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# Applying Morphological Analysis in the Control of Technical-Economic Structures

*The first part of the study presents the concepts on which the morphological analysis is based, as well as the main work phases with the steps of the optimizing algorithm, its aspects being materialized in the second part of the study by an application carried out in the tooling section of a big machine-building economic unit. The application regards the design of a new device for fastening the splinting tool to machine-tools.*

*Starting from the main constructive variants of the device components (some 960 possible combinations) with the aid of an established programme, the optimal solution is obtained for each of the five technical-economic criteria considered.*

The morphological analysis is based on the sets theory and allows for the solving of those problems implying a combination among several elements making up a whole, each element, in its turn, admitting several variants.

Ever since its working out, in 1942, the method of morphological analysis has greatly developed mainly due to its wide sphere of application for the designing of new, original products with optimal structural solutions, the choice of economical energy structures for an enterprise, section, etc., of optimal species of plants for agriculture, the choice of optimal transport networks, the choice of optimal educational structures [2], [8], [9], [11].

The method of morphological analysis consists in choosing the best combination, both technically and economically, for a certain product, process ($P$), subject to study.

We specify that $P$ is made up of "$n$" components: $C_1$, $C_2$, $C_3$, ... $C_i$, ..., $C_n$, each component admitting several constructive variants $v_{ij}$, where

"$ij$" indicates variant "$j$" of component "$i$", and $m_i$ represents the maximum number of variants of each component "$i$".

This can be synthetized as follows:

| Components | Constructive variants |
|---|---|
| $C_1$ | $V_{11}$ $V_{12} \dots V_{1j} \dots V_{1m_1}$ |
| $C_2$ | $V_{21}$ $V_{22} \dots V_{2j} \dots V_{2m_2}$ |
| $\vdots$ | $\vdots$ $\vdots$ $\vdots$ $\vdots$ |
| $C_i$ | $V_{i1}$ $V_{i2} \dots V_{ij} \dots V_{im_i}$ |
| $\vdots$ | $\vdots$ $\vdots$ $\vdots$ $\vdots$ |
| $C_n$ | $V_{n1}$ $V_{n2} \dots V_{nj} \dots V_{nm_n}$ |

Any product variant compulsorily contains all components (but of distinct variants).

We specify that each variant $V_{ij}$ is characterized by a vector ($i$ — the indices of the product component, $j$ — the variant of component "$i$" attached to it, of the form:

$$V_{ij} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \end{bmatrix}$$

where: $a_1$, $a_2$, ..., $a_k$, ... $a_q$, are absolute or relative sizes characteristic of each component variant (cost, precision, profit, etc.).

Generally, the morphological analysis is based on the following concepts:

**Definition 1.** Any possible combination of "$i$" elements variants of different $C_i$ components is called a combination of grade "$i$".

**Definition 2.** Any combination of incompatible "$i$" variants, (variants implying defficient resources, environment pollution, etc.) is called constraint of grade "$i$". The combination of grade "$i$" including combinations of lower, incompatible grades, is also incompatible. For example: the possible combination $V_{11}$ $V_{21}$ $V_{31} \dots V_{ij} \dots V_{in}$ is of grade "$n$" and is incompatible if, for instance, one of the combinations of variants $V_{21}V_{ij}$ or any other ($i = 1, n$) are incompatible.

**Definition 3.** Any possible, compatible combination of grade "$i$" is called feasible combination.

**Definition 4.** Those feasible combinations which best satisfy the multiple optimal criteria are called optimal solutions.

*The application of the morphological analysis method mainly implies the following work stages:*

— the clear and precise delimitation of the analysis object,

— the specification of the significant components of the analysis object (product, system, process, etc.),

— the partitioning of each component into possible variants,

— the analysis of the combinations of grade "2" and the drawing up of the list of constraints,

— the obtaining of the feasible solutions by a formal comparison of each possible combination of grade "*n*", with the list of constraints,

— the application of the optimal criteria on the dimensions characteristic of each feasible solution and the obtaining of the optimal solution.

The optimizing algorithm comprises the following steps:

**Step 1.** Compare the list of constraints of grade "2" to each possible combination of grade "n". If there is an empty intersection between the indices of the possible combination of grade "n" and the indices of the combinations of grade "2" on the list of constraints, this possible combination is feasible too (compatible).

