	104.607	128.039
1969	287.2	237.6	216.15	109.923	132.870
1970	307.2	265.3	277.68	114.523	134.926
1971	361.0	282.7	245.71	115.054	146.921
1972	402.8	315.9	260.12	121.443	154.851
1973	462.0	352.7	279.76	126.072	165.141
1974	528.5	398.4	298.32	133.547	177.158
1975	586.9	455.7	310.96	146.541	188.732
1976	660.6	520.1	326.79	159.154	202.148

Source: Statistical Year Book of the Socialist Republic of Romania, 1977.

On the base of data concerning the industry of the Socialist Republic of Romania during the period 1965—1976, the following concrete shape of production function was calculated:

$$Y_t = 0.9772\ K_t^{0.516\,761}\ L_t^{0.483239}\ e^{0.036377t}$$

The above equation outlines the fact that the rate of technical progress was of 3. 6% in the Romanian industry during the 1965—1976 period.

The correlation between the average labour productivity and the techsical endowement of working personnel within the industry is given by the following equation:

$$W_t = 0.9772\ K_t^{0.516761}\ exp(0.036377t)$$

The growth rate of labour productivity depending on the growth rate of endowement by fixed assets for a person in industry during 1965—1976 period is given by the following equation:

$$\frac{\dot{W}_t}{W_t} = 0.036377 + 0.516761\ \frac{\dot{K}_t}{K_t}$$

The annual average growth rate of labour productivity in the *industry of the Socialist Republic of Romania* was in 1965—1976 period of 7%. The equation of labour productivity growth outlines the fact that due to the con-

sistent policy of endowing the industry by new modern technologies of high efficiency, of the continuous increase of qualifiying the labour force, of production scientific organization, the contribution of technical progress to the annual increase of labour productivity was in 1965—1976 period of over 50%.

In order to illustrate the application mode of the model for analysing the labour productivity in a territorial profile, we shall consider the case of Sibiu county.

The data concerning the global production, the fixed assets and the number of personnel occupied in the county industry during 1966—1975 period, are presented in the following table:

Industry of Sibiu county 1965 = 100

Years	Industrial global production	Fixed assets in industry	Number of working personnel in industry
1966	115	110.6	105.1
1967	130	122.2	109.5
1968	144	135	112.2
1969	158	150	122.1
1970	172	163	115.4
1971	194	180	122.1
1972	213	198	130.4
1973	243	209	135.0
1974	288	235	142.0
1975	320	275	146.6

On the base of presented data, the following concrete shape of the presented model follows:

$$W_t = 1.007\ exp[0.051t] \cdot \frac{\dot{k}_t}{k_t}^{0.39}$$

The growth rate of labour productivity in industry depending on the growth rate of fixed assets for a person occupied during 1966—1976 period in Sibiu county is emphasized by the following equation:

$$\frac{\dot{W}_t}{K_t} = 0.051 + 0.39\frac{\dot{k}_t}{k_t}$$

Regarding the exposed model, the settlement of the circumstances that enable the labour productivity rate to outrun that of the endowment with fixed assets and that of the engaged staff in productive activities is a most important problem. Out of the above equational description it follows that the below conditions have to be satisfied in order to have the labour productivity rate outrun that of technical endowment of labour:

$$\frac{\dot{A}(t)}{A(t)} > [1 - E_K f(k_t)] \cdot \frac{\dot{k}_t}{k_t}$$

2.3. The input-output model in work units

Another important type of model of the labour productivity system is the one worked out on the base of the input-output mode, expressed in *work units* (Fig. 7)

This one enables the exact quantification of the productivity of the labour spent in all the stages of the goods production process, starting from the raw materials, and along the intermediary processing, to the finished product.

In order to enable the analysis of the achieved labour productivity in each system and subsystem of the national economy space, the working out of the input-output model in work units is started from the labour direct expenditure coefficients on good unit. These ones are computed by means of the technological coefficients and of the indicators regarding the real working of the technological coefficients and of the indicators regarding the real working time.

The input-output model in work units embeds, alike other variants of this type of model, three quadrants, each of them mirroring various sides and processes that take place in the national economy space and that depend on each other.

