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A Dynamic Model for Population Structure Planning in Sheep Farms

The paper presents a dynamic population mod l, in a sheep farm regarding the planning of population structure, in view to maximize the income (wool production, milk production and growth in weight).

In the frame of the dynamic model characteristic phenomena of the sheep farm were grasped both the dynamics and the constraints which restrict it. Thus, are described in time and space the way of development of the technological process of sheep growth, the evolution and structure of the population, as well as the inputs and outputs in and from the system.

The dynamic model was tested with real data from a farm having thus a prospection character.

1. INTRODUCTION

The question of prospecting the economic efficiency for the technological process of sheep breeding was approached on a higher level and with adequate techniques which allow a systemic approach of the farm, in view to obtain optimal performances as regards production and economic indices [1], [4].

In the frame of the dynamic model [4], phenomena specific to sheep farms were analysed, namely: growth during pasture, growth in stabulation, preparation for covering, covering, gestation, delivery, suckling, milking and wool-cutting, as well as the relation between these natural phenomena and the constraints imposed by fodder stocks and their structure, food rations and their structure per capita, shelter, money stocks for fodder and purchasing of sheep with higher biological qualities, land area, arable area, proportion of breeding rams versus ewes in the basic flock, planned wool and milk pro-

duction as well as the structure of the flock at the end of the planning horizon [5].

Thus, the development of the growth process in the frame of the sheep farm is grasped in time and space, as well as the evolution of the number of sheep — inputs and outputs into and from the system (mortality, sacrifices out of necessity, births, purchases, sales, reformation and movements from an age class to another) [6].

2. MATHEMATICAL ACCOUNT AND DESCRIPTION OF A DYNAMIC MODEL FOR THE TECHNOLOGICAL PROCESSES TAKING PLACE IN A SHEEP FARM

The mathematical model which was developed describes the evolution in time and space of the number of sheep, as well as the set of informations which links the former with other activities within the farm [2].

The leading elements for the sheep population, specific to the farm, are: mortality, sacrifices out of necessity, and birth, and they influence the evolution of population change [7].

It must be specified that the main element for the farm activity is time; by means of it the model grasps the dynamics (evolution in time) of sheep population as well as specific phenomena related to covering, gestation, delivery, suckling, milking, preparation for covering, growth during pasture, growth in stabulation [2], [3].

2.1. Dynamic equations

The dynamic equations describe the development in time and space of changes in population, starting from the initial state of the system proceeding through the stages of covering, gestation, lamb birth, milk and wool production and finishing with death and sacrifices out of necessity, scheduled sales and reformations.

$$x_1(k+1) = (1 - M^{x_1}(k) - C_1^R(k))\, x_1(k) + u_1(k) - V_1'(k) + \alpha_{51}(k)\, x_5(k) - V_1(k) \quad (1)$$

$$x_2(k+1) = (1 - M^{x_2}(k) - C_2^R(k))\, x_2(k) + u_2(k) - V_2'(k) + \alpha_{62}(k)\, x_6(k) - V_2(k) \quad (2)$$

$$x_3(k+1) = (1 - M^{x_3}(k) - \alpha_{35}(k))\, x_3(k) + \alpha_b C^{BM}(k)\, x_2(k) + u_3(k) - V_3'(k) - V_3(k) \quad (3)$$

$$x_4(k+1) = (1 - M^{x_4}(k) - \alpha_{46}(k))\, x_4(k) + \alpha_b C^{BF}(k)\, x_2(k) + u_4(k) - V_4'(k) - V_4(k) \quad (4)$$

$$x_5(k+1) = (1 - M^{x_5}(k) - \alpha_{51}(k))\, x_5(k) + \alpha_{35}(k)\, x_3(k) + u_5(k) - V_5'(k) - V_5(k) \quad (5)$$

$$x_6(k+1) = (1 - M^{x_6}(k) - \alpha_{62}(k))\, x_6(k) + \alpha_{46}(k)\, x_4(k) + u_6(k) - V_6'(k) - V_6(k) \quad (6)$$

where:

$k = 1, \ldots, 24$ number of steps
x_1 — males, basic flock (MBF)
x_2 — females, basic flock (FBF)
x_3 — males, current year (MCF)
x_4 — females, current year (FCY)
x_5 — males, preceeding year (MPY)

x_6 — females, preceeding year (FPY)
M^{x_1} — coefficient of mortality and sacrifices out of necessity for MBF
M^{x_2} — coefficient of mortality and sacrifices out of necessity for FBF
M^{x_3} — coefficient of mortality and sacrifices out of necessity for MCY
M^{x_4} — coefficient of mortality and sacrifices out of necessity for FCY
M^{x_5} — coefficient of mortality and sacrifices out of necessity for MPY
M^{x_6} — coefficient of mortality and sacrifices out of necessity for FPY
α_{35} — coefficient for the movement of MCY to MPY
α_{46} — coefficient for the movement of FCY to FPY
α_{51} — coefficient for the movement of MPY to MBF
α_{62} — coefficient for the movement of FPY to FBF
α_b — birth growth
C^{BM} — birth coefficient for MCY
C^{BF} — birth coefficient for FCY
C_1^R — reformation coefficient for MBF
C_2^R — reformation coefficient for FBF
V_1 — scheduled sales out of MBF
V_2 — scheduled sales out of FBF
V_3 — scheduled sales out of MCY
V_4 — scheduled sales out of FCY
V_5 — scheduled sales out of MPY
V_6 — scheduled sales out of FPY
V_1' — non-scheduled sales out of MBF
V_2' — non-scheduled sales out of FBF
V_3' — non-scheduled sales out of MCY
V_4' — non-scheduled sales out of FCY
V_5' — non-scheduled sales out of MPY
V_6' — non-scheduled sales out of FPY
u_1 — purchases of MBF
u_2 — purchases of FBF
u_3 — purchases of MCY
u_4 — purchases of FCY
u_5 — purchases of MPY
u_6 — purchases of FPY

$x_i(k)$ is the vector of states representing the sheep-population stock at a certain development stage during the process of growth

$u_i(k)$ is the vector of controls referring to scheduled purchases of sheep

$V_i'(k)$ is the vector of controls referring to unscheduled sales of sheep

$V_i(k)$ is the vector referring to the influence of the environment upon scheduled sales.

2.2. Constraints

Since the technological process takes place within given material conditions, within existing food and shelter capacities, the dynamics of the system is subject to a series of constraints which model the actual conditions of process development and its influence upon sheep-population dynamics.

The activity in the frame of a sheep farm takes place continuously, day by day, during 365 days. The dimension of the dynamic model would

have been to large if a day-step had been used, reason for which a 15 day step (Λt) has been choosen. Under these conditions the model has 24 steps.

Following constraints were take into account by the model:

— providing for sheep in the second part of the gestation shelter 0.8—1 sq m and paddock, 1,6—3 sq m

$$x_2(k) \leqslant b_1(k) \tag{7}$$

$$3x_2(k) \leqslant b_2(k) \tag{8}$$

— keep lambs 0—7 days of age in individual boxes or boxes with a capacity of 15—20 sheep

$$x_3(k) + x_4(k) \leqslant b_3(k) \tag{9}$$

or

$$1/20x_2(k) \leqslant b_4(k) \tag{10}$$

— sheltering and tending of mother-sheep in shelters with a capacity of 200 sheep

$$1/200x_2(k) \leqslant b_5(k) \tag{11}$$

— tending lambs 7—20 days of age by lots of 40—50, in separate compartments having 0.2 sq m per capita

$$0.2x_3(k) + 0.2\ x_4(k) \leqslant b_6(k) \tag{12}$$

— scheduled milk production

$$\sum_{K=1}^{N} 2x_2(k) \geqslant b_7(k) \tag{13}$$

— scheduled wool production

$$\sum_{x=1}^{i} (8.25\ x_1(k) + 8.25x_2(k) + 15x_5(k) + 12x_6(k)) \geqslant b_8(k) \tag{14}$$

— 100 ewes require 3 breeding rams

$$3x_2(k) - 100\ x_1(k) \leqslant 0 \tag{15}$$

— during the period 1—15 July, 30—40 ejaculates are gathered from rams of the basic flock (MBF)

$$\sum_{x=1}^{i} (30x_1(k) - 0.5x_2(k) + 40x_5(k) - 0.5x_6(k)) \geqslant 0 \tag{16}$$

— stock of money for sheep purchasing

$$\sum_{K=1}^{i}\sum_{u=1}^{i} (P_a^1u_1(k) + P_a^2u_2(k) + P_a^3u_3(k) + P_a^4u_4(k) + P_a^5u_5(k) + P_a^6u_6(k)) \leqslant b_9(k) \tag{17}$$

