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Correlation and Entropic Correlation Coefficients

This work presents some indicators of the dependence between two characteristics of a statistical collectivity based upon the entropy concept.

1. GENERAL CONSIDERATIONS

The idea of the connection between two characteristics of a statistical population appeared approximately to the end of the 19th century, under the form of a „connected variation", and it belongs to Francis Galton, founder, together with Karl Pearson, of the Anglo-Saxon school of statistics. The idea of „connected variation", then rigorously analysed by Pearson, was transposed by the latter in a mathematical language, becoming thus the correlation theory.

Both within the school mentioned above, and in the activity of other mathematicians and statisticians all over the world, there were and there are yet concerns to define and to study various indicators of the dependence between two characteristics of a statistical population.

We shall not deal now with a critical analysis of these results. We have in view to present further down a family of measures of the dependence or correlation between two random variables (phenomena, processes, characteristics), based upon the concept of entropy, a concept which led, itself, to the creation of a genuine theory with multiple applications in the most various fields, namely the mathematical theory of information (see [1] — [5], [11] — — [15].

As we shall see, the measures of dependence, based upon the concept of entropy, have some most important properties, as regarded, of course, by the persons who could use them in practice.

2. ENTROPIC CORRELATION

Let $Z = (X, Y)$ be a bi-dimensional random vector, where components are random variables, which we shall suppose as discrete, without diminishing the generality of the problem. We shall have hence, for the probabilistic distribution of the random vector Z, a correlation table, in which

$$P(X = x_i, Y = y_i) = \pi_{ij}. \; 1 \leqslant i \leqslant m, \; 1 \leqslant j \leqslant n, \; \pi_{ij} \geqslant 0,$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{ij} = 1 \tag{2.1}$$

where from we shall infer immediately the individual distributions for X and Y, namely:

$$P(X = x_i) = \sum_{j=1}^{n} \pi_{ij} = p_i; \quad P(Y = y_i) = \sum_{i=1}^{m} \pi_{ij} = q_j, \; p_i \geqslant 0, \; q_j \geqslant 0,$$

$$\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j = 1. \tag{2.2}$$

The Shannon entropies, attached to the random vector Z and to its components X and Y, are given by expressions (see [1] — [4], [11] — [14], [21]):

$$H(Z) = H(X, Y) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{ij} \log \pi_{ij} \tag{2.3}$$

$$H(X) = -\sum_{i=1}^{m} p_i \log p_i \text{ and } H(Y) = -\sum_{j=1}^{n} q_j \log q_j; \tag{2.4}$$

the logarithm having overall base two. We define the entropy of the conditioned random variable X/y_i (and analogous for Y/x_i) as having the expression (see [1], [4], [14]):

$$H(X/y_j) = -\sum_{i=1}^{m} P(X = x_i/Y = y_j) \log P\,(X = x_i/Y = y_i) =$$

$$= -\sum_{i=1}^{m} \frac{\pi_{ij}}{q_j} \log \frac{\pi_{ij}}{q_j} \tag{2.5}$$

and consequently there appears also the entropy of the conditioned random variable X/Y (analogously for Y/X) as being (see [1] — [4], [14]):

$$H(X/Y) = \sum_{j=1}^{n} q_j H(X/y_j) = -\sum_{j=1}^{n} \sum_{i=1}^{m} \pi_{ij} \log \frac{\pi_{ij}}{q_j}. \tag{2.6}$$

We can see immediately from relation (2.6) that

$$H(X/Y) = H(X, Y) - H(Y) \tag{2.7}$$

which leads us to two models of calculation of the conditioned entropy $H(X/Y)$, first time directly as in (2.6) and another time by using entropies (2.3) and (2.4). We can write analogously, also for $H(Y/X)$, the expression

$$H(Y/X) = H(X, Y) - H(X). \tag{2.7'}$$

We say that random variables X and Y are probabilistically independent, if for any $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$, there takes place relation $\pi_{ij} = p_i q_j$. As a consequence of this fact we shall have $H(X) = H(X/Y)$ and $H(Y) = H(Y/H)$, and consequently the quantity of information contained by random vector (X, Y) is equal to $H(X) + H(Y)$, i.e. with the sum of the information contained by its components (see [1] — [4]). Contrarily, hence when X and Y are not independent, the quantity of information contained together is equal to $H(X, Y)$ and satisfy relation $H(X, Y) \leqslant H(X) + H(Y)$ (see [1] — — [4]).

