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Production Sheduling in Fuzzy Conditions

The conditions in which the production process is unfolding in an enterprise are in most of the cases incompletely known at the moment when the production programme is worked out.

A number of constraints as regards the succession of operations may be fuzzily defined by technologists, some delivery terms may be established vaguely, there are not always rigurous determinations for specific consumptions, a.s.o. For this reason, the deterministic methods of production scheduling fail in some cases to take these phenomena into consideration, leading to significant deviations from the real unfolding of the production process.

This paper is concerned with problems of production scheduling by taking into consideration the fuzzy character of the parameters and variables intervening in the production system.

1. LOADING PRODUCTION CAPACITIES IN FUZZY CONDITIONS

By loading the production capacities for a given period, we mean the establishment of the manufacture targets on groups of equipments or for each equipment the range of operations they can make, the semifabs or products obtained as a result of each operation, the scheduled quantity and the necessary time to obtain this quantity.

Before passing to the fuzzification of the problem of loading production capacities, we shall define a number of elements of the model, namely:

- P — set of final products $P = \{P_1, P_2, \ldots, P_n\}$,
- i — index of a product; $i \in P \Rightarrow P_i \in P$,
- j — technological operation,
- O — set of all technological operations,
- O_i — set of operations made to obtaine product P_i,
- k — group of equipments with identical technological characteristics,

G — set of group of equipments,

G_{ij} — set of groups of equipments k which can make operation j related to product P_i,

x_{ijk} — the quantity of product P_i which is processed by operation $j \in O_i$ at the group of equipments $k \in G_{ij}$,

t_{ijk} — the time rate (the time for processing a unit of product P_i by operation $j \in O_i$ at the group of equipments $k \in G_{ij}$),

ε_{ijk} — the loss of material from product P_i by making operation $j \in O_i$ at the group of equipments $k \in G_{ij}$,

TD_k — the capacity (available time) of the group of equipment $k \in G$ for a given period of time,

Q_i — the demanded quantity from product P_i.

The problem of loading production capacities consists in solving the following system:

$$\sum_{i \in P} \sum_{j \in O_i} t_{ijk} x_{ijk}(1 + \varepsilon_{ijk}) \leqslant TD_k, \quad k \in G$$

$$\sum_{k \in G_{ij}} x_{ijk} \geqslant Q_i, \quad j \in O_i, \quad i \in P \tag{1}$$

$$x_{ijk} \geqslant 0, \quad i \in P, \quad j \in O_i, \; k \in G_{ij}$$

The first group of inequations represents capacity constraints and the second plan-production constraints. The solution of the system $X = (x_{ijk})$ $i \in P, j \in O_i, \; k \in G_{ij}$ must have positive components $x_{ijk} \geqslant 0$.

Besides the fact that the complexity of the phenomena does not surprise, the set of solutions of the system of constraints of model (1) is empty in many cases.

Considering that in many cases model (1) may be incompatible, of particular interest in production scheduling is the fuzzification of constraints (except for the nonnegativity constraints). In this case, the fuzzified model (1) is written:

$$\sum_{i \in P} \sum_{j \in O_i} t_{ijk} x_{ijk}(1 + \varepsilon_{ijk}) \lesssim TD_k, \quad k \in G$$

$$\sum_{k \in G_{ij}} x_{ijk} \gtrsim Q_i, \quad j \in O_i, \quad i \in P \tag{2}$$

$$x_{ijk} \geqslant 0, \quad i \in P, \quad j \in O_i, \; k \in G_{ij}$$

For this model Negoiță and Sularia [4] give a solution by reducing the problem to a classical mathematical programming problem, by observing limits given by the free terms. We repeat the problem by another definition of these limits and by a definition of another objective function.