P_a^i — unitary purchasing price per sheep capita

— structure of sheep population 31 December

$$\sum_{x=1}^{i}(x_1(k) + x_2(k) + x_3(k) + x_4(k) + x_5(k) + x_6(k)) = b_{10}(k) \quad (18)$$

— sheep density on land area

$$\sum_{x=1}^{i}(x_1(k) + x_2(k) + x_3(k) + x_4(k) + x_5(k) + x_6(k)) \gtreqless b_{11}(k) \quad (19)$$

— sheep density on arable area

$$\sum_{x=1}^{i}(x_1(k) + x_2(k) + x_3(k) + x_4(k) + x_5(k) + x_6(k)) \gtreqless b_{12}(k) \quad (20)$$

— available corn concentrates

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(p_1x_1(k) + p_2x_2(k) + p_3x_3(k) + p_4x_4(k) + p_5x_5(k) + p_6x_6(k)) \leqslant b_{13}(k) \quad (21)$$

p_i — specific corn consumption per sheep capita
— available oat concentrates

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(0_1x_1(k) + 0_2x_2(k) + 0_3x_3(k) + 0_4x_4(k) + 0_5x_5(k) + 0_6x_6(k)) \leqslant b_{14}(k) \quad (22)$$

o_i — specific oat consumption per sheep capita
— available lucerne hay

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(f_1x_1(k) + f_2x_2(k) + f_3x_3(k) + f_4x_4(k) + f_5x_5(k) + f_6x_6(k)) \leqslant b_{15}(k) \quad (23)$$

f_i — specific lucerne hay consumption per sheep capita
— available straw

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(g_1x_1(k) + g_2x_2(k) + g_3x_3(k) + g_4x_4(k) + g_5x_5(k) + g_6x_6(k)) \leqslant b_{16}(k) \quad (24)$$

g_i — specific straw consumption per sheep capita
— available silo-corn

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(s_1x_1(k) + s_2x_2(k) + s_3x_3(k) + s_4x_4(k) + s_5x_5(k) + s_6x_6(k)) \leqslant b_{17}(k) \quad (25)$$

s_i — specific silo-corn consumption per sheep capita
— available green stuff (rye + lucerne + Sudan grass + natural pasture + ocasional pasture)

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(m_1x_1(k) + m_2x_2(k) + m_3x_3(k) + m_4x_4(k) + m_5x_5(k) + m_6x_6(k)) \leqslant b_{18}(k) \quad (26)$$

m_i — specific green stuff consumption per sheep capita
— available mineral salts

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(sm_1x_1(k)+sm_2x_2(k)+sm_3x_3(k)+sm_4x_4(k)+sm_5x_5(k)+sm_6x_6(k))\leqslant b_{19}(k) \tag{27}$$

sm_i — specific consumption of mineral salts per sheep capita
— available nutrient units

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(un_1x_1(k)+un_2x_2(k)+un_3x_3(k)+un_4x_4(k)+un_5x_5(k)+un_6x_6(k))\leqslant b_{20}(k) \tag{28}$$

un_i — specific consumption of nutrient units per sheep capita
— ordered skins

$$\sum_{K=1}^{N}\sum_{x=1}^{i}(M^{x_1}(k)x_1(k)+M^{x_2}(k)x_2(k)+M^{x_3}(k)x_3(k)+M^{x_4}(k)x_4(k)+M^{x_5}(k)x_5(k)+$$
$$+M^{x_6}(k)x_6(k))\leqslant b_{21}(k) \tag{29}$$

— lamb weight at birth

$$\sum_{x=1}^{i}(8.8x_3(k)+8.8x_4(k)-100x_2(k))\leqslant 0. \tag{30}$$

2.3. **Goal function** (performance criterion)

The performance criterion — maximization of the farm incomes — is defined starting from the goals imposed by the activity of the sheep farm, which require the achievement of a certain population structure in view to obtain maximal planned incomes with minimal feeding expenses:

$$J=\max_{x_i,u_i}\left\{\sum_{K=1}^{N}[c_1^Tx_i(k)+c_2^Tu_i(k)]\right\} \tag{31}$$

c_1^T and c_2^T represent the incomes from, and respectively the expenses for the products of the farm.

