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Determining a Steel Rolling Programme with Stock Constraints at Heat Treatment*)

In the present study, the authors formulate a production programming model for a rolling section and present a way for its solving.

INTRODUCTION

Considering the order list of products for a given period (for example a month), the change over time matrix from an order to another on the rolling line and the specifications of the thermic treatment, the problem posed is to determine the succession of the rolling of orders by taking into account the stock constraints at the thermic treatment furnaces with a view to minimizing the rolling mill waiting time. This problem can be formulated as a travelling talesman problem (TSP) with supplementary stock constraints. The classical sravelling salesman problem belongs to the class NP-complete defined by Karp [1] a reason why the use of the Branch and Bound type algorithm with the increase in the size of the problem becomes still more inefficient.

In the following, the authors intend to present the model of the above problem and give a method of efficient solving. It is based on the method of the facetes type cutting planes [6] successfully tested and extended in the last time by Grötschel [4] and Padberg-Hong [7].

DEFINITION OF THE PROBLEM. MODEL

We consider a rolling line and a group of thermic treatment furnaces. The rolling line produces rolled products of various types and dimensions which must or not be treated thermically.

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For the rolling line, we know the change over time matrix from a type and dimension to another and the line productivity for each type and dimension. The limits of the minimum and maximum stock necessary for the furnaces for ensuring the continuity of the treatment process (in case that the rolling process is interrupted) are also given.

The list of orders to be rolled contains the steel trade-marks, the type and dimension, the delivery state (treated, not treated) the quantity in tons for each rolled product.

The arising problem is to determine a rolling programme with stock constraints for the treatment furnaces with a view to minimizing the line waiting time.

In order to model the enunciated problem, the initial orders are divided into groups and fractions so that each newly resulting order should require approximately the same rolling time. The selection of the common time unit should be made so as to obtain a reasonable dimension model (maximum 200—300 orders).

Further, a few adnotations necessary for the formal writing of the model are introduced:

$|N|$ — number of orders in the programming period including the order in hand

x_{ij} — binar variable assuming value 1 if order j follows after order i in rolling and 0 otherwise

X — matrix formed with elements x_{ij}

Π — set of all cyclic permutations of $|N|$ orders the order in hand is every time the first order.

π — a cyclic permutation of set Π

r — number of order in hand upon scheduling for the next period

R — a chosen multiple of the time unit (for example, equivalent with a day)

P — number of how many times R enters the programming period

S_{min}, S_{max} — minimum and maximum stock in tons necessary for minimum p' units of R

a_i — quantity of product i in tons rolled in the time unit

c_{ij} — time change over time from order i to order j. $(c_{ii} = 0)$

Consider $a_i' = \begin{cases} a_i - \text{quantity of product } i \text{ requiring thermic treatment} \\ 0 - \text{otherwise} \end{cases}$

and the recursive function

$$\omega(0) = r$$

$$\omega(s) = \underbrace{\pi(\ldots \pi(r))}_{s}, \qquad s > 0$$

The constraints of the model are:

$$\sum_{j=1}^{|N|} x_{ij} = 1, \quad i = 1, ..., |N| \tag{1}$$

$$\sum_{i=1}^{|N|} x_{ij} = 1, \quad j = 1, ..., |N|. \tag{2}$$

(1), (2) express the fact that each order should de followed and preceded exactly by an order.

$$\sum_{i \in S} \sum_{j \in N \setminus S} x_{ij} \geqslant 1, \quad \forall S \neq 0, \quad S \subset N \left(2 \leqslant |S| \leqslant \left[\frac{|N|}{2} \right] \right) \tag{3}$$

(3) are the loop constraints deducted from the constraints of eliminating the subtours of the form

$$\sum_{(i,j) \subset S \times S} x_{ij} \leqslant |S| - 1$$

the stock constraints for treatment furnaces are

$$S_{min} \leqslant \sum_{s=pR}^{R(p+p')} a'_{\omega(s)} \; x_{\omega(s), \omega(s+1)} \leqslant S_{max}, \; p = 1, ..., P, \quad \pi \in \Pi. \tag{4}$$

The objective function of the model defined by constraints (1), (2), (3), (4) and obviously $x_{ij} \in \{0,1\}$, for $\forall i, \forall j$ is

$$\min_{\pi \in \Pi} f(x) := \sum_{i=1}^{|N|} C_i, \; \Pi(i). \tag{5}$$

SOLVING THE MODEL

The travelling salesman problem formulated 45 years ago was the object of intense investigations in the past three decades. The interest aroused by this problem is due, on the one hand, to the wide range of concrete applications which can be written under this form and, on the other hand, to its complexity. Survey papers on this problem are those of Grötschel [4] and Burkard [5].

The method given in 1954 by Dantzig, Fulkerson and Johnson [6] resumed by Edmonds [3], Chvatal [2], Grötschel [4] and Padberg-Hong [7] appears as the most adequate one for solving large-scale (TSP) problems. It starts from a relaxation of the travelling salesman problem (for example the allocation problem) to which facetes type cutting planes are added iteratively (maximum sized-facetes of the problem polytope).

