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Methods of Forecasting the Fulfilment of Short-Range Plans

The authors of this paper present a number of methods which permit the working out of forecasts on the basis of quantitative criteria, either at enterprises or on an all-country economic scale.

To forecast the fulfilment of short-range plans, use is being made now of the data gathered by the territorial bodies of the Central Statistical Board directly from the enterprises. To this end, the way in which the plan has been fulfilled in the corresponding periods of the previous years, the seasonal character and the actual possibilities available to the enterprise are taken into account.

The econometrics and statistics have drawn up numerous models and procedures of economic forecasts used both for shorter periods *(short-term estimates)*, for average periods *(middle-range estimates)* and for longer periods *(long-range estimates)*. They have been used in both the general studies of economic conjuncture and in those of forecasts proper for at least five decades.

In the general theory of dynamic series such forecasts acquire the form of extrapolation or of econometric models whose exactness depends on several parameters. The functions describing analytically such estimates for the future and which are called predictors owe their exactness to the amount of statistical information of the past, to the statistical-mathematical analysis of this and to the size of the period for which such forecasts are made.

Generally, the problem is formulated as follows: when for a certain phenomenon (in our case, economic phenomen) there is an amount of information up to the moment t, termed by $\mathcal{I}(t)$, determine the value of the respective phenomen at the moment $t + 1, t + 2, ..., t + n$, as a function of amount of information $\mathcal{I}(t)$, with a certain error. By using the usual notations of the dy-

namic series, if at the moment t the indicator is $y(t)$, then at the moment $t + 1$, we shall have:

$$y(t + 1) = \Phi[\mathcal{J}(t + 1)] \pm \varepsilon(t + 1)$$

in which: $\varepsilon(t + 1)$ is the error (the difference between the real value and the value resulting from the forecasting of the indicator y).

Such functions may be linear, exponential, parabolic, etc. To the elaboration of the forecasting methods we must take into consideration the fact that the decomposition of a dynamic series leads to the appearance of the following components (formulated by the classical econometrics): the long-range trend which may be spotlighted by adjustment, cyclic variation, seasonal variation and the so-called aleatory deviation. *

The condition imposing itself for the determination of values $y(t + 1)$ is that of minimum, for obtaining — in the conditions of a given amount of information — a minimum value of the error. If for some phenomena the predictors of the type of linear function are the most adequate, in other cases appropriate are the predictors of the type of exponential function or the so-called multiple previsional models. The methods of regression and correlation, as well as other methods mentioned by the speciality literature, can be applied with the same success.

It appears more conspicuously that at a macroeconomic level the results are more precise, owing to the existence of richer information and also to the fact that on such a level diverse laws operate enabling the process modelling and, hence, the formulation of forecast. On a microeconomic level (as for instance, at the enterprises) such forecasts involve greater risks, with the error appearing much more important.

Under the conditions of planned economy, the forecasting methods and models acquire some characteristics which must be brought into notice. It was asserted for a long time that aleatory phenomena could not occur in the socialist planned economy and that the processes could not have a stochastic, but a determinist character (in the statistical sense of the word) bearing in mind the function fulfilled by the plan of development of the national economy. As a consequence, the forecasting methods and models could not find application. This assertion is naturally groundless in the rigorous philosophical meaning, as the aleatory phenomena occur in the planned economy as well. It is true, however, that in the planned economy the forecasts are limited to a short-range framework, owing to the fact that the lines of future development, for a longer period, are established under the plan, whose active intervention may change the development curve of a phenomenon (heavy investments, the priority development of an industry etc.). In such cases the forecasts, under the form of a general extrapolation, acquire the value of *conditional prospects* of an informative character.

The most adequate use of these methods appears to be the forecast of the plan fulfilment on shorter periods. Further we give two situations of this kind. The first one refers to the forecast in the course of the year, after several months of plan fulfilment; the second one — to the provision coinciding with

* Some of these components are valid only for the capitalist economy.

the start of the year covered by the plan. The conclusions reached are susceptible of generalization. There are, of course, other situations, asking for other methods.

1. THE FORECAST DURING THE YEAR

To estimate the output at the end of the year one must start from the achievements of the past months. On twe whole, the greater the number of months for which data are available, the more accurate the forecast is.

The computation is based on the seasonal variations recorded in the previous years. For establishing these variations use is made of the statistical data on the monthly and annual output during the last n years:

Let these data be:

$$\begin{aligned} p_{11} + p_{12} + p_{13} + \ldots + p_{1n} &= P_1, \\ p_{21} + p_{22} + p_{23} + \ldots + p_{2n} &= P_2, \\ \vdots \qquad\qquad\qquad\qquad\quad &\vdots \\ p_{n1} + p_{n2} + p_{n3} + \ldots + p_{nn} &= P_n, \end{aligned}$$

where

p_{ij} — the output in the j month of the i year;

P_i — the total output of the i year.