**Step 2.** The feasible (structure solutions) combinations are each characterized by a vector made up of the same elements (dimensions) of the vectors characterizing each constructive variant taken separately, but now at the level of the structure solution (of the whole). The elements of the vector characterizing the feasible structure solution are obtained by aggregating the same elements (dimensions) of the vectors characterizing the variants of the product components.

Following this aggregation (implying diverse relations), "p" rows of dimensions characteristic of the "p" elements of the vectors attached to the feasible solutions (cost, weight, profit, etc.) are obtained.

Obviously, the "p" rows have the elements expressed in different measure units (lei, kg., % etc.).

It is necessary to bring the elements of these rows to the possibility of operating with them, with the results of logical significance.

**Step 3.** Establish the "hierarchy" or "position" of a feasible structure solution in relation to each of the "*p*" dimensions characterizing it, in each of the "*p*" rows. The maximum "position" will go for one of the "*p*" dimensions characterizing the feasible solution to that feasible structure solution fulfilling to the greatest extent the criterion implied by each dimension "*p*". The other feasible solutions receive "positions" lower to the latter.

If several feasible solutions fulfil one of the "*p*" optimal criteria, to the same extent or almost to the same extent, they form a class of equivalence and, consequently the same "position" will be assigned to them.

Thus, the elements of the vectors characterizing the feasible solutions will be replaced by their "positions", related to each of the "*p*" dimensions.

**Step 4.** Totalize the "positions" of the same feasible solution from the "*p*" rows of "positions" of the feasible solutions. A row of the "cumulated positions" for each feasible solution is obtained. These "cumulated positions" show the usefulness of each of the "*w*" feasible solutions related to the simultaneous satisfaction of the "*p*" optimal criteria (the cost implying the minimal criterion, the profit implying the maximal criterion, etc.).

The algorithm is based on the mathematical model presented below for which the following adnotations are made:

$i$ — the index of a component of object $P$,

$r$ — the index of a dimension characterizing each variant $V_{ij}$ of component "$i$",

$l$ — the index of the feasible solution,

$n$ — the maximum number of components of object $P$,

$p$ — the maximum number of elements of the vector attached to variant $V_{ij}$ (or, of optimal criteria),

$w$ — the maximum number of feasible solutions,

$a_{ir}$ — element (dimension) of the vector characterizing each variant of component "$i$" after dimension "$r$",

$S_{1r}$ — dimension of the feasible solution related to one of the elements of vectors $V_{ij}$ characterizing each variant of the components of the feasible structure solution "1",

$S_{max}, S_{min}$ — the maximum and minimum value of row $\{S_{lr}\} l = 1, w$ for a certain $r = 1, p$.

$D$ — the length of the equivalence interval for the optimal criterion "$r$" of the feasible solutions,

$D_z$ — point marking the passage from a class of equivalence (snop) to another between $S_{max}$ and $S_{min}$ for the same "$r$",

$D_{z+1}$ — point marking the passage from an equivalence class to another placed on the left or on the right of a certain feasible solution, just as the optimal criterion implies maximum or minimum,

$z$ — the number of equivalence classes (of snops) for criterion "$r$" of the feasible solutions,

$US_{l1}$ — "position" (utility) of solution "1" for each of the criteria "$r$",

$US_1$ — "cumulated position" (general utility) of solution "1" for the "$p$" dimensions in achieving the solution,

$US_0$ — "cumulated position" going to the optimal solution in the simultaneous achievement of the "$p$" optimal criteria.

The mathematical model is of the form:

For the calculus of $S_{1r}$ consider the aggregation relations:

$S_{1r} = \sum_{i=1}^{n} a_{ir}$ — if the aggregation relation of dimensions "$a_{ir}$" for $r \in (1, p)$ is of additivity,

$S_{1r} \prod_{i=1}^{m} a_{ir}$ — if the aggregation relation of "$a_{ir}$" for $r \in (1, p)$ is of product,

$S_{1r} = mn \{a_{ir}\}_{i=1m}$ — if the aggregation relation of „$a_i r$" for $r \in (1, p)$ is of minimum.