The initial state of the system is given by

$$x_i(0)=x_0. \tag{32}$$

By solving model (1)—(32), with given $x(0)$, $V(k)$, $c(k)$, $k=1, N$, the movement of the population is obtained under the form of a set of controls and states: $x(0)$, $u(0)$, $V'(0)$; $x(1)$, $u(1)$, $V'(1)$; ... $x(N-1)$, $u(N-1)$, $V'(N-1)$; $x(N)$, which represents the evolution of the sheep population and the purchasing/selling orders for the given period.

3. TESTING OF THE DYNAMIC MODEL AND OBTAINED RESULTS

Initial state of the system:

$$x_1(0)=150$$
$$x_2(0)=4{,}550$$
$$x_3(0)=0$$

$$x_4(0) = 0$$
$$x_5(0) = 1{,}150$$
$$x_6(0) = 1{,}150$$

The free terms of the constraints:

$b_1(k) = 4{,}550$ for $k = 2-4{,}25$
$b_2(k) = 13{,}660$ for $k = 2-4{,}25$
$b_3(k) = 4{,}550$ for $k = 4$
$b_4(k) = 230$ for $k = 4$
$b_5(k) = 35$ for $k = 4-8$
$b_6(k) = 1{,}110$ for $k = 5$
$b_7(k) = 43{,}200$ for $k = 12$
$b_8(k) = 69{,}700$ for $k = 11$
$b_9(k) = 188{,}000$ for $k = 25$
$b_{10}(k) = 7{,}000$ for $k = 25$
$b_{11}(k) = 7{,}000$ for $k = 2-3 \leqslant$ and $4-24 \geqslant$
$b_{12}(k) = 7{,}000$ for $k = 2-3 \leqslant$ and $4-24 \geqslant$
$b_{13}(k) = 730{,}000$ for $k = 25$
$b_{14}(k) = 162{,}200$ for $k = 25$
$b_{15}(k) = 3{,}162{,}000$ for $k = 25$
$b_{16}(k) = 2{,}200{,}000$ for $k = 25$
$b_{17}(k) = 3{,}344{,}000$ for $k = 25$
$b_{18}(k) = 15{,}784{,}900$ for $k = 25$
$b_{19}(k) = 50{,}000$ for $k = 25$
$b_{20}(k) = 8{,}982{,}000$ for $k = 25$
$b_{21}(k) = 1{,}410$ for $k = 25$

Planned sellings:

$$x_3(12) = 1{,}340$$
$$x_4(12) = 1{,}340$$
$$x_5(12) = 1{,}050$$

Data are also available for the following:

— specific food consumption by fodder classes per sheep capita;
— mortality and sacrifications due to necessity;
— natality growth;
— natality for MCY and FCY;
— purchasing and selling prices for the various sheep categories;
— milk production per sheep within the basic flock;
— weight increase of the population during the given period;
— weight increase in wool per sheep capita during the time horizon taken into consideration;
— selling price of a skin;
— delivering price for wool;
— delivering price for milk;
— delivering price for meat (per unit of increase in weight);
— price of a foddering day per sheep capita.

The dynamic model for the technological processes pertaining to a sheep (farm was operated on a FELIX-256 computer, using the package PROGEN the improved Romanian version for OPALINE).

The operating time amounted to 6'3".

The same dynamic model was operated with a given structure at the end of the computing horizon:

$$x_1(25) = 150$$
$$x_2(25) = 4{,}550$$
$$x_3(25) = 1{,}150$$
$$x_4(25) = 1{,}150$$

Must be mentioned also other operating alternatives: free initial state and fixed final state, as well as the case in which both initial and final state are given, while sales and purchasings are free.

4. ECONOMIC MEANING OF OBTAINED RESULTS

The operation of the dynamic model led to a series of results, among which following are to be mentioned:

— the surface of shelters and paddocks meets the needs of population growth;

— the number of boxes with a 15—20 mother sheep capacity is satisfactory for the tending of 0—7 day lambs;

— the number of shelters with a 200 capita capacity for mother-sheep is adequate for the ewes of the basic flock;

— the number of compartments for 7—20 days lambs, by lots of 40—50, covers the needs of growth fot the lambs within the farm;

— as compared to the scheduled milk output of 43.200 l, an output of 44,705.577 l was obtained, which demonstrates that milk output per sheep and evolution of ewe number within the basic flock covers the scheduled production;