The contribution of each month to the annual output for the n years is computed. If we mark this relation, expressed in percentages, by $\alpha_{ij} = \frac{p_{ij}}{P_i}$, we shall have:

$$\begin{aligned} \alpha_{11} + \alpha_{12} + \ldots + \alpha_{1n} &= 100, \\ \alpha_{21} + \alpha_{22} + \ldots + \alpha_{2n} &= 100, \\ \vdots \qquad\qquad\qquad\quad &\vdots \\ \alpha_{n1} + \alpha_{n2} + \ldots + \alpha_{nn} &= 100. \end{aligned}$$

On the basis of these percentages the mean percentage for n years is determined for each month: *

$$\bar{\alpha}_j = \frac{\sum_{i=1}^{n} \alpha_{ij}}{n}, \quad \bar{\alpha}_1 + \bar{\alpha}_2 + \ldots + \bar{\alpha}_n = 100$$

as well as the cumulative mean percentages:

$$\bar{\alpha}_1;\ \bar{\alpha}_1 + \bar{\alpha}_2;\ \bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_3;\ \ldots \sum_{i=1}^{n} \alpha_j = 100.$$

* This percentage can be determined also upon the formula

$$\bar{\alpha}_j = \frac{\sum_{i=1}^{n} p_{ij}}{\sum_{i=1}^{n} P_i}.$$

This series of percentages shows the fulfilled share of the annual output (per n years on the average) in the first month, in the first two months, in the first three months, etc.

By using these percentages, the annual output for the current year can be forecast with some approximation, when the output for the first k months is known. Thus, if $\sum_{j=1}^{n} P_{n-i,j}$ production units have been turned out from a certain product in the first k months, and supposing the production rate in the current year $(n+1)$ to be the same as the mean rate in the last n years, the output for the entire year $(n+1)$ will be estimated as follows:

$$P'_{n+1} = \frac{\sum_{j=1}^{n} p_{n+1,j}}{\sum_{j=1}^{n} \bar{\alpha}_j} \times 100.$$

Generally speaking, the method of estimation of the annual output on the basis of the production rate in the preceding years can be successfully used, yielding results nearing the real ones, only in the case when there are no additional production capacities commissioned in the course of the year. Otherwise, the results obtained must be worked out anew, with a correction coefficient being applied, contingent upon the contribution to the volume of the previous output by the newly commissioned capacities.

Yet, the hypothesis of maintenance of the previous years' rate is not rigorous because, as shown by the examining of the statistical data in the previous years, the contribution of each month to the annual output varies within certain limits.

Consequently, these limits (confidence intervals) have to be found for diverse confidence levels.

For establishing the confidence limits, the mean square deviation of the production percentages in the respective month (or of cumulative percentages) in the last n years is computed:

$$\sigma_k = \sqrt{\frac{\sum_{i=1}^{n} (\alpha_{ik} - \bar{\alpha}_k)^2}{n-1}}.$$

Taking into account that the number of years considered is quite small, the distribution of the deviation is given by the Student law. Consequently, in establishing the confidence level, the confidence limits for n—1 degrees of freedom are computed, with the help of the Student criterion table.

Taking as an example the confidence level of 95%, the percentage of participation of month k can be estimated, with the following confidence interval:

$$\alpha_{n+1,k} = \bar{\alpha}_k \pm t_{95\%}\sigma_k.$$

or for the cumulative percentage for the first k months:
(in the above example σ'_k is established according to the deviations of the cumulative percentages).

The output for the whole year $n+1$, P'_{n+1} will be estimated to be within the limits:

$$\frac{\sum_{j=1}^{k} P_{n+1,j}}{\sum_{j=1}^{k} \bar{\alpha}_j + t\sigma'_k} \times 100 \leqslant P'_{n+1} \leqslant \frac{\sum_{j=1}^{k} p_{n+1,j}}{\sum_{j=1}^{k} \bar{\alpha}_j - t\sigma'_k} \times 100.$$

Numerical example:

We apply the above method in the estimation of methane output, with a strong seasonal character. The computation is based on the statistical data of monthly and annual output in the years 1959—1965. The estimate is made for the year 1966, whose effective output is known, existing thus the possibility of applying the accurate degree of estimates.