$S_{min} = min \{S_{1r}\}\ 1 = 1, w$

$$S_{max} = max\,\{S_{1r}\}\ 1 = 1,\ w \qquad \text{for each "}r\text{" of } p\ (1,)$$

$$D = \frac{S_{max} - S_{min}}{w - 1}$$

$$D_z = \begin{cases} S_{min} + z \times D & \text{for the minimum criterion,} \\ S_{max} - z \times D & \text{for the maximum criterion.} \end{cases}$$

then:

$$US_{1r} = w - z, \qquad \text{if:}$$

$S_{1r} \geqslant D_z$ and $S_{1r} < D_z + 1$ for the minimum criterion,

$S_{1r} > D_z + 1$ and $S_{1r} \leqslant D_z$ for the maximum criterion.

$$US_1 \sum_{r=1}^{P} US_{1r}$$

$$US_0 = max\,\{US_1\}\ 1 = 1,\ w \text{ or } US_0 = max\,\{US_{1r}\}\ 1 = 1,\ w$$

If: $US_0 = US_1$, where 1 (1, $w$), the optimal solution has the index "1". If this occurs several times we have multiple optimal solutions.

The optimal structure solution or the multiple optimal structures are presented in the end.

The concrete practical problems require at certain time intervals the repetition of the presented algorithm to obatin a new product structure in the event of modifying the list of constraints and of the dimensions characterizing each constructive variant.

The repetition of the algorithm, whenever the case is, requires the covering of the following phases:

— the up dating of the list of constraints (completion, replacement),

— the modifying of the dimensions characteristic of each constructive variant (adding new constructive variants, adding new components),

— the repetition of the optimizing algorithm for establishing the optimal solution in the new conditions of application,

— the comparison of the new optimal solution with the old optimal solution; following the analysis the decision is made whether the changing of the previous optimal solution is opportune or not,

— the explicite presentation of the new structure solution.

The use of the electronic computer in applying this method allows for the grasping in the study of all variants even of the less known ones, the finding of the optimal solutions with exactness and rapidity, the updating of the optimal solutions by new morphological analyses and optimizations.

We propose to exemplify those presented above in an application carried out at the tooling section of a big machine-building economic unit where the problem was raised to design a new device for fastening the splintering tool to machine tools. The device must fulfil a number of conditions for each of its components. Decision makers have provided for analysis the main constructive variants of the device components, synthetized in table 1.

TABLE 1

| COMPONENTS | CONSTRUCTIVE VARIANTS |
|---|---|
| 1 | 2 |
| $A$ = body of the device (material used) | $a_1$ = steel<br>$a_2$ = steel + plastics<br>$a_3$ = nodular cast-iron<br>$a_4$ = nodular cast-iron + plastics |
| $B$ = subsystem of fastening with the semifab incompatibilities: $a_2b_1$, $a_3b_2$. | $b_1$ = mechanical fastening with screw<br>$b_2$ = mechanical fixing by pressing<br>$b_3$ = elastic fixing (spring, bushing)<br>$b_4$ = hydraulic fastening<br>$b_5$ = pneumatic fastening |
| $C$ = subsystem of fastening with the machine tool incompatibilities: $a_2c_1$, $a_4c_1$, $b_1c_1$, $b_4c_4$. | $c_1$ = mechanical fastening with clips and screws<br>$c_2$ = mecanical fixing by pressing<br>$c_3$ = elastic fastening<br>$c_4$ = magnetic fastening |
| $D$ = subsystem of joint with the splintering tool constraints: $a_3d_3$, $b_1d_4$, $c_2d_3$, $b_2e_3$. | $d_1$ = coordinated axles<br>$d_2$ = guiding columns<br>$d_6$ = guiding bus-hing<br>$d_4$ = mechanical transmission |
| $E$ = mechanizing or automation subsystem constraints: $d_4e_1$, $b_3e_2$, $c_1e_1$, $a_3e_3$ | $e_1$ = device with cams<br>$e_2$ servo-motor<br>$e_6$ = digit control |

It results that the total number of possible combinations

$$(NCP) = 4 \times 5 \times 4 \times 4 \times 3 = 960$$

Five technical-economic criteria have been selected, namely:

