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Determination of the Optimum Operating Time of Equipment

"The improvement of knowledge and analysis methods in the field of economics may be achieved through the introduction on an ever broader scale of modern computation techniques." *

According to the principle of planning optimization, which essentially consists of obtaining maximum economic returns with minimum expenses of social labour, the determination of the optimum useful life of equipment is an important methodological factor of planning optimization in increasing the economic efficiency of basic funds and of the general productive activity.

The methodology we propose is based on the concept that the useful life of equipment depends, on the one hand, on the combined effect of physical and moral wear factors and, on the other hand, on the extrapolation of certain general trends whose importance is determined by the particular type of equipment involved. Actually, the longer the operating time of equipment is, the greater the expenses required in order to counterbalance physical wear are; in addition, the time factor contributes to moral wear, caused either by the tendency towards a diminishing reproduction value of equipment of similar construction, due to increased labour productivity in the industrial branches of machinery and equipment production (moral wear, type I), or by the development of improved and more efficient machinery (moral wear, type II).

Inasmuch as under the present technical and scientific revolutionary conditions science and technology are the main potential factors of increasing labour productivity, there is a close connection between these two types of moral wear. The speeded up improvement of technological processes and

* The Directives of the Central Committee of the Romanian Comunist Party in view of improving the national economy management and planning.

of equipment leads to an increased productivity of labour and, consequently, as a general phenomenon, to a faster moral wear. The combined effect of this moral wear process, as influenced by technical progress, is expressed through the tendency of a more rapid increase in equipment efficiency than of value.

This is why in the long run, the operation of machinery and equipment is ultimately determined by economic considerations rather than by technical ones. Thanks to maintenance and repairs, the physical duration of equipment is practically unlimited and therefore, the economic criterion is the essential limiting factor in the operating time of equipment.

We therefore distinguish between the physical operating time and the economic operating time. This is why, frequently, equipment still in good working condition is removed and replaced by new models.

In order to obviate as much as possible such phenomena which interfere with the efficient utilization of social labour as expressed in equipment, the attempt to determine an optimum operating time for equipment becomes increasingly important and timely.

PHYSICAL WEAR, AN IMPORTANT FACTOR FOR THE DETERMINATION OF AN ECONOMIC OPERATING TIME

Leaving the moral wear factors aside, the determination of the optimum operating time for equipment is for the time being made only on the basis of physical wear.

Physical wear leads not only to the depreciation of equipment, but also to the decrease of its technical and economic operating parameters. The preventive maintenance of equipment is meant precisely to restore its technical and economic parameters to a level approaching as much as possible the original state. Preventive maintenance applied at predetermined intervals, prolongs the useful life of equipment. When these operations are accompanied by technical improvements or by modernizing measures, the effect of moral wear is also somewhat counteracted.

As a rule, the "ageing" of equipment brings about a decrease in its efficiency and an increase not only in the cost of repairs, but also in working and operating costs.

Under these circumstances, when there is a pre-established programme of operation and repairs, it is possible also to calculate the optimum operating time for equipment, beyond which its utilization would lead to decreased returns from operating and maintenance and repairs costs.

In order to draw up the model for the determination of the optimum equipment operation time, taking into consideration only physical wear, we let P_0 stand for the purchasing price of a piece of equipment, and envisage that 1, 2, ..., x, ..., n repair cycles are provided, in view of obtaining the output $Q_0, Q_1, \dots, Q_x, \dots, Q_n$. *

* It may represent the number of efficiency units (number of oscillations per hour at the weaving loom; T/h/m^2 operation of the blast furnace, etc.) expressed in various units.

As a result of physical wear, it is usually found that

$$Q_0 \geqslant Q_1 \geqslant \ldots \geqslant Q_x \geqslant \ldots \geqslant Q_n \tag{1}$$

i.e., the output obtained during successive repair cycles generally decreases, and only at best remains at a constant level.

During the repair cycles, the sums spent on equipment operation, maintenance and current repairs, are

$$K_0, K_1, \ldots, K_x, \ldots, K_n$$

while through repairs cost

$$K'_0, K'_1, \ldots, K'_x, \ldots, K'_n.$$

Under present planning and financial practice, a lump sum of money assigned for repairs is equally distributed for all cycles, their administration and laying out for individual pieces of equipment to be repaired being made by the plant, depending on the respective given conditions.

Taking into account that operating, maintenance and current repair costs, as well as thorough repair costs generally never decrease but are increasing for one and the same piece of equipment, and are, at best, remaining at a constant level, the following relations are true:

$$K_0 \leqslant K_1 \leqslant \ldots \leqslant K_x \leqslant \ldots \leqslant K_n \tag{2}$$

$$K'_0 \leqslant K'_1 \leqslant \ldots \leqslant K'_x \leqslant \ldots \leqslant K'_n \tag{3}$$

Consequently, at the end of the repair cycle x, the total expenditures referring to equipment purchasing, amount to

$$C_x = P_0 + \sum_x K_x + \sum_x K'_x = P_0 + \sum_x (K_x + K'_x) \tag{3'}$$

Figure 1 shows the variation of total expenditures.

The sum total of expenditures for the respective piece of equipment at the end of each cycle is represented by: the ordinate $\overline{AA'}$ after cycle 1, the ordinate $\overline{BB'}$ after cycle 2, etc.

The sum total of output at the end of repair cycle x is $\sum_x Q_x$.