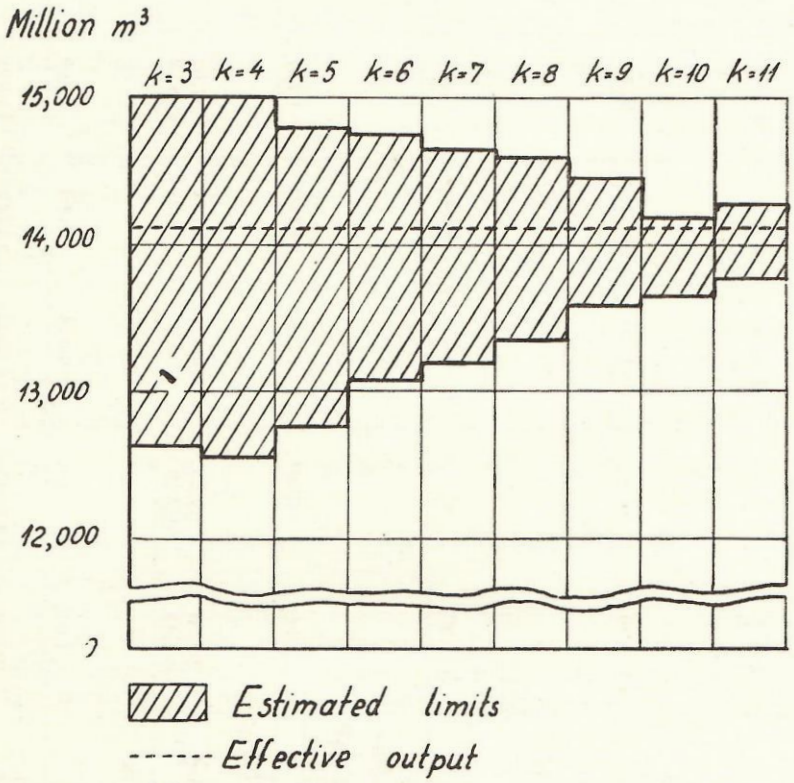

Fig. 1 Estimate of the annual methane output in 1966, after k months of execution

It results from the figures in the table that in all instances, starting with the forecasts after three months of execution, the effective output stands within the estimated limits, and these limits draw in the course of time (that is, concomitantly with the increase of the volume of data) nearer and nearer.

Table 1

	Computation of estimate after k months of fulfilment								
	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$	$k = 10$	$k = 11$
Mean percentage (1955—1965) from the annual output after k months	29.70	37.50	44.43	50.71	57.06	63.56	70.63	79.15	88.69
The square of deviations from the mean percentage $\sum_{i=1}^{n} (\alpha_{ij} - \bar{\alpha}_k)^2$	7.22	9.55	9.11	8.53	7.65	6.33	4.33	1.96	1.67
Dispersion $\sigma'_k = \sqrt{\frac{\sum_{i=1}^{n} (\alpha_{ik} - \alpha_k)^2}{n}}$	1.017	1.301	1.233	1.192	1.127	1.027	0.84	0.57	0.53
Confidence limits (95%) $\sum_{j=1}^{k} \bar{\alpha}_j \pm 3{,}45\sigma'_k$	29.70 ±2.49	37.50 ±3.19	44.43 ±3.02	50.71 ±2.92	57.02 ±8.76	63.56 ±2.52	70.63 ±2.06	79.15 ±1.40	88.69 ±1.30
Effective output in 1966 in the first k months (million cu.m.)	4,077	5,141	6.078	7,004	7,909	8,865	9,876	10,995	12,413
Estimated output for the year 1966 (million cu.m.)	12,665— —14,983	12,635— —14,984	12,809— —14,678	13,060— —14,656	13,221— —14,565	13,416— —14,523	13,599— —14,400	13,605— —14,141	13,794— —14,204

Note: The effective output was of 14,079 million cubic metres in 1966.

2. FORECASTS AT THE START OF THE YEAR

For the forecasts in the course of the year, one had to proceed from the premise that the production conditions of the current year are known and reflected in the output of several months of execution.

But, if the forecast is made at the start of the year, these conditions (the bringing into service of the new capacities, the improvement of the production and labour process, etc.) are unknown and consequently, although the production rate is known, its level keeps being unknown.

That is why in the case of forecasts at the start of the year account is taken of the plan figures established for the current year, which are supposed to be established in keeping with the new production conditions.

We shall have in this case, besides the variation of the production rate (about the mean rate) also the variation of the effective output about the plan figure.

In order to estimate the annual output at the end of the year, the way in which the plan was fulfilled in the last n years is examined, namely the percentage relations $\beta_i = \frac{P}{Q_i}$ $(i = 1, 2, ..., n)$ in which:

P_i — is the effective output in the i year;

Q_i — the plan figure corresponding to the output;

The output in the year $(n+1)$ will be appreciated at:

$$P'_{n+1} = Q_{n+1}\left(\frac{\bar{\beta}}{100} \pm \frac{t\sigma_\beta}{100}\right).$$

To estimate the monthly output account must be taken of both the variations about the plan figure, and of the variations towards the mean monthly rate.

In forecasting this output, one has to proceed for each month from the monthly plan quota $\frac{\alpha_j}{100} Q_{n+1}$ (established according to the rate), yet corrected by the mean percentage of plan fulfilment $\frac{\bar{\beta}}{100}$. It shall be agreed hence that the plan is fulfilled to the same proportion in all the months of the year. To the establishment of the confidence limits account shall be taken of both dispersion about the mean percentage of plan fulfil- ment σ_β, and of the dispersion about the mean monthly rate σ^α.