1. cost (in lei)
2. weight (in kg.)
3. gauge (cubic cm.)
4. durability (in years)
5. reliability (in %)

Corresponding to the variants of component $A$, matrix $C_1$ was attached to them for their characterization, each column of matrix $C_1$ characterizing entirely the variants of component $A$ and $C_2$, $C_3$, $C_4$, $C_5$ are matrices characteristic of components $B$, $C$, $D$, $E$:

$$C_1 = \begin{array}{c|cccc} & a_1 & a_2 & a_3 & a_3 \\ 1 & 100 & 60 & 6.5 & 30 \\ 2 & 40 & 20 & 3.8 & 1.8 \\ 3 & 35 & 40 & 50 & 40 \\ 4 & 8 & 6 & 5 & 4 \\ 5 & 0.80 & 0.35 & 0.45 & 0.26 \end{array}$$

$$C_2 = \begin{Vmatrix} 40 & 50 & 44 & 80 & 70 \\ 0.5 & 1 & 1.2 & 4 & 3 \\ 40 & 20 & 15 & 50 & 45 \\ 8 & 6 & 5 & 9 & 7 \\ 0.20 & 0.45 & 0.48 & 0.85 & 0.92 \end{Vmatrix} \quad C_3 = \begin{Vmatrix} 40 & 50 & 44 & 100 \\ 0.5 & 1 & 1.2 & 2 \\ 40 & 20 & 15 & 10 \\ 8 & 5 & 5 & 7 \\ 0.20 & 0.45 & 0.28 & 0.75 \end{Vmatrix}$$

$$C_4 = \begin{Vmatrix} 25 & 25 & 20 & 30 \\ 0.6 & 0.6 & 0.5 & 0.8 \\ 20 & 15 & 10 & 30 \\ 6 & 8 & 7 & 9 \\ 0.60 & 0.80 & 0.70 & 0.72 \end{Vmatrix} \quad C_5 = \begin{Vmatrix} 800 & 7000 & 13000 \\ 1.2 & 2 & 3 \\ 80 & 50 & 80 \\ 6 & 7 & 9 \\ 0.10 & 0.42 & 0.80 \end{Vmatrix}$$

Figure 1 presents the logical diagram of the optimizing algorithm: the programme was worked out in the FORTRAN language and was run on the FELIX C-256 computer.

Following the running of the programme, the following results were obtained:

POSITION OPTIMAL SOLUTION X (292) ACCORDING TO CRIT. 1 IS 349; cost 895 lei
POSITION OPTIMAL SOLUTION X (292) ACCORDING TO CRIT. 2 IS 350; weight 6.9 kg
POSITION OPTIMAL SOLUTION X (292) ACCORDING TO CRIT. 3 IS 345; gauge = 155 cm³
POSITION OPTIMAL SOLUTION X (292) ACCORDING TO CRIT. 4 IS 329 Durability = 4 years
POSITION OPTIMAL SOLUTION X (292) ACCORDING TO CRIT. 5 IS 350; Reliability = 0.012

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INDEX OF OPTIMAL SOLUTION IS 292

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NUMBER OF POSSIBLE COMBINATIONS = 960
NUMBER OF FEASIBLE COMBINATIONS = 351
NUMBER OF OPTIMAL CRITERIA = 5
GRADE OF ABSOLUTE UTILITY = 1755
GRADE OF UTILITY OF OPTIMAL SOLUTIONS = 1723
MINIMAL REGRET = 32

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$$X(292) = \begin{matrix} A(4) \\ B(1) \\ C(4) \\ D(2) \\ E(2) \end{matrix}$$

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The analysis of the results shows that there is a close resemblance between the "absolute" and the "heuristic" optimal obtained by calculus. The utility of the optimal solution differs from the absolute one only by a "regret" equal to 32.

The programme also provides the optimal structure of the feasible and optimal solution. Thus, it results that the body of the device will be designed out of nodular cast-iron + plastics ($A_4$); the subsystem of joint with the semifab will be carried out by mechanical fixing with screws ($B_1$); the subsystem of joint with the machine tool will be achieved by magnetic fixing ($C_4$); the fixing with the splinting tool will be achieved through guide columns ($D_2$); finally, automation will be ensured with the aid of a servo-motor ($E_2$).