A way to solve this model consists in introducing variables λ_k which should grant additional capacity for some groups of equipments and others μ_i which should diminish the ordered quantities:

$$\begin{aligned}
&\sum_{i \in P} \sum_{j \in O_i} t_{ijk} x_{ijk} (1 + \varepsilon_{ijk}) \leqslant TD_k + \lambda_k, \quad k \in G \\
&\sum_{k \in G_{ij}} x_{ijk} \gtrsim Q_i - \mu_i, \quad i \in O_i, \quad i \in P \\
&x_{ijk} \geqslant 0, \; i \in P, \; j \in O_i, \; k \in G_{ij} \\
&\lambda_k \geqslant 0, \quad k \in G \\
&\mu_i \geqslant 0, \; i \in P \\
&min \left(\sum_{k \in G} a_k \lambda_k + \sum_{i \in P} b_i \mu_i \right)
\end{aligned} \tag{3}$$

where a_k — a coefficient for granting the additionaly capacity at the group of equipments k, $a_k \geqslant 0$,

b_i — a coefficient for diminishing the ordered quantity from product P_i, $b_i \geqslant 0$,

λ_k — the additionaly capacity which can be granted at the group of equipments k,

μ_j — the quantity by which the order Q_j of product P_i can be diminished.

In some cases, in model (3) for part of the variables λ_k and μ_i constraints are introduced of the form

$$\begin{aligned}
&0 \leqslant \lambda_k \leqslant \bar{\lambda}_k, \quad k \in G' \subset G \\
&0 \leqslant \mu_i \leqslant \bar{\mu}_i, \quad i \in P' \subset P
\end{aligned} \tag{4}$$

where $\bar{\lambda}_k$ and $\bar{\mu}_i$ represent maximum feasible values established in the course of the solving of the model.

Model (3) may have several objective functions and can be solved either by the known fuzzy methods or even by deterministic methods. A global function of model (3) can be:

$$OPT \left\{ f(x_{ijk}) + (-1)^{\delta} \left[\sum_{k \in G} a_k \lambda_k + \sum_{i \in P} b_i \mu_i \right] \right\}$$

where $\delta = 0$ if $OPT = min$ and $\delta = 1$ if $OPT = max$, and $f(x_{ijk})$ a linear function.

Let $X^* = (x^*_{ijk}; \lambda^*_k; \mu^*_i)$, $i \in P$, $j \in O$, $k \in G$ be the solution of model (3). The membership grade to the observance of constraints (1) can be computed for this solution. If the exponential function is used, we obtain;

— the membership grade of solution X^* to verify the capacity constraints of model (1)

$$\eta(X^*) = exp \left(- \sum_{k \in G} a_k \lambda^*_k \right)$$

— the membership grade of solution X^* to verify plan-productio constraints of model (1):

$$\omega(X^*) = exp\left(-\sum_{i \in P} b_i \mu_i^*\right)$$

where a_k and b_i represent membership parameters which depend on th importance of the constraints.

If $\lambda_k^* = 0$, $k \in G$ and $\mu_i^* = 0$, $i \in P$ in this case model (1) is compatibl and the solution of model (3) coincides with that of model 1).

Let (u^*, v^*) be the dual solution of model (3) where

u^* — the dual variables attached to the capacity constraints;
v^* — the dual variables attached to the plan-production constraints

By taking the significance of the dual variables into account, it result that if a_k and b_i where correctly chosen, the following relations must be satis fied at least approximately:

$$a_k \cong \rho_1 u_k^*, \quad k \in G \text{ and } b_i \cong \rho_2 v_i^*, \quad i \in P. \tag{5}$$

If these relations are not satisfied to the effect that the differences ar significant, in this case corrections can be made through successive iterations until they are observed.

Another way of fuzzification consists in attaching membership function to the constraints and in constructing a global membership function F. Fo this we define for each constraint a membership function of the form.

$$f(x) = \frac{x}{x + \varepsilon} \text{ with } x > 0 \text{ and } \varepsilon \geqslant 0 \tag{6}$$

a function studied by Dumitru and Luban in [2].

For the capacity constraints we define function

$$\varphi_k = \frac{TD_k}{TD_k + \lambda_k} = \frac{1}{1 + \dfrac{\lambda_k}{TD_k}}, \quad k \in G$$

where λ_k must be as small as possible in relation to TD_k, that is $\lambda_k \ll TD_k$

For the plan-production constraints

$$\psi_i = \frac{Q_i - \mu_i}{Q_i - \mu_i + \mu_i} = \frac{1}{1 + \dfrac{\mu_i}{Q_i - \mu_i}} = 1 - \frac{\mu_i}{Q_i}, \quad i \in P \tag{7}$$

where $\mu_i \ll Q_i - \mu_i$.