The monthly output will be consequently estimated upon the formula

$$P'_{n+1,j} = \frac{Q_{n+1}}{100}\left(\bar{\beta}\,\frac{\bar{\alpha}_j}{100} \pm t\sqrt{\sigma_\alpha^2 + \sigma_\beta^2}\right).$$

Numerical example

We shall refer, also in this case, to the output of methane as an example. In the last seven years (1959—1965) the degree of plan implementation varied between 86.7 per cent and 99.28 per cent, with an average of 94.93

per cent. The plan production was established to 15,000 million cubic metres in 1966. The average square deviation of the percentages of plan fulfilment was of

$$\sigma_\beta = \sqrt{\frac{\Sigma(\beta_i - \bar{\beta})^2}{n-1}} = \sqrt{\frac{100 \cdot 3739}{6}} = \sqrt{16.729} = 4.09$$

The annual estimated output will then be of

$$P'_{n+1} = Q_{n+1}\left(\frac{\bar{\beta}}{100} \pm t\frac{\sigma_\beta}{100}\right) = 15{,}000\left(0.9493 \pm 2.45\frac{4 \cdot 09}{100}\right) =$$

$$= 15{,}000(0.9493 \pm 0.1002)\text{million cu.m.}$$

or

$$12.236 \leqslant P'_{n+1} \leqslant 15.742$$

Actually, the output was of 14,079 million cubic metres. The quite broad imits adopted were influenced by the poor implementation of the 1961 plan. If that year were not taken into consideration, we would have $\bar{\beta} = 96.305$; $\sigma_\beta = 2.07$, and the limits would be 13,685 and 15,206 million cubic metres, that is much more restricted.

The estimate of the monthly output in 1966, upon the above formula, proceeds according to the diagram on page 49.

For each month of the year, the effective output stands within the estimated limits.

3. ADJUSTMENT BY EXPONENTIAL CURVE

In the computation shown in Table 2 the forecast for the methane output in 1966 proceeds from the plan figure. In the event that this figure is unknown or has to be estimated, other bases may serve for the forecast of the annual and monthly output in the next year, namely the series of production data for the latest years.

In examining these data a certain law can be discovered in the time development of the output of the respective product and an analytical curve, which would describe most approximately the evolution of this output, can be sought. In many cases the tendency of the economic series is better described by the logistic curve, characterized by an advance in geometrical progression (that is, after an exponential curve) in the first part of the evolution, which however is changing in the course of time, the percentage rises getting smaller and smaller concomitantly with their reaching a certain limit.

For adjusting the dynamic series use is generally made of the least squares method. Practically, however, a curve which can approximate the data of the latest years must be found.

The evolution of the methane output in latest years does not adjust itself in good conditions by the logistic curve, since the percentage rises (the last three years excepted) are not specific to this curve.

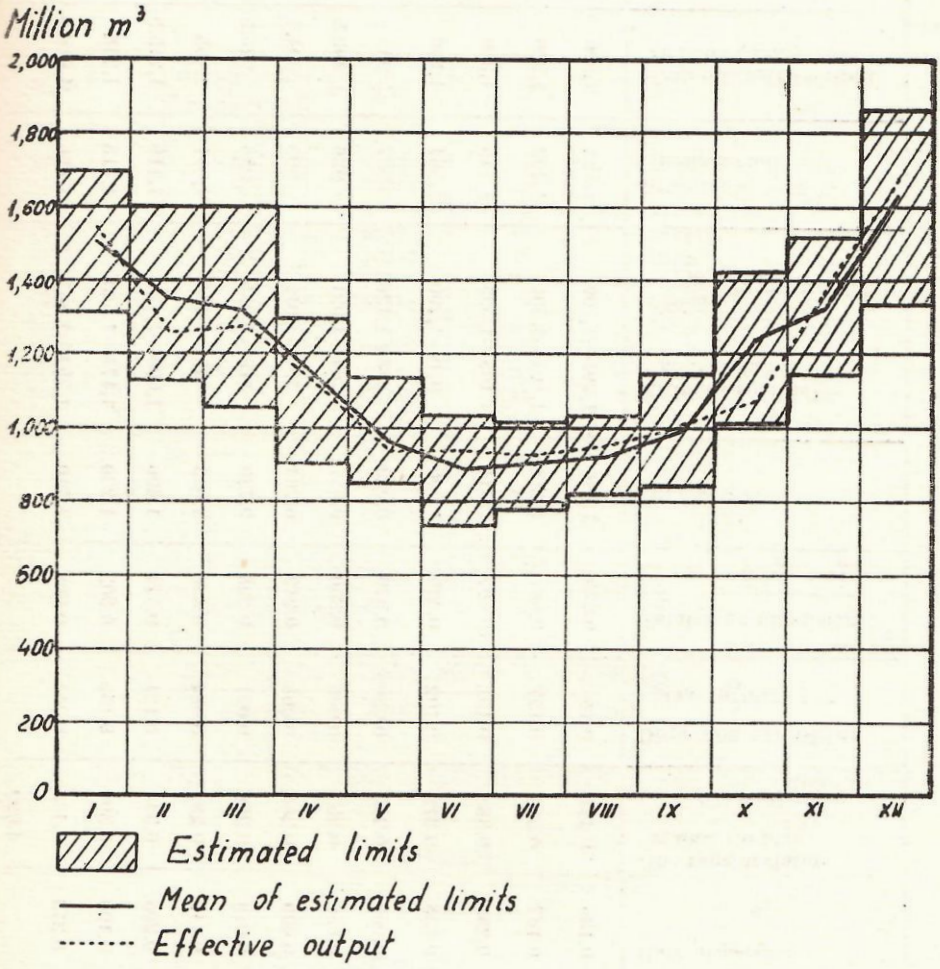

Fig. 2 Estimate of the methane output in 1966

Better results are yielded by adjusting the exponential curve, which demonstrates that we are still in the first stage of the logistic curve. *

* Practically, one cannot admit that the evolution of the methane output will continue for several years in the future at the pace of the exponential curve. This evolution may be however admitted for a few years.