To define the sides used [4], we introduce several adnotations:

Consider $E = \{(i, j) \mid i \in N, \quad j \in N, \quad i \neq j\}$

$$E(S) = \{(i, j) \in E \mid i \in S, \; j \in S, \; S \subset N\}$$

and

$S_k \subset N$, $k = 1, ..., k$ satisfying properties

a) $|S_0 \cap S_k| \geqslant 1$ for $k = 1, ..., K$

b) $|S_k - S_0| \geqslant 1$ for $k = 1, ..., K$

c) $|S_k \cap S_j| = 0$ for $1 \leqslant k < j < K$.

We call a "comb" $S = \bigcup_{k=0}^{K} E(S_k)$ with the "handle" S_0 and the "teeth" S_k, $k = 1, ..., K$.

The general form of the facetes also called inequalities "comb" is:

$$\sum_{k=0}^{K} \sum_{(i,j) \in S_k \times S_k} x_{ij} \leqslant |S_0| + \sum_{k=1}^{K} (|S_k| - 1) \left[\frac{k}{2} \right]. \tag{6}$$

These results are at the basis of the algorithm proposed by us in order to solve the given model.

We shall try to replace constraints (4) in the model which are not facetes by other constraints having this characteristic.

We denote by $\overline{\Pi}$ the set of cyclic permutations which do not satisfy constraints (4).

The system built with constraints (4) is equivalent with the system

$$\sum_{i=1}^{|N|} x_{i, \pi(i)} \leqslant |N| - 1, \quad \pi \in \overline{\Pi}. \tag{7}$$

Constraints (7) are simpler than (4) but their number is generally larger, $\overline{\Pi}$ is not known explicitely and at the same time they are not facetes.

In solving the problem while obtaining a cyclic permutation π, we can determine the set

$\bar{P} := \{p$ for π fixed and (4) unsatisified$\}$

For each $p \in \bar{P}$ we can attach a constraint given by inequality

$$\sum_{s=pR}^{R(p+p')} x_{\omega(s), \omega(s+1)} \leqslant Rp' - 1 \tag{8}$$

which eliminates the chain which does not satisfy constraint p of system (4).

Constraints (8) are facetes, see for example [7].

Using the results given in [7] and the above remark, an algorithm is formulated to solve the proposed model.

The algorithm starts with a known initial solution given by π^0.

This solution can be improved through the heuristic algorithm of Lin and Kernighan [8] (the most effective heuristic algorithm known).

The algorithm consists in:

a) generating random a (circuit) solution and determining $f_\pi(X)$;

b) trying to improve $f_\pi(X)$ by transformation, resulting X' and $f_\Pi(X')$;

c) if $f_{\Pi'}(X') < f_\pi(X)$ go to b) otherwise continue.

d) a local optimum was obtained and the process can be continued by recalling steps a)—c).

The solution given by π° is improved by using step *b* and *c* of the above algorithm where in step *b* we select only those transformations which also satisfy constraints (4). In this consists the modification of the Lin-Kernighan algorithm.

The travelling salesman algorithm with stock constraints

Step 1 Solve the model without constraints (3) and (4) with the Lin-Kernighan modified algorithm and denote by T the value of the objective function. Improve if it is possible the initial solution attached to π° (modified Lin-Kerninghan) and the value of the objective function T^m is resulting, otherwise $T^m = 0$ and go to step. 7.

Step 2 Solve by linear programming the relaxed problem (by eliminating constraints (3) and (4) and conditions $x_{ij} \in \{0,1\}$.

The value of the objective function is denoted by V_1. Put $V_2 := V_1$.

Step 3 If the solution is a Hamiltonian circuit go to step 4, otherwise go to step 5.

Step 4 If the solution satisfies constraints (4) stop, the optimal solution was obtained, otherwise if $T^m \neq 0$ generate constraints (8) and go to step (6), otherwise go to step 7.

Step 5 Calculate the stopping index $I = \dfrac{V_2 - V_1}{T^m - V_1}$ .if $I > I'$ (established a priori) go to step 7, otherwise generate the cutting plane constraints given by Grötschel [4] and add them to the previous problem.

Step 6 Solve the problem expanded by the constraints generated at step 4 or 5 as a linear programming problem. Denote the value of the objective function by V_2 and go to step 3.

Step 7 Solve the travelling salesman problem with a branch-and-bound algorithm with very good bounds T^m and V_2. If the problem has no stop solutions, the optimal solution is T^m (if $T^m \neq 0$), or the problem has no solutions (if $T^m = 0$), otherwise go to step 4.

By changing the Lin-Kerninghan algorithm and by adding step 4 to the Grötschel algorithm [4] based on the remark on inequality (8) a way of solving the travelling salesman problem with stock constraints was proposed.

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DETERMINATION D'UN PROGRAMME DE LAMINAGE DE L'ACIER AYANT DES RESTRICTIONS DE STOCK AU TRAITEMENT THERMIQUE

Les auteurs présentent dans cette étude un modèle de programmation de la production pour une tôlerie, ainsi que la manière de résoudre le modèle.

ОПРЕДЕЛЕНИЕ ПРОГРАММЫ ПО ПЛЮЩЕНИЮ СТАЛИ С УЧЕТОМ ОГРАНИЧЕНИЙ ПО ЗАПАСАМ

В настоящей разработке авторы формулируют модель программирования производства в прокатном цехе, а также способ разрешения модели.

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