The production relationships between the various parts of the goods production system (branches, subbranches, goods and groups of goods) are rendered evident in the first quadrant by means of the labour expenditures

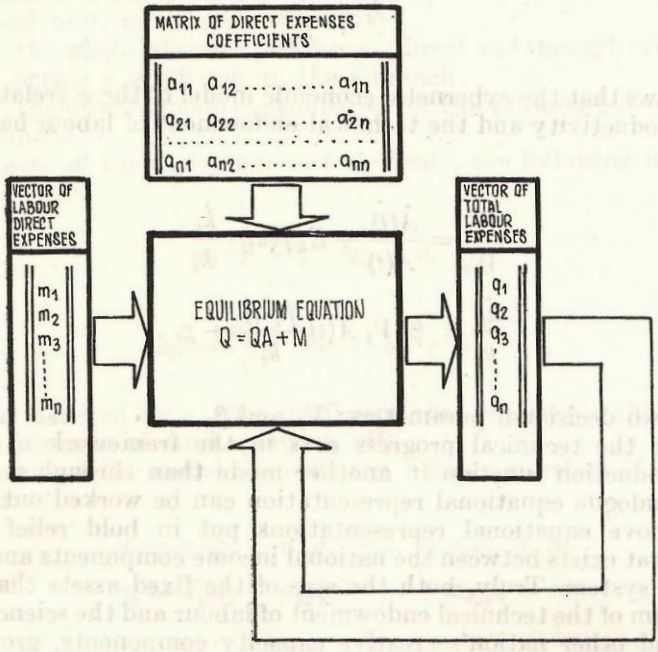


Fig. 7.

The second relation of the analysed economic-mathematical model is the *growth equation of the fixed assets:*

$$\dot{k}_t = \beta_t [V_t L_t A(t) f(k_t) + a_t K_t] - \mu_t K_t$$

The notation are the following:

V_t — the accumulation rate;

β_t — the weight of the investments meant to the goods production and services system in the whole volume of the economic-social development national fund;

a_t — the writing off coefficient;

μ_t — the fixed assets obsolescence rate.

The exposed equation contains the *decision parameters*, the accumulation rate and the weight of the investments in the fixed assets meant to the goods production and services system in the whole economic-social development national fund.

The growth equation of the technical endowment of labour is the following:

$$\dot{k}_t = \sigma_t V_t A(t) f(k_t) + g_t k_t$$

where:

$$g_t = a_t - \mu_t - l_t$$

$$l_t = \frac{\dot{L}_t}{L_t}$$

It follows that the cybernetic-economic model of the correlation between the labour productivity and the technical endowment of labour has the following form:

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{A}(t)}{A(t)} + E_K f(k_t) \cdot \frac{\dot{k}_t}{k_t}$$

$$\frac{\dot{k}_t}{k_t} = \beta_t V_t A(t) \frac{f(k_t)}{k_t} + g_t$$

and it has two decisional parameters: V_t and β_t.

In case the technical progress acts in the framework of the macroeconomic production function in another mode than through a multiplying factor, an analogue equational representation can be worked out.

The above equational representations put in bold relief the strong connection that exists between the national income components and the labour productivity system. Truly, both the size of the fixed assets that cause the growth rhythm of the technical endowment of labour and the science and training stock and other nation's creative capacity components, growth factors of the technical progresss, act upon the labour productivity growth.

materialized in raw materials, materials, a.o. By means of the first quadrant indicators, the production relations between the branches and subbranches of the goods production system are characterized.

The elements in the second quadrant express the part from the quantity of labour that was consumed in order to produce the various components of the final product: the social and individual consumption, the investments, the increasing of stocks and reserves.

In the third quadrant, the labour direct expenditures on branches, groups of labour force, forms of property are rendered evident. At the same time, in the frame of this quadrant there are mirrored, in work units, the elements of the simple reproduction of the fixed assets, namely the writing off; in some variants of the input-output mode, the writing off is included in the first quadrant.

In this way, the model shows the way the direct expenditures of alive labour is distributed in every branch for the other branches as well as for the final product. Meanwhile, it reflects the full labour consumption in each branch.

The following notations will be used in the equational representation:

X_i — the production in the i branch, monetarily expressed;

M_i — the labour direct consumption in the i branch, expressed in man-hour;

x_{ij} — the quantity from the i branch production, consumed in the production process of the j branch;

q_{ij} — the quantity from the i branch production, expressed in work units, consumed in the production process by the j branch for the working out of a good unit, monetarily expressed;

q_i — the whole labour expenditures (direct and through other branches) in order to obtain a good unit in the i branch;

m_i — the direct labour expenditures for the working out of a good unit in the i branch.