Table 2

Month j	Mean monthly rate $\bar{\alpha}_j$	Monthly quota of plan fulfilment $\bar{\alpha}_j \frac{\beta}{100}$	Rate dispersion $\sigma^2_{\alpha_j}$	The type deviation against the plan $\sigma_{1j} = \sigma_j \frac{\bar{\alpha}_j}{100}$	Dispersion against the plan $\sigma^2_{\beta j}$	Total type dispersion $\sigma_{ij} = \sqrt{\sigma^2_{\alpha j} + \sigma^2_{\beta j}}$	$t\sigma_{ij}$ t = 2.45	Estimated output (million cu.m.) $\frac{Q_{n+1}}{100}\left(\bar{\alpha}_j \frac{\bar{\beta}}{100} \pm t\sigma_{ij}\right)$	Effective output (million cu.m.)	Mean estimated output (million cu.m.)	Mean deviation against the effective output
1	10.53	10.00	0.139	0.431	0.186	0.570	1.396	1,291—1,709	1,524	1,500	— 1.6%
2	9.69	9.20	0.187	0.396	0.157	0.587	1.438	1,164—1,596	1,242	1,380	+11.1%
3	9.48	9.00	0.293	0.388	0.151	0.667	1.634	1,105—1,595	1,310	1,350	+ 3.1%
4	7.80	7.40	0.138	0.319	0.102	0.490	1.200	930—1,290	1,064	1,100	+ 3.4%
5	6.93	6.58	0.063	0.283	0.080	0.379	0.929	848—1,126	937	987	+ 5.3
6	6.28	5.95	0.056	0.257	0.066	0.350	0.857	764—1,021	926	892.5	— 3.6%
7	6.35	6.03	0.029	0.260	0.068	0.312	0.764	790—1,019	905	904.5	— 0.1%
8	6.50	6.17	0.018	0.266	0.071	0.298	0.730	816—1,035	956	925.5	— 3.2%
9	7.07	6.71	0.115	0.289	0.084	0.446	1.093	842—1,170	1,011	1,006	— 0.5%
10	8.52	8.09	0.208	0.348	0.121	0.574	1.406	1,003—1,424	1,119	1,213.5	+ 8.4%
11	9.54	9.06	0.100	0.390	0.152	0.502	1.230	1,174—1,543	1,418	1,358	— 4.2%
12	11.31	10.74	0.273	0.463	0.214	0.698	1.710	1,354—1,867	1,666	1,610.5	— 3.2%
	100.00	94.93		4.090							

The actual output is within the estimated limits in each month of the year.

Assuming that the exponential curve is $y = ab^x$, by logarithmation we obtain:

$\log y = \log a + x \log b$; $z = A + Bx (A = \log a;\ B = \log b, z = \log y)$.

Table 3

Year	x	Output millions cu.m.(y)	$\log y = z$	x^2	xz	z computed	y computed
1954	1	3,629	3.55979	1	3.55979	3.52918	3,383
1955	2	3,972	3.59901	4	7.19802	3.58062	3,807.5
1956	3	4,374	3.64088	9	10.92264	3.63207	4,286
1957	4	4,633	3.66586	16	14.66344	3.68351	4,825
1958	5	5,075	3.70544	25	18.52720	3.73496	5,432
1959	6	5,782	3.76208	36	22.57248	3.78640	6,115
1960	7	6,519	3.81418	49	26.69926	3.83785	6,884
1961	8	7,197	3.85715	64	30.85720	3.88929	7,750
1962	9	8,837	3.94630	81	35.51670	3.94074	8,724
1963	10	10,001	4.00436	100	40.04360	3.99218	9,821
1964	11	11,417	4.05756	121	44.63316	4.04363	11,057
1965	12	12,867	4.10948	144	49.31376	4.09507	12,447
	78		45.72209	650	304.50725		

$$B = \frac{\Sigma z \Sigma x - n \Sigma xz}{\Sigma x \Sigma x - n \Sigma x^2} = \frac{45.72209 \times 78 - 12 \times 304.50725}{78 \times 78 - 12 \times 650} =$$

$$= \frac{3566.32302 - 3654.08700}{6084 - 7800} = \frac{87.76398}{1761} = 0.0511445;$$

$$A = \frac{\Sigma z - B \Sigma x}{n} = \frac{45.72209 - 0.0511445 \times 78}{12} = \frac{45.72209 - 3.98927}{12} =$$

$$= \frac{41.73282}{12} = 3.47773;$$

$$z = 3.47773 + 0.0511445x;$$

$$y = 3004{,}2 \times 1.125^x.$$

If the 1966 output is computed upon this curve, we find

$z_{1966} = 4.14652$, which gives

the output $y = 14{,}014$ thousand cubic metres. The effective output was of 14,079 million cubic metres.