The ratio of the sum total of expenditures to the sum total of output is designated as the *specific expense coefficient:*

$$p_x = \frac{P_0 + \sum_x K_x + \sum_x K'_x}{\sum_x Q_x} = \frac{C_x}{\sum_x Q_x} \tag{4}$$

which can also be written as:

$$p_x = \frac{P_0}{\sum_x Q_x} + \frac{\sum_x K_x}{\sum_x Q_x} + \frac{\sum_x K'_x}{\sum_x Q_x} \tag{4'}$$

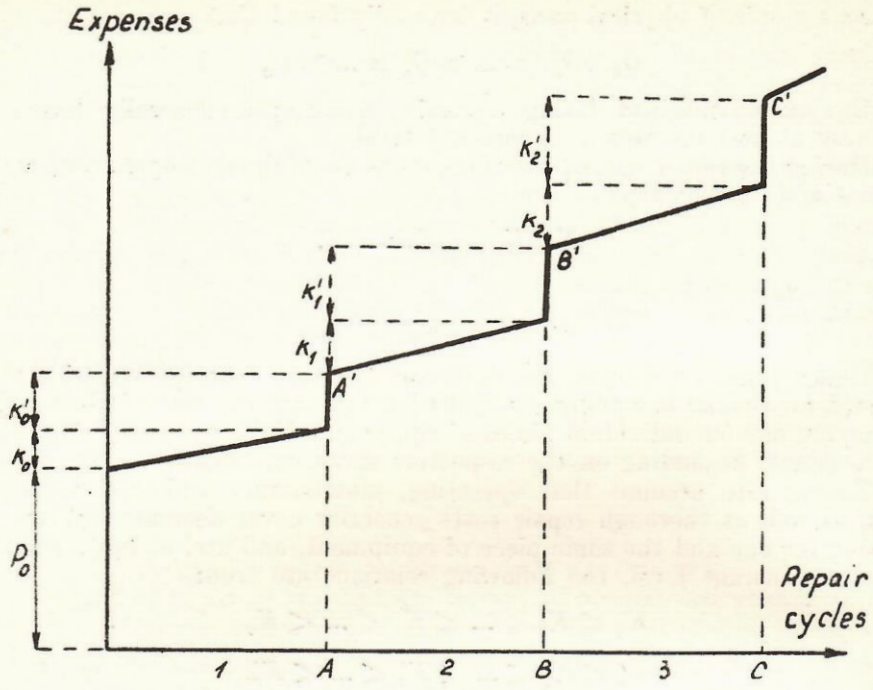

Fig. 1

This coefficient is indicating, for each cycle, the amount of expenses per unit of output, and is changing with time, as a result of the combined effect of the following factors: rate of output decrease and rate of expense increase, related to maintenance, operating and repairing equipment.

It is clear that the expenditures for the equipment are efficient until the specific expense coefficient, p_x, decreases, i.e. $p_x < p_{x-1}$. As soon as the coefficient increases, i.e. $p_x > p_{x-1}$, the expense efficiency starts to decrease. The optimum operating time of equipment is that corresponding to the minimum value of the coefficient.

Before determining the value of this optimum time, we will investigate variation of coefficient (4) and of its three constituent parts, defined in relation (4').

If the following condition is fulfilled:

$$Q_0 = Q_1 = \ldots = Q_x = \ldots\ldots = Q_n = Q$$
$$K_0 = K_1 = \ldots = K_x = \ldots\ldots = K_n = K$$
$$K_0' = K_1' = \ldots = K_x' = \ldots\ldots = K_n' = K'$$

meaning that expenses are constant for all repair and production cycles, expression (4) becomes:

$$p_x = \frac{P_0}{xQ} + \frac{K}{Q} + \frac{K'}{Q} \qquad (5)$$

As $\frac{K}{Q}$ and $\frac{K'}{Q}$ are constant in all repair cycles, coefficient p_x decreasing with the increasing number of cycles, asymptotically tends towards $\frac{K}{Q}+\frac{K'}{Q}$ (Fig. 2).

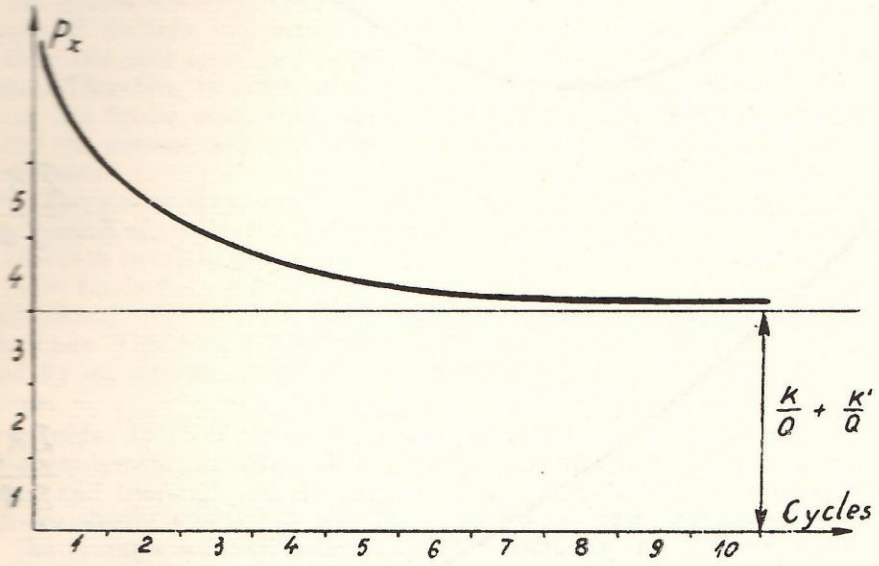

Fig. 2

In this case, coefficient p_x is not a minimum.
When the conditions

$$Q_0 > Q_1 > \ldots > Q_x > \ldots > Q_n$$
$$K_0 < K_1 < \ldots < K_x < \ldots < K_n$$
$$K'_0 < K'_1 < \ldots < K'_x < \ldots < K'_n$$

are prevailing, Fig. 3 shows the variation of the specific expense coefficient, p_x and of its constituents.

The optimum operating time of equipment is easily obtained on the basis of equation (4) or (4'). For this purpose it is necessary to know exactly the values of P_0, Q_0, K_0 and K'_0, as well as the rates of time variation of output and expenditures, which are replaced in equation (4) or (4').

The analytical examination of equation (4) may be achieved through derivating; by nullifying the derivative, the cycle where coefficient p_x is a minimum, is obtained. We shall apply this procedure later, for the analysis of the effect of moral wear.