We say that random variables X and Y are identically distributed, and we write $X = Y$ if for any pair (i, j) $1 \leqslant i, j \leqslant n$ (hence $m = n$) relation $\pi_{ij} = p_i = q_j$ takes place. Therefore, the variables are identically distributed when the matrix of the common distribution is diagonal.

We state immediately that, in this case, $H(X/Y) = H(Y/X) = 0$, resulting that $H(X) = H(Y)$, and we say also that variables X and Y are reciprocally completely dependent (see [14]).

If only one of these equalities takes place, i.e. $H(X/Y) = 0$ (or $H(Y/X) = 0$) we say that X is completely dependent on Y (or that Y is completely dependent on X) which means that X may be expressed as a function of Y (analogously, Y as a function of X) (see [5], [11], [14] — [16], [21]).

In this case, we may easily find that $H(X, Y) = \frac{1}{2}[H(X) + H(Y) + H(Y/X))]$ or that $H(X, Y) = \frac{1}{2}[H(X) + H(Y) + H(X/Y)]$.

Let us consider now the variation of the quantity of information contained by pair (X, Y), when we go over from the dependence hypothesis to the independence hypothesis of components X and Y, denoted by

$$W(X, Y) = H(X) + H(Y) - H(X, Y) \tag{2.8}$$

and which can be also expressed in other ways, equivalent between themselves, taking into account the fact that $H(X, Y) = H(X) + H(Y/X) = H(Y) + H(X/Y)$. Thus, we may write too:

$$W(X, Y) = H(X) - H(X/Y) = H(Y) - H(Y/X) \tag{2.8'}$$

or that

$$W(X, Y) = \frac{1}{2}(H(X) + H(Y) - \frac{1}{2}(H(X/Y) + H(Y/X)). \tag{2.8''}$$

Taking into account the properties of entropy (see [1] — [4], [11], [12], [14] — [16]. [21]) we can immediately evince that magnitude $W(X, Y)$ has following properties:

1) it is defined for any pair of random variables (X, Y) with common distribution (2.1);

2) it is symmetrical in X and Y, i.e. $W(X, Y) = W(Y, X)$;

3) $0 \leqslant W(X, Y) < \infty$, the equality to zero taking place if and only if X and Y are independent.

We shall say that the random variables are perfectly correlated, or reciprocally completely dependent if the entropic correlation $W(X, Y)$ coincides with the entropies of variables X, Y and (X, Y) simultaneously.

We shall say that random variables X and Y are non-correlated or independent, if entropic correlation $W(X, Y)$ is equal to zero.

4) if X is completely dependent on Y (or Y is completely dependent on X), then

$$W(X, Y) = H(X) \text{ (or } W(X, Y) = H(Y)). \tag{2.9}$$

We shall say that variable X is completely dependent on variable Y if entropic correlation $W(X, Y)$ coincides only with $H(X)$.

In this case, we must state precisely the fact that generally, when calculating magnitude $W(X, Y)$, we cannot be sure of the kind of dependence, that is to say if it is a unilateral one (either X depends completely on Y, or Y depends completely on X) or if it is a reciprocal dependence. So as to state precisely this thing, it seems necessary to determine the conditioned entropies $H(X/Y)$ and $H(Y/X)$. If one of them is zero, then we have a partial dependence, or one in a single sense.

If both are equal to zero, we are in the case reciprocally complete of the dependence, or otherwise expressed, of the maximal interdependence. If both conditioned entropies are *non-null*, we have an interdependence between X and Y or a dependence between the two characteristics, which decreases or increases according to how vary the conditioned entropies: they increase or they decrease.

5) if X and Y are reciprocally completely dependent, then and only then

$$W(X, Y) = H(X) = H(Y) = H(X, Y) \tag{2.10}$$

6) if φ and ψ are two real functions, defined in the real axis in a biunivocal way, then

$$W(\varphi(X), \psi(Y) = W(X, Y) \tag{2.11}$$

which means that magnitude (2.8) is invariant as against the biunique transformations of random variables X and Y.