Of course, this time too certain limits must be used for computing the forecast.

To this end, we shall consider the deviations of the effective data against the theoretical curve.

Table 4

Year	Theoretical curve data x'	Effective data x	Deviations % $\beta = (x - x')$	$\beta^2 = (x - x')^2$
1954	3,383	3,629	+7.27	52.85
1955	3,807.5	3,972	+4.32	17.89
1956	4,286	4,374	+2.05	4.20
1957	4,825	4,633	—3.98	15.84
1958	5,432	5,075	—6.57	43.16
1959	6,115	5,782	—5.45	29.70
1960	6,884	6,519	—5.30	28.09
1961	7,750	7,197	—7.14	50.98
1962	8,724	8,837	+1.30	1.69
1963	9,821	10,101	+2.85	8.12
1964	11,057	11,417	+3.26	10.63
1965	12,447	12,867	+3.37	11.36
				274.51

$$\sigma = \sqrt{\frac{\Sigma(x - x')^2}{n - 1}} = \sqrt{\frac{274.51}{11}} = \sqrt{24.995} \cong 5.$$

Consequently, the limits of the deviations against the theoretical curve will be included between $+ t\sigma$ and $-t\sigma$ ($t + 2.20$ for 11 degrees of freedom).

The probable output in 1966 will be included within the limits

$$14{,}014\left(1 - \frac{2.20 \times 5}{100}\right) \text{and } 14{,}014\left(1 + \frac{2.20 \times 5}{100}\right)$$

or between 12,472 million cubic metres and 15,556 million cubic metres.

The estimation of the monthly output is obtained just like in the preceding case, by replacing however, the deviations towards the plan by deviations towards the theoretical curve.

4. ANOTHER MATHEMATICAL MODEL

In the present situation, the output of methane might be estimated also on the basis of a model of the form:

$$y_{j+12i} = a_0^{zj+12i} a_1 C_j e^{zj+12i} \quad (i = 1, 2, ..., N;\ j = 1, 2, ..., 12). \qquad (1)$$

Underlying the drawing of the model there were the following hypotheses:

1. the forecast $y_{T+1}, y_{T+2i} \cdots, y_{T+0}$ is made bearing in mind the previous achievements (on 4—5 years) as well as the corresponding values of the monthly plans x_T, x_{T+1} which are presumed to be known;

2. there is close correlation between the series of achievements and the series of planned data;

3. the series of planned data and the series of achievements have the same general tendency, characterized by variations of a well determined direction and by seasonal variations of the same type (C_j)*;

4. The tendency is exponential and the seasonal variations are proportional to the general tendency.

The forecast for a certain calendar month is estimated by means of the least squares method or of other usual estimation methods.

When the least squares method is used for estimation, expression (1) is reduced to the linear form by means of logarithms:

$$\log_e y_{j+12i} = x_{j+12i} \log_e a_0 + \log_e a_1 + \log_e C_j + z_j + 12i \tag{2}$$

For reducing the notations, the logarithms of the terms in expression (2) are written with capital letters.

The parameters of expression (2), respectively $A_0, A_1, C_1, C_2, ..., C_{12}$ are estimated through the least squares method, provided that the amount of the deviation squares between the actual data and the adjusted data be minimum:

$$\sum_{i,j} (Y_{j+12i} - A_0 x_{j+12i} - A_1 - C_j)^2 = min. \tag{3}$$

The expression (3) is minimum when the derivatives in relation to A_0, $A_1, C_1, C_2, ..., C_{12}$ are reduced to zero.

$$\frac{\partial}{\partial A_1}\left[\sum_{i,j} (Y_{j+12i} - A_0 x_{j+12i} - A_1 - C_j)^2\right] = 0,$$

$$\sum_{j,i} (Y_{j+12i} - A_0 x_{j+12i} - A_1 - C_j) = 0,$$

$$\sum_{i,j} Y_{j+12i} = A_0 \sum_{i,j} x_{j+12i} + TA_1 + N \sum_j C_j. \tag{4}$$

On the strength of the theorem of null area, in case of series with rigid seasonal character $\sum_j C_j = 0$, by reducing the expression (4) with T and writing under a reduced form the terms

$$\frac{\sum_{i,j} x_{j+12i}}{T} = x ...; \quad \frac{\sum_{j,j} Y_{j+12i}}{T} = Y,$$

we shall have:

$$Y ... = A_0 x ... + A_1, \tag{5}$$

from where:

$$A_1 = Y ... - A_0 x ... \tag{6}$$

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* It is obvious that both series of data are affected by aleatory fluctuations.