Thus, if we consider only the effect of physical wear on a piece of equipment in operation, the minimum value determined by equation (4) is able to indicate the critical moment subsequent to the date when the replacement

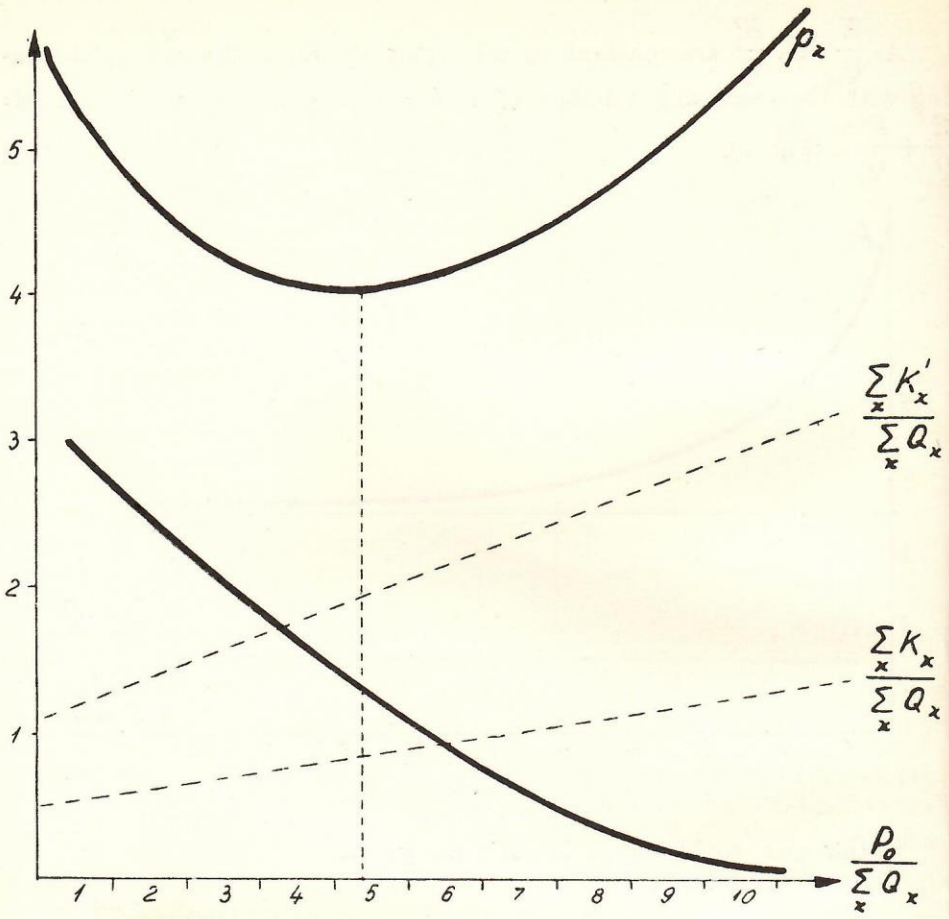

Fig. 3

of that equipment is imperative. This defficiency is removed by taking into account the effect of the time factor on maintenance and repair expenses.

ACTUALIZATION OF EXPENSES UNDER THE INFLUENCE OF THE TIME FACTOR

From the methodological standpoint, the sole consequences of physical wear are not sufficient for the optimum operating time measurement. Under present conditions, when science has become an immediate production force and technology is witnessing spectacular developments, the economic criteria have ever increasing effects on the optimum operating time of equipment. With a view of establishing an optimum operating time of equipment, it is necessary to take into account the time factor and the effects of both types of moral wear, I and II.

It is a well known fact that the time factor bears an important influence on economic activity indexes. The effect of future activities is not the same as that of the present ones. The future effect (output) is not equivalent to the present effect (output), as the value is changing. As a rule, it is decreasing, due to the increase of social labour productivity. The increase of labour productivity explains the cause of value reduction, but it does not explain why the future working expenses are less than present expenses for the same purpose. If we take into account the time factor, the present or future are less than the past ones, and consequently, the present or future have to be reduced. Therefore, in order to bring the past expenses to date with the present or the future ones, they have to be reduced and conversely, in order to level the present or future expenditures with the past ones, they have to be increased.

As the money spent for maintenance and repairing equipment results in the extension of the operating time of equipment, it is essentially representing the amounts spent in advance for the preservation of the use value and of the basic funds. * In our opinion, this means a preservation of value for an additional time interval through the extension of the operating time of basic funds. This means a money "stagnation" whose effect may be appreciated by an interest which the government also collects for the credits it bestows.

In order to compute as accurately as possible the optimum operating time of equipment, the effect of the time factor on the operating, maintenance, current and thorough repair expenditures is therefore taken into account, by materializing it in the form of the interest paid for the money spent.

The example we shall consider below refers to the calculation of the specific expense coefficient for a ring in a spinning mill, whose output is expressed in natural units (kg. thread, having the metric number 34).

The following data were obtained from the factory accounts:

— purchasing price (P_0) = 316,000 lei

— initial productivity (Q_0)

— maintenance and current repair expenses during the first cycle (K_0)

— thorough repair expenses after the first cycle (K_0')

— rate of average decrease of productivity, as a consequence of physical wear (r_0) = 10%

— rate of average increase of maintenance and current repair expenses (r_3) = 10%

— rate of average increase of thorough repair expenses (r_3') = 15%

— the interest for investments in view of preserving the initial value of basic funds (d) = 5%.

Table 1 shows the arrangement of computations.

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* The concept prevailng among economists is that the repair expenses are in fact expenses for the restoration after physical wear. In our opinion, the meaning of expenditures is to preserve the original use value. (In both cases we leave aside the modernization work carried out with the money destined to repairs).