7) $$W(X, Y) \leqslant min\ [H(X), H(Y)] \leqslant \frac{1}{2}[H(X) + H(Y)] \leqslant max\,[H(X), H(Y)] \leqslant H(X, Y) \tag{2.12}$$

with equality overall, if and only if X and Y are reciprocally completely dependent, and only to the first equality, if and only if the variable for which the minimum was reached, is completely dependent on the other one.

An analysis of the properties possessed by magnitude $W(X, Y)$ allows us to consider that this quantity may be considered as an indicator of the dependence between X and Y, which provides accurate information in the

extreme situation of the dependence. Thus, when the dependence is minimal, hence the variables X and Y are independent, $W(X, Y) = 0$ and reciprocally, and when the dependence is maximal, hence when X and Y are reciprocally completely dependent, $W(X, Y)$ reaches its maximal value, given by equalities (2.11) and reciprocally. If we make an analogy pro forma between relation (2.8) and the classical statistical covariance (see [7] — [9], [17], and taking into account the properties possessed by magnitude $W(X, Y)$, we shall call the latter entropic correlation of random variables X and Y.

3. ENTROPIC CORRELATION COEFFICIENTS

It is a sure fact that magnitude $W(X, Y)$, denominated by us entropic correlation, provides precise information about the dependence of random variables X and Y. The enough great range of values this correlation can take, seems nevertheless to be a drawback in utilization, a thing otherwise established for the classic correlation too. We are thus led to bring this magnitude to a form, able to create the possibility of a smaller set of values, also as a consequence of some surer appreciation and of some narrower results.

Starting from relation (2.12), let us divide the first member of the relation by any of the following four members. We shall deduce hence that, for any pair of random variables (X, Y), in which neither of both is constant with probability one, the following inequalities take place:

$$0 \leqslant \frac{W(X, Y)}{H(X, Y)} \leqslant \frac{W(X, Y)}{max[H(X), H(Y)]} \leqslant \frac{W(X, Y)}{\frac{1}{2}[H(X) + H(Y)]} \leqslant$$

$$\leqslant \frac{W(X, Y)}{min\,[H(X), H(Y)]} \leqslant 1 \tag{3.1}$$

the equality with zero taking place for all fractions, if and only if X and Y are independent, and the equality with one taking place, if and only if X and Y are reciprocally completely dependent.

Let us denote the ratio of relation (3.1) as follows:

$$R_1(X, Y) = \frac{W(X, Y)}{H(X, Y)}; \quad R_2(X, Y) = \frac{W(X, Y)}{max[H(X), H(Y)]}; \quad R_3(X, Y) =$$

$$= \frac{W(X, Y)}{\frac{1}{2}[H(X) + H(Y)]}; \quad R_4(X, Y) = \frac{W(X, Y)}{min[H(X), H(Y)]}. \tag{3.2}$$

We shall call magnitudes $R_i(X, Y)$, $1 \leqslant i \leqslant 4$, defined by relation (3.2), entropic correlation coefficients of random variables X and Y.

As a consequence of the properties of the magnitudes defined by them, the entropic correlation coefficients $R_i(X, Y)$ have the following properties:

1) they exist for any pair of random variables (X, Y), not any of them being constant with probability one;

2) they are symmetrical in X and Y, i.e. $R_i(X, Y) = R_i(Y, X)$, $1 \leqslant i \leqslant 4$;

3) $0 \leqslant R_1(X, Y) \leqslant R_2(X, Y) \leqslant R_3(X, Y) \leqslant R_4(X, Y) \leqslant 1$,

the equality of all coefficients to zero taking place simultaneously if and only if X and Y are probabilistically independent, and their equality to one takes also place simultaneously, if and only if X and Y are reciprocally completely dependent;

4) they are invariant to any biunique transformation of random variables X and Y.

Consequently, we wanted to find an indicator more accurate than the entropic correlation starting from it, and we became four entropic correlation coefficients.

The question raised by the practice can be immediate. Which from those coefficients must be taken into consideration? We shall select $R_1(X, Y)$, for two reasons of practical order. The first reason is connected to the somewhat symmetrical form of its expression, and the second reason, and the most important one, would be the fact that $R_1(X, Y)$, as the smaller from all four coefficients, offers for comparison the smallest scale of values of the standard dependence indicators, and this in the situation of a dependence not being in the extreme situation, when all coefficients do coincide.

