C. V. NEGOIȚĂ
AL. C. ȘTEFĂNESCU
Institute for Management and Informatics

A Categorical Approach to Fuzzy Optimization

In the following we show that optimization can be viewed as a problem of internal logic in a topos.

1. PREREQUISITES

1.1 Definition A category E is a topos if

(i) E has pullbacks

(ii) E has exponentials, that is for any object A, $(-x\,A)\colon E \to E$ has a right adjoint, denoted $(-)^A$ The adjunction is denoted ev:

$$B^A \times A \xrightarrow{ev} B$$

(iii) E has a subobject classifier $T \to \Omega$ where T is the terminal, so that for any monomorphism $i_A\colon A \to X$ in E there is a unique map $\varkappa_A\colon X \to \Omega$) "characteristic function of i_A") making

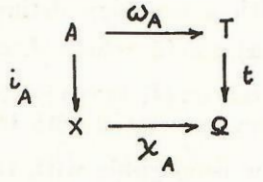


Fig. 1

a pullback.

1.2 Observation Through axiom (iii) the correspondence $i_A \to \varkappa_A$ is a bijection between *Sub* X and *Hom* (X, Ω)

1.3 Observation Any map with domain T is a monomorphism. In particular, true: $T \to \Omega$ is monic. Pulling a monic back along a map gives a monic.

So a monic i_A defined by pullback is viewed as the subobject of X clasified by $\varkappa_A$.

1.4 Observation In any topos there exist the monomorphisms $\Lambda: \Omega \times \Omega \to \Omega$, $\mathsf{V}: \Omega \times \Omega \to \Omega$ and $\to: \Omega \times \Omega \to \Omega$ such that $(\Omega, \mathsf{V}, \Lambda, \to)$ satisfies the axioms for a Heyting algebra and defines the logic of the topos. For any object X in the topos E the elements of $Hom\ (X, \Omega)$ are predicates on X.

1.5 Observation For any morphism $f: X \to Y$ in a topos E one can define the mappings

$$f^{-1}: SubY \to SubX$$

$$\exists f: SubX \to SubY$$

$$\forall f: SubX \to SubY$$

where f^{-1} is the application which associates every subobject Y' of Y with the subobject $f^{-1}(Y')$ of X given by pullback.

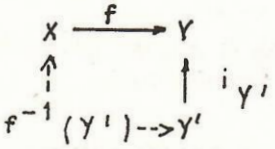


Fig. 2

If we consider the category associated with the poset $(SubX, \leqslant$ then $f^{-1}, \exists$ and $\forall f$ are covariant functors.

1.6 Observation The main example of a topos is Set, the category of sets, where the subobject classifier is $\Omega = \{0, 1\}$, $B^A = Hom\ (A, B)$ $ev_{A,B} = B^A \times \times A \to B$ is given by $ev_{A,B}(f, x) = f(x)$ for all $f \in B^A \times \in A$ and if $A \subseteq X$, then $\varkappa_A: X \to \Omega = \{0,1\}$ is the characteristic function of A.

2. EMBEDDING THE CATEGORY OF FUZZY SETS IN THE TOPOS OF *I*-SETS

Let be $I = [0, 1]$ with a topology defined by the open sets

$$D(I) = \{[0, \alpha)/\alpha \in L\} \text{ where } L = [0, 1] \subseteq R$$

2.1 Definition A presheaf of sets on $(I, D(I))$ is a covariant functor $X: \mathfrak{D}(I) \to$ Set where $\mathfrak{D}(I)$ is a category associated with the ordered set $(D(I), \supseteq)$

2.2 Observation $(D(I), \supseteq)$ is isomorphic with the ordered set $(L, \geqslant)$. Consequently a presheaf of sets is given by a family of sets $(X_\alpha)_{\alpha \in L}$, and for every $\alpha, \beta \in L$ with $\alpha \geqslant \beta$ an application $\rho^X_{\alpha\beta}: X_\alpha \to X_\beta$ is given such that if $\alpha \geqslant \beta \geqslant \gamma$ then $\rho^X_{\alpha\gamma} = \rho^X_{\beta\gamma} \cdot \rho^X_{\alpha\beta}$ and $\rho^X_{\alpha\alpha} = 1_{X_\alpha}$ we shall use the notation $X = (X_\alpha, \rho^X_{\alpha\beta})$