Table 1

Repair cycles	Purchasing price (lei)	Output obtained $\frac{Q_0}{(1+r_0)^x}$	$\sum_x \frac{Q_0}{(1+r_0)^x}$	$K_0\left(\frac{1+r_3}{1+d}\right)^x$	$\sum_x \left(\frac{1+r_3}{1+d}\right)^x$	$K_0'\left(\frac{1+r_3'}{1+d}\right)^x$	$\sum_x K_0'\left(\frac{1+r_3'}{1+d}\right)^x$	$P_0 + \sum_x K_0\left(\frac{1+r_3}{1+d}\right)^x + \sum_x K_0'\left(\frac{1+r_3'}{1+d}\right)^x$	p_x
A	1	2	3	4	5	6	7	8	9
I	316,000	195,120	195,120	25,000	25,000	68,700	68,700	409,700	2.099
II	316,000	177,381	372,501	26,200	51,200	75,227	143,927	511,127	1.372
III	316,000	161,256	533,757	27,450	78,650	82,371	226,298	620,948	1.163
IV	316,000	146,700	680,457	28,750	107,400	90,203	316,501	739,901	1.087
V	316,000	133,644	814,101	30,125	137,525	98,722	415,223	868,748	1.067
VI	316,000	121,192	935,293	31,575	169,100	108,065	523,288	1,008,388	1.078
VII	316,000	110,237	1,045,530	33,090	202,190	118,300	641,588	1,159,778	1.109

The equation for obtaining p_x, in which the expenses are also actualized, will have the form:

$$p_x = \frac{P_0 + K_0 \sum_x \left(\frac{1+r_3}{1+d}\right)^x + \sum_x K_0' \left(\frac{1+r_3'}{1+d}\right)^x}{\sum_x \frac{Q_0}{(1+r_0)^x}} \tag{6}$$

As K_0, K_0' and Q_0 are constants, this equation may be written

$$p_x = \frac{P_0 + K_0 \sum_x \left(\frac{1+r_3}{1+d}\right)^x + K_0' \sum_x \left(\frac{1+r_3'}{1+d}\right)^x}{Q_0 \sum_x \frac{1}{(1+r_0)^x}} \tag{6'}$$

or by decomposing in factors,

$$p_x = \frac{P_0}{Q_0 \sum_x \frac{1}{(1+r_0)^x}} + \frac{K_0 \sum_x \left(\frac{1+r_3}{1+d}\right)^x}{Q_0 \sum_x \frac{1}{(1+r_0)^x}} + \frac{K_0' \sum_x \left(\frac{1+r_3'}{1+d}\right)^x}{Q_0 \sum_x \frac{1}{(1+r_0)^x}} \tag{6''}$$

Thus, by applying any of the above equations, an expression is obtained which is affected by two categories of constant quantities; on the one hand, the initial data referring to productivity, maintenance expenses and the two types of repair expenses of the first cycle and, on the other hand, the average rates pointing to the average intensity of time change of expenses and productivity of the respective equipment. These average rates have been obtained on the basis of data found in the record card of the equipment taken as an illustration.

It may be easily seen that the optimum operating time of this machinery is 5 repair cycles, as $p_5 = 1.067$ is the minimum value. Beyond this cycle, the efficiency of maintenance, operating and current and thorough repair expenses, decreases. The fact is evidenced through the increase of expenditures per unit of output, within the cycle.

Subsequently, the idea of determining the optimum operating time is developed, by considering the influence of the two types of moral wear.

It will therefore be necessary to operate in the equation serving to compute the specific expense coefficient, in order to emphasize these aspects and, particularly, their effects.

MORAL WEAR AS A FACTOR HAVING THE EFFECT OF DECREASING THE OPERATING TIME

Based on the data collected from the actual practice for the equipment being discussed (the ring), for a period prior to the moment referred to in the computation, that is data on the replacement value of the equipment established by the new estimation made in 1964, the average rate of reducing the price for the respective time interval could be calculated. *

In the relation arrived at, it is considered that this computated average rate holds true, even after the re-estimation, at least for some time, during which the standardized operating time of an equipment set going may be integrated. Moreover, it is considered that the productivity increase of a new piece of equipment, reflecting moral wear type II, is maintained as a trend for the same time interval, acting in an indirect way towards hastening the disuse of the respective equipment.

In other words, considering all other conditions as being constant, the advent of equipment with increased productivity is the source of an average rate of productivity reduction of the existing equipment in use.

The combined effect of these two sorts of influences will or, at least, should lead to such specific expense coefficients as to speed up the advent of the moment when a minimum occurs. Thus, not only the physical wear, whose effects have been examined, is considered but also the effects of technical progress. Therefore the effects of moral wear may be definitely materialized by establishing some amortization quotas, different from those in force, that is, higher amortization standards, differentiated on the basis of types or groups of equipment for which the above-mentioned parameters (the changing rates of prices and productivity) are still effective.

The average rates are applicable in the example taken, as we are considering cycles, instead of years. (Cycles are definitely established, each corresponding to a certain number of operating hours. The year duration of a cycle is therefore prescribed by this standardized number, as well as by the scheduled shift operation of the respective piece of equipment.)

If r_1 stands for the average decreasing rate of the purchasing price, and r_2 for the average decreasing rate of the equipment productivity, as a result of moral wear type II, and taking into account that P_0, Q_0, K_0 and K'_0 are constant quantities, the relation to be used for computing the specific expense coefficient may be written:

$$p_x = \frac{\dfrac{P_0}{(1+r_1)^x} + K_0 \sum_x \left(\dfrac{1+r_3}{1+d}\right)^x + K'_0 \sum_x \left(\dfrac{1+r'_3}{1+d}\right)^x}{Q_0 \sum_x \left[\dfrac{1}{(1+r_0)\,(1+r_2)}\right]^x} \qquad (7)$$

* In our country, the prices are fixed between two new appraisals. It is, however, not difficult to compute the average rate of equipment value reduction during the respective period, as there is a difference between the initial value and the replacement value, resulting from the new estimation (moral wear, type I).

which may be decomposed in factors:

$$p_x = \frac{\frac{P_0}{(1+r_1)^x}}{Q_0 \sum_x \left[\frac{1}{(1+r_0)(1+r_2)}\right]^x} + \frac{K_0 \sum_x \left(\frac{1+r_3}{1+d}\right)^x}{Q_0 \sum_x \left[\frac{1}{(1+r_0)(1+r_2)}\right]^x} +$$

$$+ \frac{K_0' \sum_x \left(\frac{1+r_3'}{1+d}\right)^x}{Q_0 \sum_x \left[\frac{1}{(1+r_0)(1+r_2)}\right]^x} \qquad (7')$$

There are many ways of using this relation by means of which p_x is determined. In the following we shall use two of these, namely:

a) the derivative method, and

b) the tabular method (which served, in fact, to compute p_x in the first case).