4. PRACTICAL WORKING FORMS OF THE ENTROPIC MEASURES OF DEPENDENCE

As in other problems of statistics, the present one does not work with theoretical probabilistic distributions and, as a consequence, the entropy and all magnitudes constructed above with the help of the entropy, are no longer the theoretical ones. We work in practice with absolute frequencies or with relative frequencies of the values taken by the characteristics, and all magnitudes we construct with their help are sample magnitudes. But we know that, in the conditions of some accurate measurements, for a sufficiently great number of observations, the relative frequencies well estimate the theoretical distribution of the considered random variables. Consequently, when using frequencies instead of practical probabilities, not too great errors are made.

Let us suppose that n observations are made on characteristics X and Y of the statistical population, and that they lead to the following situations:

— characteristic X takes n_1 separate values, and characteristic Y takes n_2 separate values;

— pair (x_i, y_j) was observed f_{ij} times $1 \leqslant i \leqslant n_1$, $1 \leqslant j \leqslant n_2$;

— value x_i was observed g_i times, $1 \leqslant i \leqslant n_1$;

— value y_j was observed h_j times, $1 \leqslant j \leqslant n_2$.

Between the numbers f_{ij}, g_i and h_j, called absolute frequencies of the pair (x_i, y_j), and the values x_i and y_j, respectively, take place relations:

$$g_i = \sum_{j=1}^{n_2} f_{ij}; \quad h_j = \sum_{i=1}^{n_1} f_{ij} \quad \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} f_{ij} = n. \tag{4.1}$$

If we divide in relations (4.1) each equality to n, we shall obtain the estimations of distributions (2.1) and (2.2), which we will denote as being:

$$\hat{p}_i = \frac{g_i}{n} = \sum_{j=1}^{n_2} \frac{f_{ij}}{n}; \quad \hat{q}_j = \frac{h_j}{n} = \sum_{i=1}^{n} \frac{f_{ij}}{n}; \quad \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \frac{f_{ij}}{n} = \sum_{i=1}^{n_1} \sum_{j=2}^{n_2} \hat{\pi}_{ij} = 1. \tag{4.2}$$

We shall keep the notations introduced in paragraphs two and three for the entropic correlation $W(X, Y)$ and for the entropic correlation coefficients $R_i(X. Y)$, so as not to render the problem more complicated. We understand that, instead of the theoretical probabilities (2.1) and (2.2), we have their estimations (4.2). Consequently, taking into account relations (2.3), (2.4) and (2.6), in which we shall replace probabilities (2.1) and (2.2) by their estimations (relative frequencies) given by relations (4.2), we shall deduce immediately that

$$W(X, Y) = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \frac{f_{ij}}{n} \log \frac{n f_{ij}}{g_i h_j}. \tag{4.3}$$

Developing this relation, we become:

$$W(X, Y) = \log n - \frac{1}{n} \left[\sum_{i=1}^{n_1} g_i \log g_i + \sum_{j=1}^{n_2} h_j \log h_j - \right.$$

$$\left. - \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} f_{ij} \log f_{ij} \right]. \tag{4.4}$$

We shall find, through analogous substitutions in expressions (3.2), the corresponding working expressions of the entropic correlation. Considering, for instance, the first entropic correlation coefficient, $R_1(X, Y)$, we shall deduce that its working form is

$$R_1(X, Y) = \frac{\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \frac{f_{ij}}{n} \log \frac{g_i h_j}{n f_{ij}}}{\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \frac{f_{ij}}{n} \log \frac{f_{ij}}{n}} \tag{4.5}$$

By developing it, we become

$$R_1(X, Y) =$$

$$= \frac{n \log n - \left[\sum_{i=1}^{n_1} g_i \log g_i + \sum_{j=1}^{n_2} h_j \log h_j - \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} f_{ij} \log f_{ij} \right]}{n \log n - \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} f_{ij} \log f_{ij}}. \tag{4.6}$$

The developed form, both for the entropic correlation $W(X, Y)$, and for the entropic correlation coefficient R_1 (X, Y), presents interest, as concerns the calculation accuracy, and we are obliged to approximate lesser, hence to make fewer and smaller errors.