By means of the above exposed elements, the following indicators can be defined:

$$m_i = \frac{M_i}{X_i}; \quad q_{ij} = a_{ij} \cdot q_j$$

$$b_{ij} = \frac{x_{ij}}{X_j}; \quad a_{ij} = b_{ij} + c_{ij}$$

It was denoted by c_{ij} the coefficient that represents the specific consumption of means of work (writing off).

By means of the model elements the following system of equations can be written:

$$q_1 = a_{11}q_1 + a_{21}q_2 + \ldots + a_{n1}q_n + m_1$$

$$q_2 = a_{12}q_1 + a_{22}q_2 + \ldots + a_{n2}q_n + m_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$q_n = a_{1n}q_1 + a_{2n}q_2 + \ldots + a_{nn}q_n + m_n$$

In order to write the system of equations in a matricial form, it shall be denoted by A — the matrix of the direct expenditures of objects of work and means of work, by Q — the line-vector of the full labour expenditures, and by M — the line-vector of the direct labour expenditures, namely:

$$\left\| \begin{array}{l} a_{11}\, a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21}\, a_{22} \ldots\ldots\ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ a_{n1}\, a_{n2} \ldots\ldots\ldots a_{nn} \end{array} \right\|$$

$$Q = (q_1, q_2, \ldots\ldots, q_n)$$

$$M = (m_1, m_2, \ldots\ldots, m_n)$$

The system of equations is written in a matricial form as follows:

$$Q = QA + M$$

It follows that:

$$Q = M(E - A)^{-1}$$

wherein by E was denoted the unit-matrix.

Denoting by α_{ij} the elements of the inverse matrix $(E - A)^{-1}$, the following mathematical relations that provide the whole size of alive and materialized labour, consumed on a good unit in every branch or subbranch, monetarily expressed, are obtained:

$$q_j = \sum_{i=1}^{n} \alpha_{ij} m_i; \quad j = 1, 2, ..., n$$

The full labour expenditures for the working out of the production in the i branch are quantified in the following relations:

$$Q_i = q_i\, X_i = \sum_{j=1}^{n} \alpha_{ij} m_j X_i$$

The infered mathematical relations put in bold relief the functional dependence between the whole labour consumption in every branch and the size of the direct labour expenditures in all the goods production and services system components.

The falling down of the direct labour expenditures on good unit in a certain branch has good effects, that are spread in all the branches of the national economy. Truly, the infered mathematical relations show that the falling down with one unit of the direct labour expenditures in the i branch, leads to the falling down with α_{ji} units in the full labour consumption in the j branch. This falling down is an effect of the propagation phenomenon which takes place in the economic space, according to the objective feedback law. The effect of the falling down with one unit of the direct labour expenditures

in a certain i branch at the level of the national economy is quantificated in the following formula:

$$\theta_i = \sum_{j=1}^{n} \alpha_{ji}$$

The labour productivity analysis based upon the above indicators, obtained by processing the input-output model in work units, allows the *rendering evident of the directions of action in order to obtain the permanent falling down of the labour expenditure.*

The wide-scale promotion of technical progress, the utilization of improved technical methods, as well as a better organization of labour and production lead, apart from the saving of alive labour expenditures, to the falling down of the unitary consumption of raw materials. These, in their turn, because of the propagation phenomenon, lead to the labour productivity growth in all the branches of the economy and finally, in all the general cybernetic system.

In order to quantify the influence the falling down of the direct labour expenditures in a certain i branch has upon other branches of the economy, the following mathematical relations will be used:

$$\frac{\partial \alpha_{kl}}{\partial b_{ij}} = - \alpha_{ki} \alpha_{ji}$$

It follows that the effect, at the level of the whole national economy space, of the falling down with one unit of the unitary consumption b_{ij} in the i branch is quantified by the following relation:

$$E_{ij} = \sum_{k=1}^{n} \sum_{l=1} \alpha_{ki} \cdot \alpha_{jl}$$

The ever growth of labour productivity in the frame of the general cybernetic system, the most efficient utilization of labour resources are vectorial issues of many economical and social factors of outstanding complexity.

In order to identify the optimal growth trajectories of labour productivity in every general cybernetic system component and in the whole activity developing in the national economy space, the labour productivity system also uses other models that contribute to the settlement of a reasonable structure of the staff, to the scientific organization of production and labour, a.o.

The cybernetic analysis of the economic-mathematic models presented above reveal the outstanding importance of the labour productivity in the functioning mechanism of the material economy system.