The first method allows a more detailed analysis of this relation by largely using mathematic apparatus for this purpose.

The constants in this equation are positive, ipso facto, and $r_3 > d$ and $r_3' > d$.

It may be seen that the sums constituting the numerator are in fact sums of geometrical progressions, having the constant ratios of $\frac{1+r_3}{1+d}$ and $\frac{1+r_3'}{1+d}$, respectively.

Knowing that the sum of a geometrical progression is given by the expression $S = a\frac{1-q^n}{1-q}$, where a is the first term, q is the constant ratio and n the number of terms, and replacing by the expression taken from our problem and developing, the two sums in the numerator of the ratio (7) become:

$$\sum_x \left(\frac{1+r_3}{1+d}\right)^x = \frac{1+r_3}{d-r_3} - \frac{1+r_3}{d-r_3}\left(\frac{1+r_3}{1+d}\right)^x \qquad (8)$$

and

$$\sum_x \left(\frac{1+r_3'}{1+d}\right)^x = \frac{1+r_3'}{d-r_3'} - \frac{1+r_3'}{d-r_3'}\left(\frac{1+r_3'}{1+d}\right)^x \qquad (9)$$

Similarly, the sum in the denominator may be written:

$$\sum_x \left[\frac{1}{(1+r_0)(1+r_2)}\right]^x = \frac{1}{r_0 r_2 + r_0 + r_2}\left[1 - \left(\frac{1}{(1+r_0)(1+r_2)}\right)^x\right] \qquad (10)$$

These sums have been computed by choosing arbitrarily x, so that they may be applied whatever the value of x is: $x = 1, 2, \ldots, n$.

By introducing the above results in equation (7), we obtain:

$$p_x = \frac{\dfrac{P_0}{(1+r_1)^x} + K_0\dfrac{1+r_3}{d-r_3} - K_0\dfrac{1+r_3}{d-r_3}\left(\dfrac{1+r_3}{1+d}\right)^x + K_0'\dfrac{1+r_3'}{d-r_3'} - K_0'\dfrac{1+r_3'}{d-r_3'}\left(\dfrac{1+r_3'}{1+d}\right)^x}{\dfrac{Q_0}{r_0r_2+r_0+r_2}\left[1-\left(\dfrac{1}{(1+r_0)(1+r_2)}\right)^x\right]} \tag{11}$$

In this rather extended and cumbersome expression, the constant quantities may be replaced as follows:

$$A = K_0\frac{1+r_3}{r_3-d}; \quad A_1 = K_0'\frac{1+r_3'}{r_3'-d}; \quad B = \frac{O_0}{r_0r_2+r_0+r_2}.$$

As K_0, K_0' and Q_0 are positive and $r_3 > d$, it follows that these constants are positive too and may initially be computed from the definite parameters of the problem. In this case, equation (11) may be written:

$$p_x = \frac{P_0\dfrac{1}{(1+r_1)^x} + A\left(\dfrac{1+r_3}{1+d}\right)^x + A_1\left(\dfrac{1+r_3'}{1+d}\right)^x - (A+A_1)}{B\left[1-\left(\dfrac{1}{(1+r_0)(1+r_2)}\right)^x\right]} \tag{11'}$$

The diagram representing this expression and containing, as we have seen, many exponential curves is illustrative.

We previously specify that:

$$\frac{1+r_3}{1+d} > 1, \quad \frac{1+r_3'}{1+d} > 1, \quad \frac{1}{(1+r_0)(1+r_2)} < 1 \text{ and } \frac{1}{1+r_1} < 1 \tag{12}$$

The combination of these exponential curves leads to the cumulative curve p_x which, as can be seen, admits a minimum. This is particularly important for the economic interpretation of the phenomenon being analyzed (Fig. 4).

We shall now proceed to ascertain, for the above mentioned ring, whose main parameters have been previously shown, the cycle during which a minimum is reached, based on equation (11). It must be pointed out that in considering the particular condition of the equipment surveyed for a number of years, as well as by comparing it with new types of equipment having a superior productivity, it has been possible to evaluate the two average rates needed for the computation of the minimum, as affected both, by physical and moral wear.

We have thus estimated an average rate of price decrease (r_1) of 5% and an average decreasing rate of productivity, as a result of moral wear type II (r_2) amounting to 12%.