We compute the derivative of expression (3) in relation to A_0:

$$\frac{\partial}{\partial A_0}\left[\sum_{i,j}(Y_{j+12i} - A_0 x_{j12i} - A_1 - C_j)^2\right] = 0,$$

$$\sum_{i,j} x_{j+12i}(Y_{j+12i} - A_0 x_{j+12i_2} - A_1 - C_j) = 0,$$

$$\sum_{i,j} x_{j+12i} Y_{j+12i} = A_0 \sum_{i,j} x_{j+12i} + A_1 \sum_{i,j} x_{j+12i} + \sum_{i,j} C_j x_{j+12i}$$

Finally, we compute the partial derivative in relation to C_j:

$$\frac{\partial}{\partial C_j}\left[\sum_{i,j}(Y_{j+12i} - A_0 x_{j+12i} - A_1 - C_j)^2\right] = 0,$$

$$\sum_{i}(Y_{j+12i} - A_0 x_{j+12i} - A_1 - C_j) = 0, \tag{7}$$

$$\sum_{i} Y_{j+12i} = A_0 \sum_{i} x_{j+12i} + NA_1 + NC_j.$$

The equation (7), after a number of simplifications and separations of terms, is brought to the form:

$$C_j = \frac{\sum_{i,j} Y_{j+12i}}{N} - A_0 \frac{\sum_{i,j} x_{j+12i}}{N} - A_1 \tag{8}$$

on the basis of which the values of C_j are estimated.

By substituting for A_1 in (8) the expression equivalent to $\frac{\sum_{i,j} Y_{j+12i}}{N} = Y_j$; $\frac{\sum_{i,j} x_{j+12i}}{N} = x_j$ we shall have:

$$C_j = Y_j - Y - A_0(x_j - x\,..) \tag{9}$$

Replacing in (6) the expression of C_j and A_1 and assigning A_0 as common factor, we have:

$$A_0 = \frac{\sum_{i,j} x_{j+12i}(Y_{j+12i} - Y_j)}{\sum_{i,j} x_{j+12i}(x_{j+12i} - x_j)}. \tag{10}$$

Numerical example

For estimating the output of methane in the months of November and December 1966, we shall use the series of monthly data of the last 48 months (November 1962—October 1966):

$$Y \ldots = 2.98189i; \quad \sum_{i,j} x_{j+12i}(Y_{j+12i} - Y_j) = 372.40969,$$

$$x \ldots = 1015i; \quad \sum_{i,j} x_{j+12i}(x_{j+12i} - x_j) = 1{,}155{,}973.$$

On the basis of the above values we have computed the parameters:

$$A_0 = 0.00032; \; A_1 = 2.65709$$

The value of $C_1, C_2, \ldots, C_{12}$ constants is written in column 3 of table 5.

Table 5

Month	Y_j	x_j	C_j
1	3.11168	1,283	+0.04403
2	3.06196	1,154	+0.03559
3	3.06051	1,189	+0.02294
4	2.97171	983	+0.00006
5	2.90922	879	—0.02915
6	2.87665	758	—0.02300
7	2.87799	801	—0.03542
8	2.89403	838	—0.03122
9	2.92252	916	—0.02769
10	2.99622	1,065	—0.00167
11	3.00958	1,047	+0.01745
12	3.09055	1,257	+0.03122

The estimated output for November 1966 is of 1,373 million cubic metres.

$$Y_0^{11} = (0.00032 \times 1447) + 2.65709 + 0.01745 = 3.13758.$$

The antilogarithm of 3.13758 = 1,373

Similarly, the forecast for December 1966 — 1,575 million cubic metres — has been computed.

$$Y_0^{12} = (0.00032 \times 1{,}590) + 2.65709 + 0.03122 = 3.19711$$

The data on the output of gas in November and December 1966 are aleatory magnitudes, with a given probability within certain limits.

The limit forecasting error * $E(y_0^p - y)^2$ depends in the present case on the proportions in which varies the fulfilment of the plan during the whole period and in the respective months.

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* The forecast error was computed upon the formula:

$$E(y_0^p - y_0)^2 = \sigma_z^* \left[1 + \frac{1}{T} + \frac{(x_0 - x \ldots)^2}{\sum_{i,j}(x_{j+12i} - x \ldots)^2} \right]$$

where:

$$\sigma_z^{*2} = \frac{\sum_{i,j}(y_{j+12i} - y_{j+12i})^2}{T - 1}.$$

By making use of the tables of the Student function for $\nu = 46$* corresponding to a 0.975 probability, we find $t = 2.014$.

Hence, with a probability of 0.975 one can be sure that the real value of the methane gas output delivered in November 1966 is within the limits

$$1{,}373 - (2.014 \times 63) \leqslant y_0 \leqslant 1{,}373 + (2.014 \times 63),$$

respectively

$$1{,}246 \leqslant y_0 \leqslant 1{,}500$$

The predictable value of the methane output delivered in November 1966 is of 1,376 million cubic metres, while the real value of the output stands at 1,418 million cubic metres; the predictable error is consequently of 45 million cubic metres, representing some 3 per cent of the methane gas output delivered in November 1966.

If we add to the output of methane delivered in the first ten months of 1966 the estimated output of November and December 1966, we shall have 13,942 million cubic metres of methane with an error amounting to only —0.98 per cent for the whole year.

