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Cybernetic Models for Two De Broglie's Formulas *

We defined in [1] the physical control (with negative feedback) using the system:

$$\frac{\partial \varepsilon}{\partial t} + f(\varepsilon, S; t) = 0$$

$$\frac{\partial S}{\partial t} + g(\varepsilon, S; t) = 0 \qquad (1)$$

where

$\varepsilon = \varepsilon(q_1, q_2, \ldots, q_n; p_1, p_2 \ldots, p_n; t)$ is the entropy of the system;

$S = S(q_1, q_2, \ldots, q_n; p_1, p_2, \ldots, p_n; t)$ is the action of the system; and the condition (1') — that the trajectory defined by equation (1) be stable or invariant as against a group of (continuous) transformations.

The functions (or functionals) f and g may be considered as the Wignerian Σ and Hamiltonian H, thus extending the Hamilton-Jacobi's formalism for describing the physical phenomena and the laws of nature, by using in (1) a trajectory function (of action type) and a state function (of entropy type).

The necessity of minimal consideration of the two types of functions for describing the behaviour of physical systems, and particularly the physical control, results both from the necessity of reflecting within the cybernetic model (1) the effects of the principles of stationary action, of stationary entropy, of minimal potential energy, of the second principle of thermodynamics etc., and also of the reflecting of the functional side and of the structural one of the behaviour of a system, etc. [2].

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Correspondences are established in a series of theorems (Hilbert, Klein, Wigner, Weyl and especially E. Noether) between the properties of invariance (symmetry) of a system and the conservation theorems, respectively between magnitudes which conserve themselves and which, on the one hand, have associated prime integrals of the equation of state and, on the other hand, may be considered as parameters of state, as they, being able to be observed and measured, allow the experimental verification of the results, by using the equation of state (the law of motion).

Correspondences are emphasised within the E. Noether's theorems between a principle of stationarity (principle of "minimal action"), expressed through an integral invariant and ρ symmetry properties, or ρ magnitudes which are conserved, respectively, the number ρ being the number of parameters of group λ_i, with respect to which we have the invariance.

We consider the functional which depends on trajectory (of action type):

$$J = \int_{x_1}^{x_2} f(x, \quad u(x) \cdot \frac{\partial u}{\partial x} \, dx \tag{2}$$

invariant to a continuous group of transformations with ρ parameters, and we obtain ρ relations (conservation laws) of the form:

$$\sum \psi_i \delta u_i = \operatorname{div} \vec{B} \tag{2'}$$

where ψ_i and $\vec{B}$ are obtained within E. Noether's theorems through an algorithm.

In the case of a physical system which evolves in time, the principles of optimum must be associated at least to conservation laws, so as to describe both the functional side and the structural side of the system; by associating a function of state to the physical state, when the parameters vary in time, the function of state may reach extreme values, which means the stationarity or the equilibrium of the system (as against the parameters of state).

We can consider consequently that, for a system which evolves in time, the invariant of the structure on various stable levels leads us to principles of extremum or conditions of stationarity, associated to magnitudes which are conserved and which characterise the invariant organization or structure on a certain level of development. It is thus necessary, so as to describe both the functional side and the structural side of the behaviour of a system, to achieve a correspondence between two sequences, strictly increasing associated to numbers and, for instance, to groups of transformations, respectively; this correspondence can be carried out by model (1) and condition(1').

Particularly, the invariance of the trajectory of a quantic system in the space of states (the trajectory being described, for instance, by Schrödinger's equations, resulting from (1) and (1')), leads to relations:

$[\hat{H}, \hat{\Sigma}] = 0$ (see also relations (15) further down).

Hence, within the physical system evolving in time, both the conditions of extremum (wherein functions of action type appear) and the conditions of stationarity (wherein there appear functions of entropy type, and also magnitudes, which are conserved and which describe the conservation

of the structure — of the organization — on various stable levels of the development) correspond to the invariance (symmetry) properties.

If, particularly, we consider in (2): $J = S(\varepsilon)$, $f = L(\varphi)$ and $x = t$, then the condition of stationarity of the variation, which is obtained from (2), means the invariance of the Hamilton-Jacobi type equations (that define together with the invariance condition the physical control), with respect to the canonical transformations, and reciprocally.

We take into account the relation of complementarity between structure and determination, between the structural side and the functional side, which characterizes any system and which is reflected in the principle of complementarity in physics, in the dual methods in mathematics etc., and the duality of the modelling sides of any system: the state and the motion law, respectively (i.e. the equation of state, optimality criteria, conditions of invariance, of stationarity, of stability, the principle of dialectical determinism etc.). It results herefrom, as a modelling criteria of the states, *the necessity of describing the dual (structural and functional) characteristics of any system*, of achieving the correspondence between mathematical structures (groups of transformations) and numbers (analytical methods, equations of state), so as to describe both synchronic, entropic aspects, and diachronic, energetical aspects.