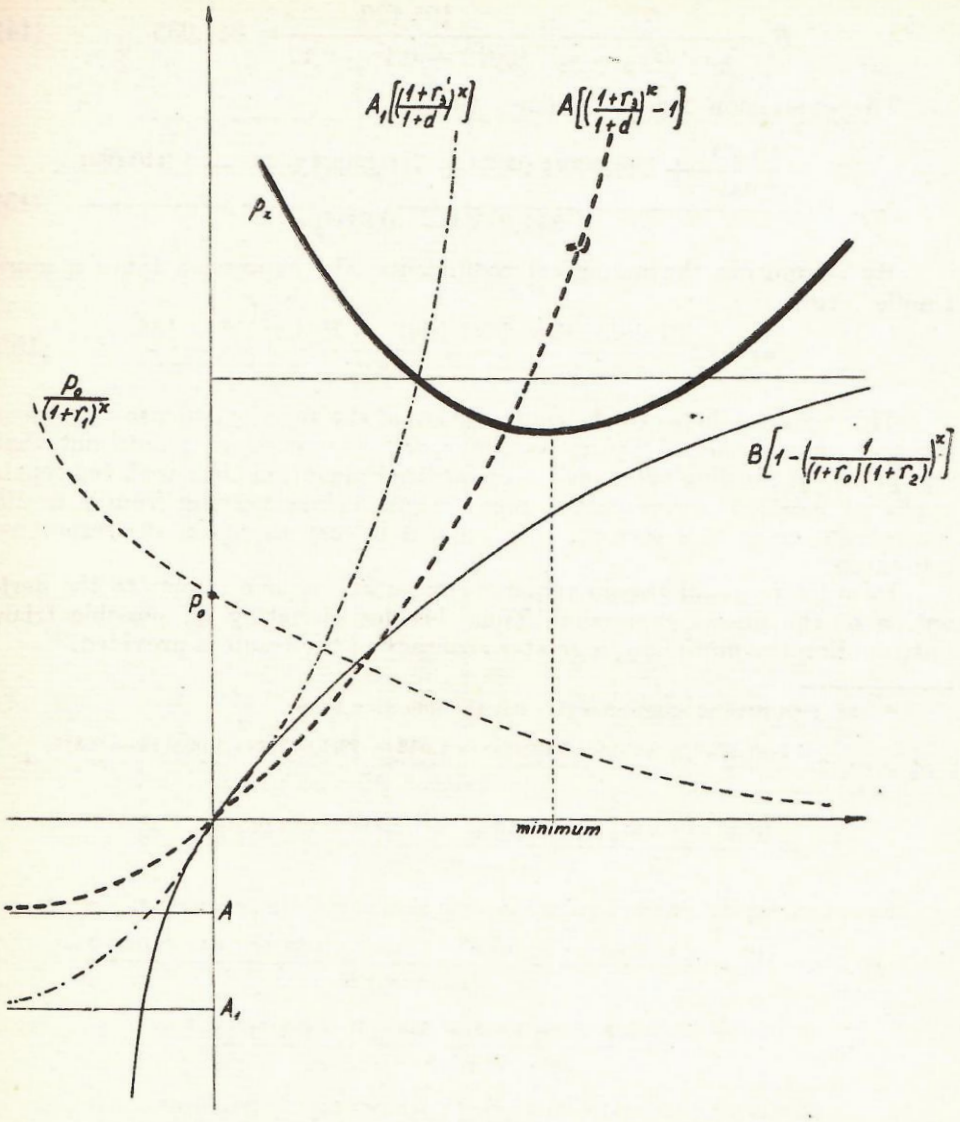

Fig. 4

In the case of our equation, we compute first the constants A, A_1 and B as follows:

$$A = K_0 \frac{1 + r_3}{r_3 - d} = 25{,}000 \frac{1.1}{0.05} = 550{,}000 \tag{13}$$

$$A_1 = K_0 \frac{1 + r_3'}{r_3' - d} = 68{,}000 \frac{1.15}{0.1} = 790{,}050 \tag{13'}$$

$$B = \frac{Q_0}{r_0 r_2 + r_0 + r_2} = \frac{195{,}120}{0.012 + 0.1 + 0.12} = 841{,}035 \qquad (14)$$

The expression for p_x becomes thus:

$$p_x = \frac{\dfrac{316{,}000}{(1.05)^x} + 550{,}000(1.048)^x + 790{,}050(1.095)^x - 1{,}340{,}050}{841{,}035[1 - (0.81)^x]} \qquad (15)$$

By computing the numerical coefficients, the expression takes a more simple form,

$$p_x = 0.12\,\frac{31.6(0.95)^x + 55(1.048)^x + 79(1.695)^x - 134}{1 - (0.81)^x} \qquad (15')$$

This equation helps to the computation of the specific expense coefficient for any value assumed by x. As the practical experience points out that equipment of the ring type has a standardized operation time with ten repair cycles at most, it means that x may assume values varying from 1 to 10. By computing in this way $p_1, p_2, \ldots, p_{10}$, it is easy to retain the respective minimum.

In order to avoid these extensive computations, one resorts to the derivative of the above expression. Thus, besides obviating all possible trials until finding the minimum, a greater accuracy of the result is provided. *

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* The derivative of expression (15) has the following form:

$$p'_x = 0.012\,\frac{[31.6(0.95)^x \ln 0.95 + 55(1.048)^x \ln 1.048 + 79(1.095)^x \ln 1.095]\,[1 - 0.81^x]}{[1 - (0.81)^x]^2} +$$

$$+ \frac{[(0.81)^x \ln 0.81]\,[31.6(0.95)^x + 55(1.048)^x + 79(1.095)^x - 134]}{[1 - (0.81)^x]^2}$$

By eliminating the natural logarithms on the basis of the well known relation, we obtain:

$$p'_x = 0.012\,\frac{[(31.6 \times 2.3026 \log 0.95)\,(0.95)^x + (55 \times 2.3026 \log 1.048)\,(1.048)^x] +}{[1 - (0.81)^x]^2}$$

$$\frac{+ (79 \times 2.3026 \log 1.095)\,(1.095)^x]\,[1 - 0.81^x]}{[1 - (0.81)^x]^2} +$$

$$+ \frac{(2.3026 \log 0{,}81)\,(0.81)^x\,[31.6 \times 0.95^x + 55 \times 1.048^x + 79(1.095)^x - 134]}{[1 - (0.81)^x]^2}$$

The necessary computations lead to more simple expression, that is:

$$p'_x = 0.012\,\frac{2.575(1.048)^x + 7.16(1.095)^x + 28.2(0.81)^x}{[1 - (0.81)^x]^2} -$$

$$- \frac{[1.62(0.95)^x + 5.13(0.77)^x + 12.26(0.85)^x + 23.76(0.887)^x]}{[1 - (0.81)^x]^2}$$

The computation shows that the derivative is annulled for $x = 3.5$.