If a composition operator of a product type is considered the followin global membership grade is defined:

$$F = \prod_{k \in G} \varphi_k \prod_{i \in P} \psi_i$$

with

$$F = \prod_{k \in G} \frac{1}{1 + \frac{\lambda_k}{TD_k}} \prod_{i \in P} \frac{1}{1 + \frac{\mu_i}{Q_i - \mu_i}}$$

respectively

$$F = \prod_{k \in G} \frac{1}{1 + \frac{\lambda_k}{TD_k}} \prod_{i \in P} \left(1 - \frac{\mu_i}{Q_i}\right)$$

$$\text{If } \frac{\lambda_k}{TD_k} \ll 1 \text{ and } \frac{\mu_i}{Q_i - \mu_i} \ll 1$$

$$F \cong \frac{1}{1 + \sum_{k \in G} \frac{\lambda_k}{TD_k} + \sum_{i \in P} \frac{\mu_i}{Q_i - \mu_i}} \tag{8}$$

respectively

$$F \cong \frac{1 - \sum_{i \in P} \frac{\mu_i}{Q_i}}{1 + \sum_{k \in G} \frac{\lambda_k}{TD_k}} \tag{9}$$

By pursuing a membership grade F maximum ($0 \leqslant F \leqslant 1$) it results that in both cases the minimizing of a rational function is obtained (from (8)):

$$min\left(\sum_{k \in G} \frac{\lambda_k}{TD_k} + \sum_{i \in P} \frac{\mu_i}{Q_i - \mu_i}\right) \tag{10}$$

respectively (from (9)):

$$min\left(\sum_{k \in G} \frac{\lambda_k}{TD_k} + \sum_{i \in P} \frac{\mu_i}{Q_i}\right). \tag{11}$$

The second criterion (11) leads to a linear programming model similar to (3) in which penalties a_k, respectively b_i where established with the aid of relations:

$$a_k = \frac{1}{TD_k}, \quad k \in G \text{ and } b_i = \frac{1}{Q_i}, \quad i \in P. \tag{12}$$

It results from those shown above that a fuzzy algorithm for computing the compromise solution of system (3) can be the following:

Step 1. Let a_k and b_i are initialled with the aid of relations (12) and $\rho_1 = \rho_2 = 1$.

Step 2. System (3) is solved and the primal solution $(x^*_{ijk};\ \lambda^*_k;\ \mu^*_i)_{i\in P,\ j\in 0,\ k\in G}$ and the dual solution $(u^*_k,\ \varphi^*_i)_{k\in G,\ i\in P}$ are obtained.

Step 3. If $\lambda^*_k = 0$, $k \in G$ and $\mu^*_i = 0$, $i \in P$ go to step 7.

Step 4. If $a_k \cong \rho_1 u^*_k$, $k \in G$ and $b_i \cong \rho_2 \varphi^*_i$, $i \in P$ go to step 7.

Step 5. From the analysis of the resulting solution constraints of the form (4) are introduced in model (3) for those capacities for which supplementation is difficult as well as for priority orders or orders which must not be given up.

Step 6. Consider

$$\rho_1 = \frac{\sum_{k\in G} \frac{a_k}{u^*_k}}{|G|}, \quad \rho_2 = \frac{\sum_{i\in P} \frac{b_i}{\varphi^*_i}}{|P|},$$

$$a_k = \rho_1 u^*_k,\ k \in G,\ b_i = \rho_2 \varphi^*_i,\ i \in P$$

and repeat the algorithm from step 2.

Step 7. Print solution $X^* = (x^*_{ijk};\ \lambda^*_k;\ \mu^*_i)_{i\in P,\ j\in 0,\ k\in G}$.

Obs 1. The repetition of algorithm at step 2 can be made by reoptimizing the vector of the free terms.

Obs 2. The aspects linked with the convergence of the algorithm are not studied yet.

2. DISTRIBUTING PRODUCTION ON TIME PERIODS AND EQUIPMENTS

Generally, the problems on production scheduling can be tackled with the aid of industrial dynamics techniques (of a Forrester type). Either these techniques are explicitely applied or the application of these technique is not rendered evident, the relations intervening in the models on production scheduling are of three types: rhythm, level and auxilliary equations.

2.1. *Fuzzification of rhythm and auxiliary equations.*

The rhythm equations are used to establish every day (of rank t) the quantities of semifabs distributed by a section A to other sections s. Let S — set of all production sections.