The selection made for defining the physical control (a function of action type and a function of entropy type) is thus justified, because the description of phenomena, of systems' behaviour, needs the consideration, at least of the two types of functions.

We consider that the variation of the entropy of a system or of the "information" from a system (at the level of Clausius or Boltzmann measures respectively, associated to entropy, and of Hartley or Shannon measures, respectively, associated to the quantity of information, we have the known relation $\varepsilon_0 = \varepsilon_1 - I$), shows us the direction of evolution in time of the system (the sequence of states), and the action specifies the trajectory (from a family of trajectories). It results hence that there is a proportionality between the variation o entropy (information) and the time interval in which this variation takes place [3].

The relation of proportionality between entropy and local time may be illustrated in a series of concrete cases. The disintegration of radioactive elements, for instance, is submitted to the law:

$$N = N_0 e^{-\lambda t}$$

The probability (p) associated to a state with N non-disintegrated nuclei is proportional to N/N_0, so that we obtain $\ln p \sim t$.

It results, from the definition of ε given in [1], that $\varepsilon \sim t$.

We obtain $\delta \int d\varepsilon = 0$ from the fundamental equation of thermodynamics (deduced from (1) in [1]) and the Fermat's principle of extremum

$$\delta \int dt = 0.$$

We have, on the other hand, for the action, the principle of Maupertuis or Hamilton (principle of stationarity)

$$\delta S = 0.$$

We have, according to [1], $\bar{p} = \nabla S$ and $\bar{\pi} = \nabla \varepsilon$; $\bar{p}$ and $\bar{\pi}$ are tangent to the real trajectory (from the space of states for which S is minimal and ε is maximal).

As a matter of fact, [1] evinced that the trajectory of a material point in the real space E_3 is determined by equation (1). According to those equation we obtain the Lagrangian

$$\mathfrak{L} = \vec{p} \cdot \vec{r} - H \tag{3}$$

and the Shannonian

$$\mathfrak{S} = \vec{\pi} \cdot \vec{r} - \Sigma. \tag{3'}$$

The determination of the trajectory and of the stationary states is made by imposing conditions $\delta S = 0$ and $\delta \varepsilon = 0$, of Euler-Lagrange type:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathfrak{L}}{\partial \dot{q}_k}\right) - \frac{\partial \mathfrak{L}}{\partial q_k} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathfrak{S}}{\partial \dot{q}_k}\right) - \frac{\partial \mathfrak{S}}{\partial q_k} = 0 \qquad k = 1, 2, 3. \tag{4}$$

The parametric form of the common solution of this equation system is $\vec{r} = \vec{r}(t)$, consequently there must be a relation of the type:

$$\mathfrak{L} = f(\mathfrak{S}).$$

We obtain, by replacing in (4):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathfrak{L}}{\partial \dot{q}_k}\right) - \frac{\partial \mathfrak{L}}{\partial q_k} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial f}{\partial \mathfrak{S}} \cdot \frac{\partial \mathfrak{S}}{\partial \dot{q}}\right] - \frac{\mathrm{d}f}{\partial \mathfrak{S}} \cdot \frac{\partial \mathfrak{S}}{\partial \dot{q}_k} =$$

$$= \frac{\mathrm{d}^2 f}{\partial \mathfrak{S}\, \mathrm{d}t} \cdot \frac{\partial \mathfrak{S}}{\partial \dot{q}_k} + \frac{\mathrm{d}f}{\mathrm{d}\mathfrak{S}}\left\{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathfrak{S}}{\partial \dot{q}_k}\right) - \frac{\partial \mathfrak{S}}{\partial q_k}\right\} = 0.$$

By taking into account the second equation from (4), the second term from the above equation is anulled, hence:

$$\frac{\mathrm{d}^2 f}{\partial \mathfrak{S}\, \mathrm{d}t} = 0,$$

and it results herefrom

$$\mathfrak{L} = f = \alpha \mathfrak{S} + \beta, \tag{5}$$

α and β being constant. As there exist the relations extending in our case the canonic system of Hamilton

$$p_k = \frac{\partial \mathfrak{L}}{\partial \dot{q}_k}, \quad \pi_k = \frac{\partial \mathfrak{S}}{\partial \dot{q}_k}$$

we obtain from (5) (through deriving as against q_k):

$$p_k = \alpha \pi_k. \tag{6}$$

Let us consider the canonic equations systems (of Hamilton-Wiener type):