CONCLUSIONS

The methods and procedures proposed for the plan forecast are obviously based on methods of analysis of the dynamic series for periods representing fractions of the calendar year (quarters, months, etc.). These periods have to be selected so as to express the most characteristic features of the evolution of the predictable economic phenomena and to correspond at the same time to the plan periods. The exactness of estimates depends mainly on the amount of information concerning the evolution of the past economic phenomena; consequently, the "incertitude" appears as a mean of the variations of the past series. In this situation, no hypothesis has been introduced in connection with the probable changes in the future evolution of the pehomenon. However, the forecast is not appreciated as sure, as in the case of the determinist models, so that to each possible achievement a probability may be added; its magnitude depends on the variations of the series, respectively on the proportions to which the previous plan has been fulfilled.

For distancing the action of the factors due to unique situations, to exceptional events, it is recommended to ground the forecasts on an amount of information recorded in a longer period of time. The excessive variations may not eventually be taken into consideration in the computation.

On the basis of the above values we have computed the parameters.

The models envisaged in (1), (2) and (4) may be considered models with connection variables, because the estimates are closely correlated to the plan tasks.

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* In the tables of the Student distribution, to $\nu = 40$ degrees of freedom corresponds $t = 2.021$ at a probability of 0.975 probability. For $\nu = 46$ degrees of freedom, $t = 2.014$ was computed by interpolation, corresponding to a probability of 0.975.

The results of forecasts in the case of models with connection variables depend, as a consequence, not only on the way in which the plan has been fulfilled in the past, but also on the scrupulous way in which the plan tasks are established for the coming periods.

For the model in (4) the fundamental hypotheses underlying the elaboration of the model must be fulfilled, at least approximately. For verifying these hypotheses diverse non-parameter graphs and tests of signification are used.

The model may imply essential adaptations in case of the series with mobile seasonal variations and trends which can be approximated by a high degree polynom.

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METHODES D'EVALUATION DE L'ACCOMPLISSEMENT DU PLAN PAR PETITES PERIODES

Les méthodes d'évaluation de l'exécution du plan peuvent être assimilées, dans certaines conditions et avec certaines reserves, aux méthodes et aux modèles prévisionnels (à court et moyen terme). L'extension de la période à plusieurs années mènerait à des notions de prévision à long terme, ce qui dépasse le cadre d'application efficace des méthodes de ce genre.

Les périodes pour lesquelles la question se pose d'employer ces méthodes sont le mois, le trimestre et le sémestre, leur limite supérieure étant l'année (en tant qu'unité du calendrier et du plan). Les méthodes proposées partent de l'examen du matériel statistique empirique de plusieurs années, concernant un nombre de produits importants en leur expression naturelle, ainsi que d'autres indices.

L'utilisation de plusieurs courbes et adaptations, en fonction de l'évolution remarquée des indices en question, permet de saisir une certaine régularité à caractère de «loi de comportement». Dans certains cas, où les variations saisonnières sont plus accentuées, il est nécessaire d'utiliser des procédés suplémentaires qui tiennent compte de cette circonstance. Pour mesurer les différents liens remarqués et l'effet à retardement des investissements achevés, les auteurs préconisent l'emploi de la méthode du «lag correlation». Les méthodes proposées, en première aproximation, permettent de faire des estimations sur l'accomplissement du plan aux indices élémentaires. Pour ce qui est des indices de valeur, qui soulèvent des problèmes plus complexes, des études ont été amorcées, afin de trouver des modalités adéquates.

МЕТОДЫ ПРЕДВАРИТЕЛЬНОЙ ОЦЕНКИ ВЫПОЛНЕНИЯ ПЛАНА НА НЕБОЛЬШИЕ ПЕРИОДЫ

Методы предварительной оценки выполнения плана могут быть уподоблены при определенных условиях и с определенными ограничениями, методам и моделям предвидения (на короткий срок и на средний срок). Распространение периода на большее число лет привело бы к понятию предвидения на долгий срок, что, однако, переходит рамки эффективного применения подобных методов.

Периодами, для которых ставится вопрос применения этих методов, являются месяц, квартал, полугодие и их высшим лимитом является год (как календарная единица плана). Предлагаемые методы исходят из наблюдения эмпирического статистического материала за несколько лет, касающегося нескольких важных продуктов, в их натуральном выражении, и других показателей.

Применение многих кривых и подгонок, в зависимости от наблюдаемой эволюции соответствующих полазателей, ведет к выявлению некоторых закономерностей, носящих характер «законов поведения». В отдельных случаях, с наибольшим сезонным характером, необходимо использование некоторых дополнительных методов, учитывающих это обстоятельство. Для измерения различных наблюдаемых связей, а также и запоздалого эффекта законченных вложений, рекомендуется использование метода соотношений (lag correlation). Предложенные методы, на первый взгляд, позволяют предвидеть выполнение плана по элементарным показателям. Для стоимостных показателей, которые ставят более сложные проблемы, проводятся исследования с целью найти соответствующие способы.