Let $\widetilde{CZ}^s_t$ be the average daily demand of a beneficiary section s at section A (which, in general, is the same every day t). The quantity $\widetilde{W}^s_t$ can be computed until the day t which section A must deliver to the beneficiary section s, in keeping with the average daily demands $\widetilde{CZ}^s_t$. We obtain:

$$\widetilde{W}^s_t = \sum_{\tau=1}^{t} \widetilde{CZ}^s_\tau,\ s \in S,\ t = 1, 2. \ldots, T. \tag{13}$$

Initially $\widetilde{W}^s_0 = 0$, $s \in S$.

Let q_t^{*s} be the quantity obtained by section A for section s on day t and W_t^{*s} the quantity obtained by section A for section s from the beginning of the plan period until day t. We obtain:

$$W_t^{*s} = W_{t-1}^{*s} + q_{t-1}^{*s},\ s \in S,\ t = 1, 2, \ldots, T. \tag{14}$$

Initially $W_t^{*s} = 0,\ s \in S$.

Let q_t^s be the necessary quantity for section A to supply to section s on day t so as to settle the possible deviations from the average daily demand (within the t days which elapsed since the beginning of the plan period).

The size of q_t^s can be established as follows:

$$q_t^s = \widetilde{CZ}_t^s + max\ (\widetilde{W}_{t-1}^s - W_t^{*s},\ 0). \tag{15}$$

The quantity which section A can supply to section s on day t (that is the production rate of section A for section s on day t) is calculated with equation:

$$CZ_t^s = \frac{\alpha_s q_t^s}{\sum\limits_{s \in S} \alpha_s q_t^s} \cdot \widetilde{CZ}_t^A \tag{16}$$

where $\widetilde{CZ}_t^A$ represents the average production which section A obtains on day t, and α_s an importance coefficient which is granted to section s.

Relations (16) give the possibility of calculating the rate scheduled on day t for section s. These rhythm equations generally underload the installations. Moreover, the importance coefficients α_s intervens which have a subjective character and cannot be rigorously established. Nevertheless, the deviations introduced by this relation are corrected through the scheduling process. For this reason, it results that it is only natural to fuzzify relation (16), that is:

$$CZ_t^s \cong \frac{\alpha_s q_t^s}{\sum\limits_{s \in S} \alpha_s q_t^s} \widetilde{CZ}_t^A \tag{17}$$

It is also possible to proceed in another way, too. A feasible membership grade μ_a^s is estabilshed for each section, for satisfying the daily demand of each section s. The more important the section is, the greater μ_a^s and, through this daily distribution they try not to surpass this membership grade. If

$$\sum_{s \in S} q_t^s \leqslant \widetilde{CZ}_t^A$$

there is, obviously no problem.

$$\sum_{s \in S} q_t^s > \widetilde{CZ}_t^A$$

they try to make the distribution so as to observe the minimum feasible membership grades for each section. Let $\mu(x_t^s)$ be the membership grads of a quantity x_t^s distributed to section s at time t for satisfying demand q_t^s. By using the exponential function we have

$$\mu(x_t^s) = exp(-k_s(q_t^s - x_t^s)),\ s \in S \tag{18}$$

where k_j represents a membership parameter. From the condition of achieving at least the minimum membership grade, we obtain:

$$exp\ (-\ k_s(q_i^s - x_i^s)) \geqslant \mu_a^s, \quad s \in S \tag{19}$$

from where:

$$q_i^s + \frac{1}{k_s} \ln \mu_a^s \geqslant x_i^s, \quad s \in S. \tag{20}$$

Let λ be a coefficient which multiplied by the left term of relation (20) would achieve the equilibrium, by a total for all sections, with the achievements of section A. It results:

$$\lambda = \frac{CZ_i^A}{\sum\limits_{s \in S} \left(q_i^s + \frac{1}{k_s} \ln \mu_a^s \right)}.$$

In this case, quantity x_i^s is given by relation:

$$\tilde{x}_i^s = \lambda \left(q_i^s + \frac{1}{k_s} \ln \mu_a^s \right). \tag{21}$$

Obviously, the quantities obtained through relation (17) differ from those obtained through relation (21), however, in both cases the rhythm equations were fuzzified.