2.3 Definition $I^\wedge$ is the category whose objects X, Y are presheaves of sets on $(I, D(I))$ and $Hom(X, Y)$ is the set of functorial morphisms from X to Y

2.4 Observation If $X = (X_\alpha, \rho^X_{\alpha\beta})$ and $Y = (Y_\alpha, \rho^Y_{\alpha\beta})$ then $f: X \to Y$ is defined by a family of mappings $(f_\alpha)_{\alpha \in L}$ with $f_\alpha: X_\alpha \to Y_\alpha$ such that for all $\alpha, \beta, \in L$ with $\alpha \geqslant \beta$ we have $\rho^Y_{\alpha\beta} \circ f_\alpha = f_\beta \circ \rho^X_{\alpha\beta}$

2.5 Proposition $I^\wedge$ is a topos

This statement is a consequence of the fact that for each category C the category of presheaves of sets over C is a topos. In our case $C = \mathfrak{D}(I)$.

In the following we shall describe the elements appearing in definition (1.1) and in observation (1.5) for the topos $I^\wedge$

2.6 Pullback in I^

Let X, Y, Z, f, g be objects and morphisms in $I^\wedge$ and the diagram

$$X \xrightarrow[f]{} Z \xleftarrow[g]{} Y$$

The pullback of this diagram is (P, p^X, p^Y) with

$$P = (p_\alpha, \rho^P_{\alpha\beta}), \quad p_\alpha = X_\alpha Z_\alpha Y_\alpha$$

$$p^X = (p^X_\alpha)_{\alpha \in L}, \quad p^X_\alpha = P_\alpha \to X_\alpha$$

where for all $\alpha \in L$, $(X_\alpha \times Y_\alpha, p^X_\alpha, p^Y_\alpha)$ is the pullback (in Set) of the diagram $X_\alpha \xrightarrow{f_\alpha} Z_\alpha \xleftarrow{g_\alpha} Y_\alpha$ and for $\alpha \geqslant \beta$ $\rho^P_{\alpha\beta}: p_\alpha \to p_\beta$ is defined by the property of universality of the pullback p_β in Set aplied to the pair $(\rho^X_{\alpha\beta} \circ p^X_\alpha, \rho^Y_{\alpha\beta} \circ p^Y_\alpha)$ from the diagram

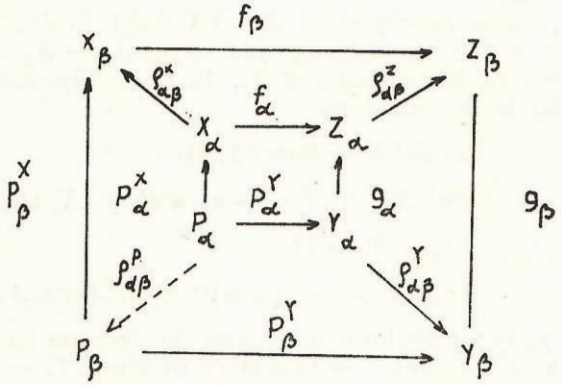


Fig. 3

if g is monomorphism then $p_\alpha \cong f_\alpha^{-1}(g_\alpha, (Y_\alpha))$

2.7 Exponentiation in I^

Let be X, Y as in (2.6). For $\alpha \in L$ let us consider $h^\alpha \in Ob\, I^\wedge$ defined by $h^\alpha = (h^\alpha_{\alpha'}, \rho^{h^\alpha}_{\alpha'\beta})$ with

$$h^\alpha_{\alpha'} = \begin{cases} \{\cdot\} & \text{if } \alpha' \leqslant \alpha \\ \theta & \text{if } \alpha' > \alpha \quad \{\cdot\} \text{ is a singleton} \end{cases}$$

and for $\beta \geqslant \alpha'$ $\rho^{h^\alpha}_{\beta\alpha'}: h^\alpha_\beta \to h^\alpha_{\alpha'}$ is identity if $\beta \leqslant \alpha$ and empty if $\beta > \alpha$. The object $Y^X = ((Y^X)_\alpha, \rho^{Y^X}_{\alpha\beta})$ is defined by

$$(Y^X)_\alpha = Hom_{I^\wedge}(h^\alpha \times X, Y) \text{ for all } \alpha \in L$$