Table 2

Repair cycles	Amended purchasing price $\frac{P_0}{(1+r_1)^x}$	Amended output resulting from physical wear and moral wear, type II $\frac{Q_0}{[(1+r_0)(1+r_2)]^x}$	$\sum_x \frac{Q_0}{[(1+r_0)(1+r_2)]^x}$	$\sum_x K_0\left(\frac{1+r_3}{1+d}\right)^x$	$\sum_x K_0'\left(\frac{1+r_3'}{1+d}\right)^x$	$\frac{P_0}{(1+r_1)^x} + \sum_x K_0\left(\frac{1+r_3}{1+d}\right)^x + \sum_x K_0'\left(\frac{1+r_3'}{1+d}\right)^x$	p_x
A	1	2	3	4	5	6 = 1 + 4 + 5	7 = 6/3
I	316,000	195,120	195,120	25,000	68,700	409,700	2.099
II	300,952	158,376	353,496	51,200	143,927	496,079	1.403
III	286,750	128,622	482,118	78,650	226,298	591,698	1.227
IV	273,120	104,398	586,516	107,400	316,501	697,021	1.188
V	260,082	84,761	671,277	137,525	415,223	812,830	1.211
VI	247,843	68,801	740,078	169,100	523,288	940,231	1.270
VII	236,173	55,844	795,922	202,190	641,588	1,079,951	1.357

As we can see in Table 2, the minimum value is located in the midst of the fourth repair cycle, having an advance of one and a half cycle, as compared to the value of p_x resulting from relation (6). This may be materialized in a definite number of years and months, by knowing accurately the duration of a cycle.

The effect of moral wear, as evidenced by the two rates (r_1 and r_2), has therefore lead to a hastened reaching of the minimum point, that is the primary purpose suggested in approaching this problem.

In our opinion, this example is illustrative for the viability of the relation serving as a basis for determining the amended specific expense coefficient; it may be thus successfully applied to *all* sorts of equipment for which the necessary parameters pointed out in the introduction are known, particularly the two rates numerically defining the expression of moral wear.

Moreover, we consider necessary to emphasize the fact that this relation may be particularly applied to equipment having a naturally measurable productivity. For other types of equipment we shall give and illustrate in the following paragraphs another method.

The computation method previously shown supposes the prior establishment of the general relation (11′) which, by derivation or by assuming for x the successive values from 1 to 10, gives the minimum point, materialized in our example at $3^1/_2$ cycles.

A very similar result may be reached by the tabular method used in the first part of the paper, leading to the determination of unamended values for p_x.

Part of the necessary data are collected from the table computed on this occasion, following which we have only to emphasize the effect of the constants r_1 and r_2. The arrangement of the respective data is illustraded in Table 2.

We find that p_x reaches a minimum in the fourth repair cycle, a fact that is moreover proved by the derivative method. The increased degree of accuracy of this method results also from the fact that it allows the exact location of the minimum within the cycle, in years and even in months, an achievement which the tabular method is not able to accomplish.

In exhausting the example taken for analysis and pointing out once more the assumptions considered in computing — which in our opinion are realistic, as a result of their checking up in practice — we may conclude by admitting that the effect of technical progress, expressed by the two types of moral wear, is measurable. This conclusion is mathematically substantiated, as was shown in this paper, without avoiding approximations.

Some pieces of equipment may produce during a time interval, completely different use values, that cannot be totalized by the natural method nor by the natural-conventional method. In this case we are obliged to resort to the value method. A series of economic reasons, generated by economic reality result in the fact that the time accumulation of the value production of such equipment and its use in computing the specific expense coefficient is not self-evident and is even misleading.

The determination of the critical moment for this sort of equipment, that is the moment when its replacement becomes necessary, could be made

starting from data referring to the time progress of expenditures caused by maintenance, operating and current and thorough repairs.

Assuming the periods (years) separating the dates of well defined expenditures to be equal and taking into account the monotonous time increase of these expenditures ($C_1 > C_0$, $C_2 > C_1$, ..., $C_{n+1} > C_n$), designating by P_0 the purchasing price of the equipment, and by d the interest for the amount of money, it is shown that:

$$\Gamma_n = \frac{P_0 + \sum_{i=1}^{n} a^{i-1} C_i}{1 - a^n} \tag{16}$$

where Γ_n = actualized accumulated expenditures, and

$$a = \frac{1}{1+d}.$$

The choice of the most favourable replacement moment for a piece of equipment is shown by minimizing expression (Γ_n), which leads to the inequality:

$$C_{n+1} > \frac{P_0 + C_1 + C_2 a + C_3 a^2 + \ldots + C_n a^{n-1}}{1 + a + a^2 + \ldots + a^{n-1}} \tag{17}$$

In other words, a piece of equipment may be replaced when the costs of the subsequent period become greater than the weighted sum of expenditures already performed.

In the following paragraphs we shall illustrate the application of this expression for a lathe in a machine-building plant, with $P_0 = 48{,}500$ lei and $d = 5\%$. The maitenance and repair costs are shown in Table 3, referring to the 12 years of standardized operating time of the respective lathe.

Table 3

Years	Maintenance, operating and repair expenses for the lathe	a^{i-1}	$C_i a^{i-1}$	$P_0 + \Sigma C_i a^{i-1}$	Σa^{i-1}	$\frac{P_0 + \Sigma C_i a^{i-1}}{\Sigma a^{i-1}}$
A	1	2	3	4	5	6
1	2,400	1	2,400	50,900	1	50,900
2	10,600	0.952	10,091.2	60,991.2	1.952	31,245
3	11,600	0.906	10,509.6	71,500.8	2.858	25,018
4	12,900	0.863	11,132.7	82,633.5	3.721	22,207
5	13,600	0.821	11 165.6	93,799.1	4.542	20,651
6	14,400	0.782	11,260.8	105,059.9	5.324	19,733
7	15,300	0.744	11 383.2	116,443.1	6.068	19,189
8	16,200	0.708	12.469.6	128,912.7	6.776	19,025
9	18,700	0.674	12,703.8	141,616.5	7.450	19,009
10	20,900	0.642	13,417.8	155,034.3	8.092	19,159
11	25,000	0.611	15,275.0	170,309.3	8.703	19,569
12	30,000	0.582	17,460.0	187,769.3	9.285	20,223

The table shows that the ninth year of operation is the adequate moment for replacing the lathe, as beyond that year maintenance and repair costs become inefficient.

The above mentioned methods and models assume that data referring to the productivity of the various pieces of equipment, as well as those showing the magnitude of operating, maintenance and repair costs, are known.