5. NUMERICAL APPLICATIONS

A.5.1. Let us assume that the observations made by us on a population collectivity, concerning two of its characteristics lead us to the following correlation table:

X \ Y	1	2	3	g_i
-1	2	2	2	6
0	2	2	2	6
1	2	2	2	6
h_j	6	6	6	18

After calculating the entropic correlation, we shall infer that $W(X, Y) = \log 18 - \frac{1}{18}(18 \log 6 + 18 \log 6 - 18 \log 2) = 0$ which was expected, as the two characteristics are independent. In this case, if we calculate also the classical correlation coefficient (see [7] — [10]), we shall also find that $\rho(X, Y) = 0$.

A.5.2. Let us suppose that the 18 observations from the previous example led to the following correlation table:

X \ Y	1	2	3	g_i
-1	6	0	0	6
0	0	6	0	6
1	0	0	6	6
h_j	6	6	6	18

We shall have in this case:

$$W(X, Y) = \log 18 - \frac{1}{18}(18 \log 6 + 18 \log 6 - 18 \log 6) = \log 3$$

and $H(X, Y) = \log 3$, and we shall deduce that $R_1(X, Y) = 1$.
We find analogously that $R_2(X, Y) = R_3(X, Y) = R_4(X, Y) = 1$.

When calculating the classical correlation coefficient, we shall find that $\rho(X, Y) = 1$.

A.5.3. Let us suppose that, in the previous table, $f_{11} = 4$, $f_{22} = 6$ and $f_{33} = 8$, and the remainder are zero, hence

X \ Y	1	2	3	g_i
-1	4	0	0	4
0	0	6	0	6
1	0	0	8	8
h_j	4	6	8	18

We remark immediately that $W(X, Y) = H(X) = H(Y) = H(X, Y)$ and $R_1(X, Y) = 1$.

When calculating the classical correlation coefficient, we find also that $\rho(X, Y) = 1$.

A.5.4. Let us suppose, considering the previous example, that variable Y takes values -1, 1 and 2.

X \ Y	-1	1	2	g_i
-1	4	0	0	4
0	0	6	0	6
1	0	0	8	8
h_j	4	6	8	18

We found immediately that $W(X, Y) = H(X) = H(Y) = Y(X, Y)$ and consequently $R_i(X, Y) = 1$. When calculating however the classical correlation coefficient, we find that $\rho^2(X, Y) = 24/25 \neq 1$.

A.5.5. Let us suppose that the common distribution of variables X and Y is given by the following correlation table:

X \ Y	0	1	g_i
-1	0	4	4
0	8	0	8
1	0	4	4
h_i	8	8	16

We shall find immediately that $H(X) = 3/2 = H(X, Y)$ and $H(Y) = 1$. It results hence that $H(X/Y) = 1/2$, $H(Y/X) = 0$, $W(X, Y) = 1$ and

$$R_1(X, Y) = \frac{2}{3}.$$

We deduce hence that Y is completely dependent on X and we remark, indeed, that $Y = X^{2k}$. As X cannot be expressed uniquely as a function of Y (the transformation connecting them is not injective), we have not a reciprocally complete dependence, but only a unilateral dependence.

Nevertheless, if we calculate the classical correlation coefficient, we shall find that $\rho(X, Y) = 0$, and this means that X and Y are not correlated, which doesn't correspond to reality.

A.5.6. Let us assume that the common distribution of variable X and Y is given by the following table:

X \ Y	0	1	4	g_i
0	4	0	0	4
1	0	6	0	6
2	0	0	8	8
h_j	4	6	8	18

We remark immediately that $H(X, Y) = H(X) = H(Y) = W(X, Y)$ and we find immediately that $H(X/Y) = H(Y/X) = 0$. Hence X and Y are

reciprocally completely dependent. We remark moreover that $Y = X^2$ or $X = \sqrt{Y}$.

After calculating the classical correlation coefficient, we find that $\rho(X, Y) = \frac{53}{55}$, which points to a powerful correlation between X and Y.

A.5.7. The situation of the chemical fertilizer consumption by hectare, and of the mean wheat production by hectare, in a number of 24 countries, in 1976, is given further down (according to the Statistical Year Book of the S.R. of Romania 1978).