$$\dot{p}_k = -\frac{\partial H}{\partial q_k}. \tag{7}$$

$$\dot{q}_k = \frac{\partial H}{\partial p_k};$$

$$\dot{\pi}_k = -\frac{\partial \Sigma}{\partial \dot{q}_k}, \tag{7'}$$

$$\dot{q}_k = \frac{\partial \varepsilon}{\partial \pi_k}.$$

We note that

$$\frac{\partial H}{\partial p_k} = \frac{\partial \Sigma}{\partial \pi_k}.$$

We obtain, consequently, from (6) and (7):

$$\frac{\partial H}{\partial p_k} = \alpha \frac{\partial \Sigma}{\partial p_k}$$

hence function $H - \alpha \Sigma$ is independent as against p_k. We can select $H = \alpha \Sigma$ in the simplest case. It results from (1)

$$\frac{\partial}{\partial t}(S - \alpha \varepsilon) = 0.$$

We can consider particularly $S = \alpha \varepsilon$ and $\alpha = \gamma \frac{\hbar}{k}$ (for homogenizing the units of measure) hence we obtain de Broglie's relation [3]

$$\frac{S}{\hbar} = \frac{\varepsilon}{k} \qquad (\text{if } \gamma = 1) \tag{8}$$

or more generally

$$\frac{S}{\hbar} = \gamma \frac{\varepsilon}{k}. \tag{8'}$$

Relation (8) or (8′) expresses the equilibrium between the increase of entropy and the decrease of action, their duality. This relation also suggests, within the duality wave-corpuscle, that the corpuscular aspect (particle's trajectory) is associated to the parameter of state — the action, and that the undulatory aspect of the particles in motion is associated to the parameter of state-the entropy.

We consider the interaction, the final cause of motion, as having four sides: information (entropy), matter in restricted sense (energy), in space and time, and the criterion represented by the duality principle for drawing up the models. We shall state precisely further down relation (8′), taking into account the dual projective metrical properties of the space.

If we select two fixed points I, J on a straight line, then the hyperbolic distance (in projective geometry) between any two points A and B on the straight line, different from I and J, is expressed (fig. 1) by:

$$d_{AB} = \beta \ln \frac{AI/AJ}{BI/BJ}$$

where β is a constant.

If we consider that points A and B are in the interval (I, J), then the probability that a random point from interval (I, J) be in interval (A, B) is given by relation $p = \dfrac{AB}{IJ}$ [5].

If point J is shifted to α, we have:

$$d_{AB} = \beta \ln \frac{AI}{BI} = \beta \ln \frac{AI}{BA + AI} = -\beta \ln \left(1 + \frac{AB}{AI}\right).$$

This expression suggests a relation of the type

$$d_{AB} = \beta \ln f(p).$$

We can associate to entropy, in the projective space, the hyperbolic measure:

$$\varepsilon = d_{AB} = -\beta \ln f(p).$$

We associate to action the elliptical (parabolic) measure, as the projective properties of space are dual to the metric properties, in the sense of diachronic-synchronic complementarity (determination-structure). The relation between the hyperbolic and the parabolic (elliptical) measure is, according to [6]

$$\|\sigma\|_{\varepsilon} = i\|\sigma\|_{s}. \tag{9}$$

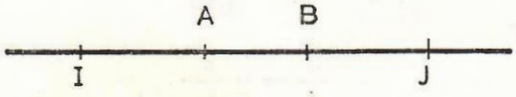


Fig. 1

We use relation (9) between measures as standard of the magnitudes, and relation (8′) can be written:

$$\frac{\varepsilon}{k} = i\frac{S}{\hbar} \text{ for magnitudes, or according to (1) } \frac{\Sigma}{k} = i\frac{H}{\hbar} \tag{10}$$

(for functions).

Particularly:

$$\frac{\bar{\pi}}{k} = i\frac{\bar{p}}{\hbar}. \tag{10'}$$

Relation (10) was considered in the modul [4] by Louis de Broglie, as we showed in the first part of the paper.

Relations (10) and (10′) justify the correspondence between the quantic operators and the associated physical magnitudes as a particular case of the duality objects-relations, magnitudes-operators, numbers-mathematical structures etc.