There are, however, circumstances when these figures are not known. In this case, mathematical functions approximating the time progress of these figures are found.

These functions are designated as

$$Q = f_1(x), \quad K = f_2(x) \quad \text{and} \quad K' = f_3(x).$$

In actual practice they may assume the most diversified forms.

For instance, machine efficiency decreases from cycle to cycle by a constant quantity s, i.e. $f_1(x) = Q_{i-1} - sx_i$; the function expressing expenses for thorough repairs is parabolic, $f_3(x) = a + bx_i^2$ (where a and b are the approximated function parameters), and the maintenance, operating and current repair costs are proportional to the machine productivity $f_2(x) = cf_1(x)$, where c is a proportionality factor.

In this case, the efficiency coefficient of expenses (specific expense coefficient) assumes the form:

$$p_i = c + \frac{P_0}{\Sigma(Q_{i-1} - sx_i)} + \frac{\Sigma(a + bx_i^2)}{\Sigma(Q_{i-1} - sx_i)} \tag{18}$$

There are still other models for determining — at least approximately — the volume of these equipment maintenance, operating and repair expenses. The approximated functions may be linear, exponential, etc. The equipment characteristics, as well as those of the respective industrial branch, will certainly determine the choice of the most appropriate function. The method itself is based on the possibilty of extr apolatintg the recorded results for future conditions. It is evidently impossible to obtain a perfect accuracy; but, as compared to the existing methodology, we consider that by taking into account factors such as physical wear, the time factor and its effect on equipment maintenance and operating expenses, as well as moral wear, it is possible to draw the operating time determination of equipment — by the use of the above models mathematically substantiated — close to its optimum point and, on this basis, amortization quotas can be more scientifically substantiated.

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COMMENT DETERMINER LES MEILLEURES LIMITES DE FONCTIONNEMENT DE L'EQUIPEMENT

L'ouvrage aborde un aspect particulièrement important pour l'accroissement de l'efficacité de l'activité économique, à savoir, l'utilisation la plus rationnelle des équipements dont sont dotées les entreprises.

Partant de principes théoriques connus et vérifiés dans la pratique, à savoir, que le rendement de l'équipement baisse au fur et à mesure de son utilisation dans le temps, parallèlement à l'augmentation des dépenses d'entretien et de réparation, les auteurs présentent des modèles permettant de fixer le moment critique où le remplacement de l'équipement s'impose.

L'analyse économique est fondée sur des calculs mathématiques englobant une quantité suffisante de données fournies par les entreprises.

Pour fixer les meilleures limites de fonctionnement de l'équipement, les auteurs tiennent compte tantôt de la seule usure physique, tantôt des deux aspects de l'usure (physique et morale).

Le meilleur moment pour remplacer l'équipement est dicté par le coefficient des dépenses spécifiques, cet indice étant calculé comme le rapport entre le total des dépenses d'entretien et de réparations cumulées et le volume de la production cumulée.

Ce qui retient particulièrement l'attention est la façon d'aborder la question de l'usure morale sous ses deux aspects. La relation mathématique servant à calculer en ce cas le coefficient des dépenses spécifiques prend en considération autant la création d'équipements similaires meilleur marché, que de la création d'équipements nouveaux, à caractéristiques supérieures.

La conclusion théorique vérifiée dans la pratique est que le moment critique intervient après un nombre moindre de cycles de reparations.

Les auteurs s'occupent ensuite du meilleur terme de fonctionnement des équipements ayant pour objet une production hétérogène, le fonctionnement de ceux-ci étant différent par rapport aux équipements à production homogène.

L'illustration graphique de l'ouvrage permet de faciliter la compréhension des questions posées.

La conclusion générale de l'ouvrage est que pour fixer les normes d'amortissement de l'équipement, il faut tenir compte de l'intesité du processus d'introduction du progrès technique dans la production, processus qui se traduit par l'apparition permanente de l'usure morale. C'estce qui rend impérieusement nécessaires de justes normes d'amortissement dans toutes les branches de l'économie nationale.

УСТАНОВЛЕНИЕ ОПТИМАЛЬНЫХ СРОКОВ ФУНКЦИОНИРОВАНИЯ ОБОРУДОВАНИЯ

В работе затрагивается исключительно важная сторона вопроса увеличения эффективности экономической деятельности, — вопрос наиболее рационального использования оборудования, которым оснащены предприятия.

Исходя из известных теоретических принципов, проверенных на практике, согласно которым по мере использования во времени отдача оборудования уменьшается параллельно с ростом расходов на содержание и ремонт, авторы представляют модели, на основе которых можно определить критические моменты, когда необходима замена оборудования.

Экономический анализ основывается на математических расчетах, на обработке достаточного количества данных, собранных на предприятиях.

Вопрос определения оптимальных сроков функционирования оборудования рассматривается или с точки зрения физического износа, или с точки зрения обоих видов износа (физического и морального).

Оптимальный момент замены оборудования диктуется коэффициентом специфических расходов — показателем, расчитанным как отношение между общей суммой расходов на содержание и ремонт и объемом всего производства.

Однако особенно обращает на себя внимание метод, каким анализируется проблема морального износа в его двух формах. Математическое отношение, при помощи которого рассчитывается коэффициент расходов, специфических в данном случае, учитывает как появление похожего оборудования, но более дешевого, так и появление нового оборудования, с более высокими характеристиками.

Теоретическим и практически проверенным выводом является то, что критический момент замены появляется после меньшего числа циклов ремонта.

Затем рассматривается вопрос установления оптимального срока функционирования для оборудования, производящего гетерогенную продукцию, отличающегося от оборудования, производящего однородную продукцию.

Работа иллюстрируется при помощи графика, что облегчает ее понимание.

Общим выводом, к которому приходят авторы к концу исследования, является то, что при установлении норм амортизации для оборудования нужно учитывать интенсивный процесс внедрения технического прогресса в производство, процесс проявляюйщийся в постоянном появлении морального износа. Все это требует правильного установления норм амортизации во всех отраслях народного хозяйства.