Y, chemical fertilizer consumption kg/ha / X, wheat production, quintal (100 kg) by ha	Under 25	25–50	50–75	75–100	100–125	125–150	150–175	Over 175	g_i
Under 10	0	1	1	0	0	0	0	0	2
10–15	4	1	0	0	0	0	0	0	5
15–20	1	0	0	0	0	0	0	0	1
20–25	0	1	0	0	0	0	0	1	2
25–30	0	0	0	1	1	1	1	0	3
30–35	0	0	0	1	0	0	0	2	3
Over 35	0	1	0	0	0	0	1	6	8
h_j	5	4	1	2	1	1	1	9	24

We shall obtain

$$H(X) = \log 24 - \frac{1}{24}(2 + 5 \log 5 + 2 + 6 \log 3 + 24) = \log 24 - \frac{1}{24} \times$$

$$\times (5 \log 5 + 6 \log 3 + 28)$$

$$H(Y) = \log 24 - \frac{1}{24}(5 \log 5 + 8 + 2 + 9 \log 9) = \log 24 - \frac{1}{24} \times$$

$$\times (10 + 5 \log 5 + 18 \log 3)$$

$$H(X, Y) = \log 24 - \frac{1}{24}(8 + 2 + 6 \log 3) = \log 24 - \frac{1}{24} \times (16 + \log 3)$$

$$W(X, Y) = \log 24 - \frac{1}{24}(22 + 10 \log 5 + 18 \log 3)$$

$$R_1(X, Y) = \frac{24 \log 24 - (22 + 10 \log 5 + 18 \log 3)}{24 \log 24 - (16 + 6 \log 3)} =$$

$$= \frac{56 + 6 \log 3 - 10 \log 5}{56 + 18 \log 3} = 0{,}43.$$

It results that in 1976, there was, between the quantity of fertilizers applied by hectare and the mean wheat production by hectare, a sufficiently powerful dependence.

6. CONCLUSIONS

The entropic measure of the dependence between random characteristics X and Y varies, as can be easily seen, both as form, and as concerns the content, as against the classical statistical correlation.

1. a) The classical correlation coefficient $\rho\ (X, Y)$ expresses the degree of „understanding", or of „agreement" between the values taken by random variables X and Y, with certain weightings.

b) The entropic correlation coefficient $R_1\ (X, Y)$ expresses the degree of deviation from the probabilistic independence of the values which the two random variables X and Y would take.

2. a) If X and Y are probabilistically independent, then $\rho\ (X, Y) = 0$, but the reciprocal is not true.

b) The entropic correlation coefficient $R_i\ (X, Y) = 0$ if and only if random variables X and Y are probabilistically independent. Hence, the reciprocal is also true.

3. a) The classical correlation coefficient $\rho\ (X, Y)$ is invariant only with linear transformations of variables X and Y, and it is tightly connected to the type of studied random (quantitative or qualitative) variables.

b) The entropic correlation coefficient $R_1\ (X, Y)$ is invariant for any biunique transformation of variables X and Y, and as it is defined by means of the probabilistic distributions of variables X and Y, it can be successfully used, no matter whether the random variables are quantitative or qualitative.

4. a) As it depends also on the values of variables X and Y, the classital correlation coefficient $\rho\ (X, Y)$ expresses only the connection between these variables.

b) As it depends only on the probabilistic distribution of random variables X and Y, the entropic correlation coefficient $R_1\ (X, Y)$ expresses the conection between two families of random variables, represented by X and Y.

5. a) From the way it is defined, the classical correlation coefficient $\rho(X, Y)$ expresses the connection between only two variables, and it may be extended so as to measure the total connection itself, between several random variables (see [17]).

b) Due to the manner it is defined, the entropic correlation coefficient $R_1\ (X, Y)$ can be easily extended so as measure the total dependence between several random variables, and moreover, between, random vectors, a situation frequently met with in the statistical practice (see [12], [14]).

Taking into account the above consideration, we can immediately establish the superiority of the entropic measurements of the dependence, as against the classical correlation, a fact which recommends them, as a matter of fact, for practical applications.

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CORRELATION ET COEFFICIENTS DE CORRELATION ENTROPIQUE

On présente quelques indicateurs de la dépendance entre deux caractéristiques d'une collectivité statistique ayant pour base le concept d'entropie

КОРРЕЛЯЦИЯ И КОЭФФИЦИЕНТЫ ЭНТРОПИЧЕСКОЙ КОРРЕЛЯЦИИ

В этой работе приведены некоторые показатели зависимости между двумя характеристиками статистической совокупности, основаные на понятии энтропии.

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