From (18) and (21)

$$\mu(x_i^s) = (\mu_a^s)^\lambda\ exp(-k_s q_i^s(1 - \lambda)).$$

And from condition (19) it results that

$$\mu_a^s \leqslant \mu(q_i^s) \tag{22}$$

In case that (22) is not observed, system (19) is incompatible with the solution of the balance equation

$$\sum_{s \in S} \tilde{x}_i^s = \tilde{C} Z_i^A.$$

Among the auxiliary equations which can be fuzzified are those concerning the establishment of the urgency with which a product i must be obtained.

Let QR_i^t be the quantity from product P_i left to be manufactured at moment t and T_i the delivery term for product P_i given by sections s.

An urgency index z_i is defined with the aid of relation:

$$z_i = \frac{T_i - t}{QR_i^t} \left(1 - \frac{QR_i^t}{\sum\limits_{i \in P} QR_i^t} \right). \tag{23}$$

The fuzzy character of this index is obvious and many modalities of fuzzification can be introduced. Thus, before applying relation (23), the very computing of the elements intervening in this relation can be fuzzified. For

example, the delivery terms are most often given in the form "around term T_i^* Thus, a new fuzzified auxiliary equation can be introduced, of the form:

$$T_i \cong T_i^* \tag{24}$$

for which the membership grade is

$$\mu(T_i) = e^{-k_i|\,T_i - T_i^*\,|} \tag{25}$$

where T_i^* is a delivery term in keeping with a frame-programme.

Relation (23) can also be fuzzified

$$z_i \cong \frac{T_i - t}{QR_i^t}\left(1 - \frac{QR_i^t}{\sum\limits_{i \in P} QR_i^t}\right). \tag{26}$$

Another way of fuzzification may intervene even in the process of charting indices z_i which will be used to establish a fuzzy priority.

The indices z_i is charted increasingly, in a fuzzy sense, namely it is considered that the order relation between two terms can be achieved only if the difference between them is significant and, otherwise they are considered in an indifference relation. We can consider that two indices z_u and z_v are an indifference relation, that is they do not differ significantly if

$$|\,z_u - z_v\,| \leqslant \rho \frac{z_u + z_v}{2}, \quad \rho \in [0.05\,;\,0.15]. \tag{27}$$

It results that if two terms are in a relation of indifference another index is still necessary in order to obtain a correct hierarchization. To this purpose, a distance d_{uv} between two technologies of some orders u and v can be defined. This distance can be defined as a Hamming or as an Euclidian distance.

Let d_{uv} = the number of almost identical operations in the technologies of products P_u and P_v.

These distances d_{uv} are put in a decreasing order. It is advisable to launch products with almost identical technologies, giving the possibility of a minimum time for the preparation of installations, the formation of batches and facilitating the passing from a technology to another.

Similarly, other indices can also be fuzzified, the most important of which is the distributing intensity m_{ik}^t of product P_i at time t, on the group of equipments k whose computing relatoin is:

$$m_{ik}^t = \left[\frac{QR_i^t}{N_{ijk}(T_i - t)}\right] + 1 \tag{28}$$

where

QR_i^t — the quantity left to be manufactured from product P_i at time t;
T_i — delivery term for product P_i;
N_{ijk} — the production rate of the group of installations k, operation j for obtaining product P_i.
$[x]$ — integer part of x.

In the first place, a relation of type (24) can be applied for the delivery term. The fractional side can also be valued:

$$f_{ik}^{t} = \frac{QR_i^t}{N_{ijk}(T_i - t)} - \left[\frac{QR_i^t}{N_{ijk}(T_i - T)}\right] \tag{29}$$

by generating a random number ν uniformly distributed between 0 and 1. then a boolean variable δ can be computed, defined as follows:

$$\delta = \begin{cases} 0 & \text{if } \nu > f_{ik}^{t} \\ 1 & \text{if } \nu \leqslant f_{ik}^{t} \end{cases}$$

In this case, relation (28) becomes:

$$m_{ik}^{t} = \left[\frac{QR_i^t}{N_{ijk}(T_i - t)}\right] + \delta. \tag{30}$$

2.2. *Models of distributing production on time periods and equipments*

a) Let us consider a section of finished products. The distribution of production on time periods and equipments consists in establishing the fabrication targets for the periods (month, decade) of a considered plan (quarter of a year, month).

