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Updating Chronicles in the Systems Theory

1. In the money circulation speed theory, Maurice Allais has introduced an updating theory which has been extended by R. Vallée to the theory of multidimensional dynamic systems and which will be presented in the following, after specifying the terminology.

By chronicle we mean a $t \to u(t)$ function defined on R and with values in a vectorial space. An observation operator 0 makes an observed chronicle $v = 0u$ correspond to u, provided that 0 is causal, namely $v(t)$ depends only on values taken previously by $u(t)$. Considering that operator 0 is linear, we write

$$v(t) = \int_{t_0}^{t} N(t, \tau)u(\tau)\mathrm{d}\tau, \quad t \geqslant t_0. \tag{1}$$

We are now able to specify what we mean by updating a chronicle. Generally, the observer has no access to chronicle u but only to the image chronicle v which results from the observation process. The term updating is used in economy to express the equivalence between certain goods ascribed at moment t and a temporal allocation subsequently distributed to t of goods of the same nature. It is considered that in general, the allocation of the same goods at present and a few years later does not necessarily exercise the same attraction on a given economic agent. Updating permits to replace the future allocation by a present equivalent thus obtaining a certain projection of the future into the present. Obviously, updating can be conceived in an inversely temporal sense, namely of equating past situations at present.

2. In our present vectorial case, the vectorial value $v(\tau)$ of the chronicle perceived at moment $t \geqslant \tau$ is distortedly transferred at the present moment t, in the form $\varnothing(t, \tau)\, v(\tau)$, where $\varnothing$ is a quadratic matrix. One infers from condition that $v(t')$ is the same either it is perceived directly or through a previous observation in t

$$\varnothing(t', t)\ \varnothing(t, \tau) = \varnothing(t', \tau) \quad \tau, t, t' \in R,$$

thus

$$\varnothing(t, t) = I \qquad \varnothing(t, t') = \varnothing(t', t),$$

where I is a unit quadratic matrix.
By taking into account the real condition that the effect is annuled at a great distance, it is inferred that for $t - \tau \to \infty$

$$\lim \varnothing\ (t, \tau) = 0,$$

where 0 is a nul quadratic matrix.
3. In case that time is discrete, the relation between v and u becomes in the scalar case

$$v(t_i) = \Sigma N(t_i, \tau_j)\ u(\tau_j), \tag{2}$$

where t_i and τ_j are moments taken equidistantly or not. Relation (2) can be put in the matrix form

$$V = N^T U$$

where V and U are column matrices and N has convenient dimensions; index T marks the transposition.

In the particular case in which matrix N is quadratic, U is obtained immediately by relation

$$U = (N)^{T-1} V,$$

which permits the immediate calculation of column matrix U.

In the scalar case, we can write denoting by $v(t_n) = v(n)$, etc.,

$$\begin{aligned} v(1) &= u(1) \\ v(2) &= a_{21}u(1) + a_{22}u(2) \\ v(3) &= a_{31}u(1) + a_{32}u(2) + a_{33}u(3). \end{aligned}$$

We write in a concise form

$$v = Lu$$

where L is an inferior triangular matrix. Resulting from the observation data v is the immediate determination of u.

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MISE A JOUR DES CHRONIQUES DANS LA THÉORIE DES SYSTEMES

Dans la théorie de la vitesse de circulation de la monnaie Maurice Allais a introduit une théorie d'actualisation, appliquée par R. Vallée à la théorie des systèmes dynamiques multidimensionnels et qui sera présentée dans cet ouvrage après la précision de la terminologie.

ПРИВЕДЕНИЕ К НАСТОЯЩЕМУ МОМЕНТУ КРОНИК В ТЕОРИИ СИСТЕМ

В теории скорости оборота монеты, Морис Але включил теорию актуализации (приведения к настоящему моменту), применённую Р. Вале к теории многомерных динамичских систем. Это будет далее изложено после уточнения терминологии.

20. 09. 1981