Let $\vec{k}$ be the wave vector, hence

$$\vec{p} = \hbar\vec{k}.$$

The correspondence

$$-i\hbar\nabla \leftrightarrow \bar{p} \tag{11}$$

is justified through (10):

$$-i\hbar\nabla\frac{\varepsilon}{k} = -i\hbar\frac{\bar{\pi}}{k} = \bar{p}$$

where k is Boltzmann's constant. We have also

$$\vec{\pi} = ik\vec{k}. \tag{12}$$

The correspondence

$$\hat{H} = i\hbar\frac{\partial}{\partial t} \leftrightarrow H \tag{13}$$

is justified through (10):

$$i\hbar\frac{\partial(\varepsilon/k)}{\partial t} = -\frac{\partial S}{\partial t} = g(\varepsilon, S; t).$$

As

$$g(\varepsilon, S; t) = H$$

we have

$$\frac{\partial S}{\partial t} + H = 0.$$

The correspondence

$$\hat{\Sigma} = -k\frac{\partial}{\partial t} \leftrightarrow \Sigma \tag{14}$$

is justified through (10):

$$-k\frac{\partial}{\partial t}\left(i\frac{S}{\hbar}\right) = -\frac{\partial \varepsilon}{\partial t} = f(\varepsilon, S; t).$$

As

$$f(\varepsilon, S; t) = \Sigma$$

we have

$$\frac{\partial \varepsilon}{\partial t} + \Sigma = 0.$$

On the basis of correspondences (13) and (14) it results that operators $\hat{H}$ and $\hat{\Sigma}$ verify the relation of type (10) established for magnitudes and functions:

$$\frac{\hat{\Sigma}}{\hbar} = i\frac{\hat{H}}{\hbar}. \tag{15}$$

The obtained results justify the correspondence between measures and operators from the quantic mechanics. In system (1) the equation

$$\frac{\partial \varepsilon}{\partial t} + \Sigma = 0,$$

is associated to the undulatory aspect of motion, having as propagation speed

$$v_\varepsilon = \frac{\Sigma}{\pi} \ (\varepsilon = \text{const}).$$

Indeed

$$d\varepsilon = \frac{\partial \varepsilon}{\partial t}\,dt + \frac{\partial \varepsilon}{\partial q}\,dq = 0.$$

Hence

$$v_\varepsilon = \frac{dq}{dt} = -\left(\frac{\partial \varepsilon}{\partial t}\right)\Big/\left(\frac{\partial \varepsilon}{\partial q}\right) = \frac{\Sigma}{\pi}.$$

In system (1) the equation

$$\frac{\partial S}{\partial t} + H = 0$$

is associated to the corpuscular phenomena, to the trajectory described by the particle. The particle displacement speed on the trajectory is $v_s = \frac{H}{p}$ (inferred analogically).

We have

$$H = pv_s = mv_s^2$$
$$\Sigma = \pi v_\varepsilon = m_\varepsilon v_s^2 \tag{16}$$

where m_ε is the mass associated to the information quantum; the "elementary quantity" of information k (Boltzmann's constant) was associated in [1] as measure, to the information quantum.

We have, from (10), by using the relation between functions:

$$\frac{m_\varepsilon v_\varepsilon^2}{k} = i\frac{mv_s^2}{\hbar} \tag{17}$$

It results from (10) and (10'):

$$\frac{H}{p} = \frac{\Sigma}{\pi} \text{ hence } v_s = v_\varepsilon.$$

We have consequently from (17)

$$m_\varepsilon = i\left(\frac{\hbar}{k}m\right). \tag{18}$$

It results that the quanta of information associated to the undulatory aspect of information are tachyions, hence they can have speeds superior to the speed of light. Indeed the phase speed of a wave pack [4] is $v_\varepsilon = v_\varphi = \frac{\upsilon}{1/\lambda} = \frac{h\nu}{h(1/\lambda)} = \frac{H}{p} = \frac{mc^2}{mv_s} = \frac{c^2}{v_s} \geqslant c$; at "equilibrium" as we saw above, $v_\varepsilon = v_s = c$.

REFERENCES

1. CONSTANTINESCU P., NICOLAU ED., NICOROVICI A.: *Sisteme cibernetice, modele-invarianță. Cibernetică și progresul societății.* (Cybernetic systems, models, in, variance. Cybernetics and progress of society), Editura politică, 1980.
2. CONSTANTINESCU P.; *Modelling of Organization and Management by the Information Processing System.* Economic Computation and Economic Cybernetics Studies and Research, 1980, nr. 3 (XIV).
3. SACHS M.: *Conceptul de timp în fizică și cosmologie* (Concept of time in physics and cosmology). La Recherche, no. 86, february, 1968.
4. DE BROGLIE LOUIS: *La thermodinamique de la particule isolée* (Thermodinamique cachée de la particule). Gauthier-Villers, Paris, 1964.
5. ONICESCU O.: *Numere și sisteme aleatoare.* (Numbers and random systems). Collection: Theory of Probabilities, vol 3. Editura Academiei R.S.R., București, 1962.
6. REDEI L.: *Foundation of euclidian and non-euclidian geometries—According to F. Klein,* Akadémiki Kiadò, Budapest, 1968.

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