With the notations in the first paragraph of this paper, the model of allocating production on periods and equipments is the following:

$$\begin{gathered} \sum_{i \in P} \sum_{j \in O_i} t_{ijk} x_{ijk} (1 + \varepsilon_{ijk}) \leqslant TD_{kl}, \; k \in G, \; l = 1, 2, \ldots, m \\ \sum_{l=1}^{m} \sum_{k \in G_{ij}} x_{ijkl} \geqslant Q_i - \mu_i, \quad i \in P, \quad j \in O_i \\ x_{ijkl} \geqslant 0, \; i \in P, j \in O_i, \; k \in G_{ij}, \quad l = 1, 2, \ldots, m \\ \mu_i \geqslant 0, \; i \in P \\ min\left(\sum_{i \in P} a_i \mu_i\right) \end{gathered} \tag{31}$$

where

x_{ijkl} — is the quantity of product P_i which is processed through operation $j \in O_i$ at the group of equipments $k \in G_{ij}$ in period l.

TD_{kl} — the available capacity (time) of the group of equipments $k \in G$ in period l.

m — the number of periods of the considered plan.

Coefficients a_i, $i \in P$ of the objective function are subunitary and positive sizes and are calculated, as follows:

$$a_i = exp\,(-k_i Q_i), \; i \in P \tag{32}$$

where k_i represent membership parameters which depends on the importance of product $i \in P$.

b) Let us consider a section A which produces semifinished products for other sections. Let S be the set of these beneficiary sections.

In this case the model of distributing production in section A on time periods and equipments is the following:

$$\sum_{i\in P}\sum_{j\in O_i} t_{ijk}x_{ijkl}(1+\varepsilon_{ijk}) \leqslant TD_{kl},\ k\in G,\ l=1,2,\ldots,m$$

$$\sum_{l=1}^{m}\sum_{j\in G_{ij}} x_{ijkl} \geqslant Q_i-\mu_i,\quad i\in P,\quad j\in O_i$$

$$\sum_{i\in Ps}\sum_{k\in G_{ij}} x_{ijkl} \geqslant \widetilde{W}_l^s-\nu_{sl},\quad j\in O,\quad s\in S,\ l=1,2,\ldots,m$$

$$x_{ijkl}\geqslant 0,\quad i\in P,\quad j\in O_i,\quad k\in G_{ij},\quad l=1,2,\ldots,m \qquad (33)$$

$$\mu_i\geqslant 0,\ i\in P$$

$$\nu_{sl}\geqslant 0,\ l=1,2,\ldots,m,\ s\in S$$

$$min\left(\sum_{i\in P} a_i\mu_i+\sum_{s\in S}\sum_{l=1}^{m} b_{sl}\nu_{sl}\right)$$

where

P_s — the subset of products of section A allocated to section s. $\bigcup_{s\in S} P_s = P$, thus each index i implies a membership to a subset P_s.

$\widetilde{W}_l^s$ — the quantity cumulated for period l of the production which is allocated to section s and is calculated with relation (13).

Coefficients a_i are calculated with relation (32) and the coefficients b_{sl} are determined from (6) as follows:

$$b_{sl}=\frac{\widetilde{W}^s_l}{\widetilde{W}^s_l+(1-\alpha_i)},\quad s\in S,\ l=1,2,\ldots,\ \mathrm{m} \qquad (34)$$

where α_i is an importance coefficient of section s.

By solving problems (31) and (33), the fabrication targets are established on groups of equipments, as well as the range of operations they can do, the semi-manufactured goods or products obtained after each operation, the quantity and period in which each product must be manufactured.

c) Let us consider that a given moment t of a considered plan period, a production programme should be worked out in section A for the period $[t, t+\tau]$, $\tau>0$. In this case, we solve the following model:

$$\sum_{i\in P}\sum_{i\in O_i} t_{ijk}x_{ijk}(1+\varepsilon_{ijk}) \leqslant TD_{k\tau},\ k\in G$$

$$\sum_{k\in G_{ij}} x_{ijk} \geqslant QR_i^t-\mu_i,\ i\in P,\ j\in O_i$$

$$\sum_{i\in P_s}\sum_{k\in G_{ij}} x_{ijk} \geqslant \widetilde{x}_l^s-\eta_s,\ s\in S,\ j \text{ fixed},\ j\in O_i \qquad (35)$$

$$x_{ijk} \geqslant 0, \quad i \in P, \quad j \in O_i, \quad k \in G_{ij}$$

$$\mu_i \geqslant 0, \; i \in P$$

$$\eta_s \geqslant 0, \; s \in S$$

$$min\left(\sum_{i \in P} a_i \mu_i + \sum_{s \in S} b_s \eta_s\right)$$

where

x_{ijk} — the quantity of product i made through operation j on the group of equipments k in the period $[t, t + \tau]$,