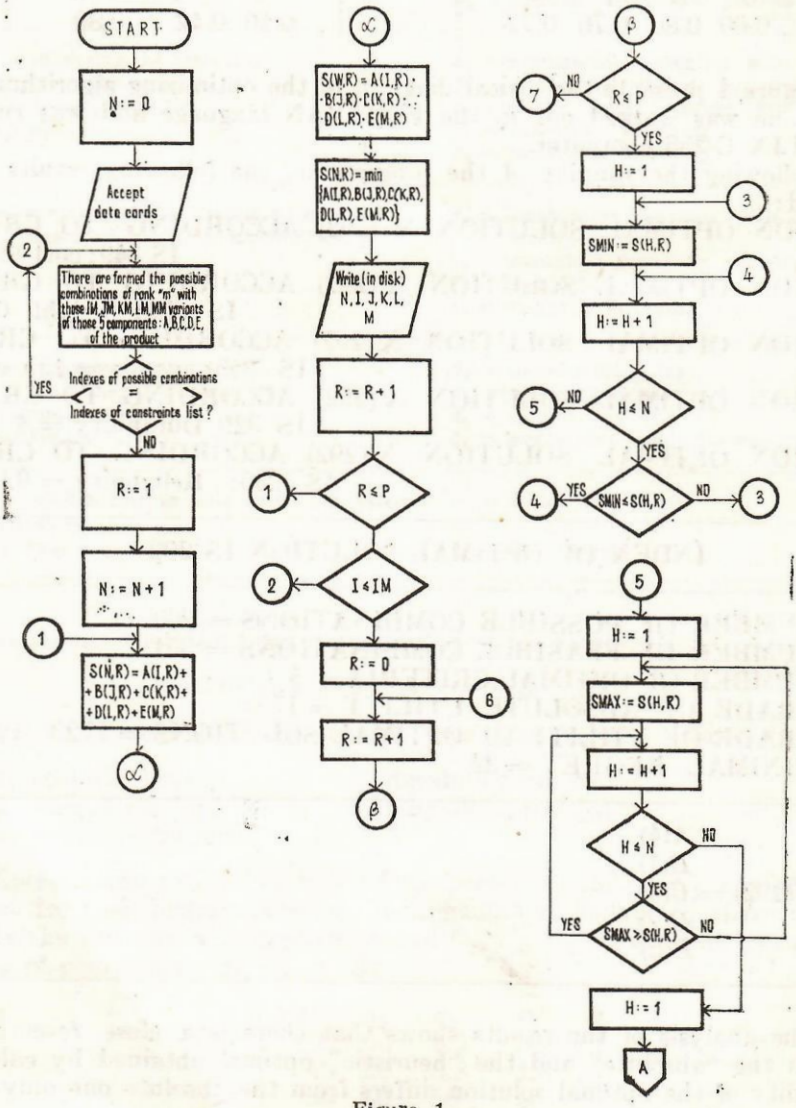



Figure 1.