— as compared to the scheduled wool output of 69,700 kg, 67,855.845 kg were obtained, which shows that wool output per sheep and evolution of wool-yielding sheep do not reach the scheduled value. A negative influence is induced by the high mortality index which reduces the number of wool-yielding sheep, as well as a back correlation between the planned value and the mean output of wool per sheep capita;

— the ratio of three breeding rams per hundred ewes within of basic flock is observed;

— gathering of ejaculate from the breeding rams provides an adequate quantity of seminal material for the artificial insemination of the ewes within the basic flock;

— the money stock scheduled for sheep purchasing, i.e. 188,000 lei, was spent completely. In the case of closed farm systems these funds are, generally, not stipulated;

— the planned structure of the sheep population — 7,000 sheep at 31 December, is reached and thus realized;

— sheep density on land area and arable area registers a lowering for the first steps (due to mortality and sacrifications due to necessity), then a peak at the fifth step and afterwards a steady decrease;

— the available 730,000 kg of corn concentrates exceed the required 549,908 kg, due to changes in population structure and to the used corn consumption per sheep capita;

— the available 162,200 kg of oat concentrates exceed the required 68,083 kg, due to changes in farm population an to the used oat consumption per sheep capita;

— the available 3,162,000 kg of lucerne-hay exceed the 3,035,884 kg resulting from the model, due to changes in farm population and to the used lucerne-hay consumption per sheep capita;

— the available 2,200,000 kg of straw exceed the 1,446,870 kg resulting from the model, due to changes in population dynamics and to the used straw consumption per sheep capita;

— the available 3,344,000 kg of silo-corn exceeds the resulting 2,257,836 kg, due to changes in population dynamics and to the used silo-corn consumption per sheep capita;

— the available 15,784,900 kg of green-stuff exceeds the required quantity resulting from the computations;

— the available computed NU — 8,982,000 — exceeds the resulting required value (representing the transformation of each fodder class into NU);

— the scheduled figure of 1,410 ordered skins is only partly reached (728 skins). To substantiate this figure double mortality indices were taken into consideration.

We consider as relevant the fact that, in decision-making, the model enables a dialogue which provices useful information as regards adequacy of available fodder (by classes), achievement of planned targets (physically and in value) and of the value for the performance criterion.

The performance criterion reaches a value of 108,366,160 lei.

5. CONCLUSIONS

Obtained results demonstrate that the dynamic model for the sheep breeding farm meets the practical requirements and allows a substantiated decision-making. It also allows to optimize the farm system by correlating imposed indices, fodder stocks and initial structure of farm population with imposed final structure, taking into account the development of the technological process.

We mention the possibility to make the above presented model also fit for other animals farms, which will cntail an increase of their efficiency due to the fact that managers will be faced with several alternatives for decision substantiation.

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UN MODELE DYNAMIQUE CONCERNANT LA PLANIFICATION DE LA STRUCTURE DES EFFECTIFS DES FERMES D'OVINES

On présente un modèle dynamique de population dans une ferme d'ovinés, concernant la planification de la structure des effectifs dans le but de la maximisation des revenus (la production de laine, la production de lait et l'augmentation du poids). En présentant le modèle dynamique, on a surpris aussi les phénomènes spécifiques à une ferme d'ovinés, tant en ce qui concerne la dynamique que pour les restrictions. Ainisi, on a supris en temps et espace le modèle du déroulement du processus technologique de croissance des ovinés, l'évolution et la structure des effectifs, ainsi que les entrées et les sorties du système. Donc le modèle dynamique a été mis à l'épreuve pour une ferme ayant un caractère prospectifs.

ДИНАМИЧЕСКАЯ МОДЕЛЬ ПЛАНИРОВАНИЯ СТРУКТУРЫ СОСТАВА ОВЦЕВОДЧЕСКИХ ФЕРМ

В работе показана динамическая модель для общеводческой фермы, применяемой при планировании структуры эффективов с целью максимизации доходов (производства шерстии, молока и вессового роста). В рамках динамической модели были отражены явления специфичные для овцеводческой фермы, так в динамике, как и в ограничениях. Таким образом, отражаются во времени и в пространстве спссоб развертывания технологического процесса овцеводства, эволюция и структура состава, а также входы и выходы системы. Динамическая модель была проверена на одной ферме и имеет проспективный характер.

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