$TD_{k\tau}$ — the available time in period $[t, t + \tau)$ for the group of equipments $k \in G$,

η_s — the quantity by which we can diminish the production allocated to section s in period $[t, t + \tau]$,

$\tilde{x}_i^s$ — the quantity allocated to section s in the period $[t, t + \tau]$ and given by relation (21),

QR_i^t — the quantity left to be made from product i at moment t.

Coefficients a_i are calculated with relation

$$a_i = exp\,(-k_i z_i), \quad i \in P \tag{36}$$

where from (26) we have

$$z_i = \begin{cases} 0 & \text{if } T_i - t \leqslant 1 \\ \dfrac{T_i - t}{QR_i^t}\left(1 - \dfrac{QR_i^t}{\sum_{i \in P} QR_i^t}\right) & \text{if } T_i - t > 1 \end{cases}$$

Coefficients b_s, are calculated with function (6) thus

$$b_s = \frac{\tilde{x}_i^s}{\tilde{x}_i^s + (1 - \alpha_s)}, \quad s \in S \tag{37}$$

where α_s is an importance coefficient associated to section s.

It is noted that if the urgency index is high through (36), a small coefficient a_i is resulting and through model (35) product P_i will be postponed.

In the objective function of (35) they try to include coefficients a_i into interval (0,1/2) and coefficients b_i into interval (1/2,1).

Considering that at moment t the dimensions intervening into model (35) are updated, we can associate a membership grade to the fulfilment of the production plan for the considered period. In this sense, we define this global membership grade obtained by applying the product operator:

$$G(t) = exp\left(-\sum_{i \in P} a_i \mu_i\right) \tag{38}$$

where a_i and μ_i, $i \in P$ are calculated at each moment t.

$G(t)$ is a growing function, considering that the production plan is constantly reduced, as it is carried out. At the end of the plan period, $G(t)$ will give an index for plan fulfilment.

Dynamic aspects of the problem of production scheduling have also been included into model (35) in comparison with models (31) and (33). By solving model (35), we can pass to the solution of the problem of scheduling on equipments.

If t is the start point for a period l and τ the size of a period l, the model (35) solves the problem of distributing production on time periods and equipments given by model (33).

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LA PROGRAMMATION DE LA PRODUCTION DANS DES CONDITIONS FUZZY

Les conditions de déroulement du processus de production dans une entreprise sont le plus souvent incomplètement connues au moment de l'élaboration du programme de production. Ainsi, les technologues peuvent définir fuzzy un nombre de restrictions concernant la succession des opérations; on peut vagement établir certains termes de livraison; l'on ne peut pas toujours faire une détermination rigoureuse de la consommation spécifique e.a. C'est pourquoi, les méthodes de détermination de la programmation de la production prennent à tort en considération ces phénomènes, fait qui mène à des équarts importants par rapport au déroulement réel du processus de production.

Les auteurs s'occupent des problèmes de la programmation de la production, tenant compte du caractère fuzzy des paramètres et des variables du système de production.

ПРОГРАММИРОВАНИЕ ПРОИЗВОДСТВА ПРИ УСЛОВИЯХ ФУЗЗИ

Условия, при которых развертывается процесс производства в рамках предприятия, чаще всего известны недостаточно в момент, когда разрабатывается производственная программа. Таким образом, некоторое число ограничений, касающихся очередности операций может быть определенно технологами как фузии, а также могут быть расплывчато установлены сроки поставок, и не всегда может быть сделано строгое определение удельного расхода и т.д. По этим причинам, методы программирования производства учитывают такие явления, приводя к значительным отклонениям от реального хода производственного процесса.

Настоящая работа трактует проблемы программирования производства, учитывая характер фуззи параметров и переменных производственной системы.

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