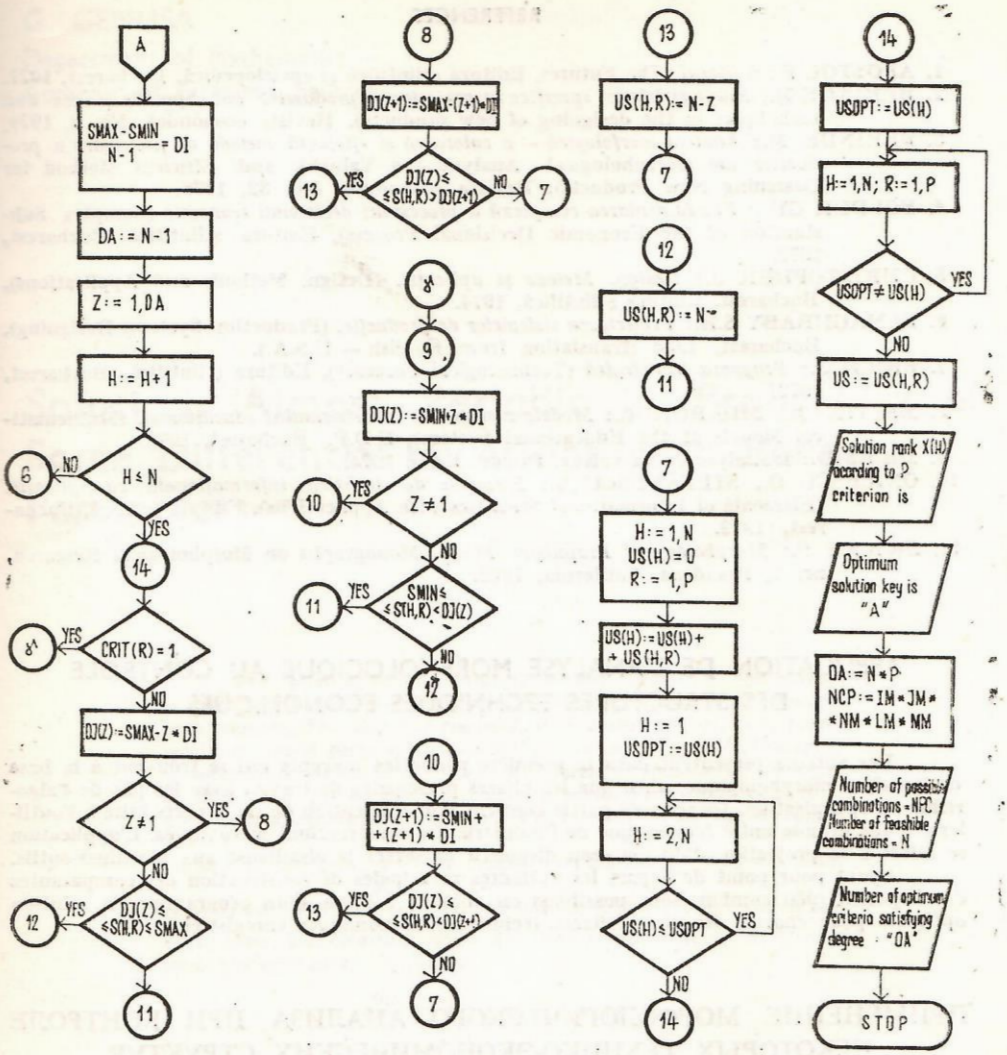



Figure 1 (continuation)

The application is being implemented in the analysed tooling section, the results being of real use to the technical-economic decision makers of the economic unit.

The optimizing algorithm which we present may be extended with very good economic effects in the machine-building industry with a big serial or mass production.

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## APPLICATION DE L'ANALYSE MORPHOLOGIQUE AU CONTROLE DES STRUCTURES TECHNIQUES ECONOMIQUES


Les auteurs présentent dans la première partie les concepts qui se trouvent à la base de l'analyse morphologique, ainsi que les phases principales de travail avec les pas de l'algorithme d'optimisation. La seconde partie contient une application de ces aspects faite à l'outillerie d'une grande unité économique de l'industrie des constructions mécaniques. L'application se réfère à la projection d'un nouveau dispositif de serrer la cisailleuse aux machines-outils.

Ayant pour point de départ les variantes principales de construction des composantes du dispositif (960 combinaisons possibles) on obtient, à l'aide d'un programme, la solution optimale pour chacun des cinq critères techniques économiques enregistrés


## ПРИМЕНЕНИЕ МОРФОЛОГИЧЕСКОГО АНАЛИЗА ПРИ КОНТРОЛЕ НЕКОТОРЫХ ТЕХНИКО-ЭКОНОМИЧЕСКИХ СТРУКТУР


В первой части работы представляются понятия на которых ссновывается морфологический анализ, а также основные рабочие этапы и шаги оптимизирующего алгоритма, а затем, во второй части, конкретизируются анализируемые аспекты при помощи эксперимента, проведённого в инструментальном цехе крупного машиностроительного предприятия. Это относится к проектированию нового приспособления для прикрепления режущего инструмента к станкам.

Исходя из основных вариантов построения компонентов этого приспособления (960 возможных комбинаций), определяется оптимальное решение по каждому из пяти технико-экономических критериев при помощи специально разработанной программы.



26.XI.1979