For $\alpha \geqslant \beta$ $\rho^{YX}_{\alpha\beta}: (Y^X)_\alpha \to (Y^X)_\beta$ is given by

$$Hom_{I^\wedge}(h^\alpha \times X, Y) \ni \varphi \to \rho^{YX}_{\alpha\beta}(\varphi) = \varphi \circ (h_{\beta<\alpha} \times 1_X)$$

where $h_{\beta\leqslant\alpha}: h^\beta \to h^\alpha$ if the functorial morphism in $I^\wedge$ induced by the morphism $\beta \to \alpha$ from $\mathfrak{D}(I)$. Consequently, $f^\alpha \in (Y^X)_\alpha$ has the form $f^\alpha = (f^\alpha_{\alpha'})_{\alpha' \in L}$ with $f^\alpha: X_{\alpha'} \to Y_{\alpha'}$, if $\alpha' \leqslant \alpha$ and $f^\alpha_{\alpha'}$ empty if $\alpha' > \alpha$ such that $\rho^Y_{\alpha'\alpha''} \circ f^\alpha_{\alpha'} = f_{\alpha''} \circ \rho^X_{\alpha'\alpha''}$ for all α', α'' with $\alpha \geqslant \alpha' \geqslant \alpha''$

2.8 Evaluation in $I^\wedge$

We shall define $ev_{X,Y}: Y^X \times X \to Y$ by $(ev_{X,Y})_\alpha: (Y^X)_\alpha \times X_\alpha \to Y_\alpha$

$$(Y^X)_\alpha \times X_\alpha \ni (f^\alpha, x) \to (ev_{X,Y})_\alpha (f^\alpha, x) = f^\alpha_\alpha(x)$$

2.9 Classification, subobjects and predicates in $I^\wedge$

The terminal object in $I^\wedge$ is $T = (T_\alpha, \rho^T_{\alpha\beta})$ where for all $\alpha \in L$ $T_\alpha = \{\cdot\}$ and for $\alpha \geqslant \beta$ $\rho^T_{\alpha\beta}: T_\alpha \to T_\beta$ is identity. Therefore, following (2.8) we have $T = h^1$.

The classifier object Ω is defined by $(\Omega_\alpha, \rho^\Omega_{\alpha\beta})$ $\Omega_\alpha = [0, \alpha]$ for all $\alpha \in L$ and for $\alpha \geqslant \beta$ $\rho^\Omega_{\alpha\beta}: \Omega_\alpha \to \Omega_\beta$ is defined by $\Omega_\alpha \ni \gamma \to \rho^\Omega_{\alpha\beta}(\gamma) = min(\gamma, \beta)$ with "*min*" in $[0, 1], \leqslant)$

Let (A, i_A) be a subobject of $X = (X_\alpha, \rho^X_{\alpha\beta})$ in $I^\wedge$. We can suppose that for every $\alpha \in [0, 1]$, $A_\alpha \subseteq X_\alpha$ and $(i_A)_\alpha: A_\alpha \to X_\alpha$ is canonical inclusion. Therefore, we have $\rho^X_{\alpha\beta}(A_\alpha) \subseteq A_\beta$. In $I^\wedge$ a bijection between $SubX$ and $Hom(X, \Omega)$ is described by

$$\varkappa: SubX \to Hom(X, \Omega)$$

$$Sub\ X \ni (A, i_A) \mapsto \varkappa_A \in Hom(X, \Omega)$$

$$(\varkappa_A)_\alpha: X_\alpha \to \Omega_\alpha$$

$$(\varkappa_A)_\alpha(x) = sup\ \{\alpha' \in [0, \alpha] / \rho^X_{\alpha\alpha'}(x) \in A_{\alpha'}\}$$

It follows that χ_A is a morphism in $I^\wedge$ and the diagram from definition (1.1) (iii) is a pullback in $I^\wedge$ (true: $T \to \Omega$ is given by $\text{true}_\alpha: T_\alpha \to \Omega_\alpha$, $\text{true}_\alpha(\cdot) = \alpha$). Further,

$$S: Hom(X, \Omega) \to SubX$$

is given by

$$Hom(X, \Omega) \ni p \mapsto S(p) \in Sub\ X$$

$$S(p) = ((S(p))_\alpha,\ \rho^{S(p)}_{\alpha\beta}),\ (S(p))_\alpha = p_\alpha^{-1}(\alpha)$$

and for $\alpha \geqslant \beta$ $\rho^{S(p)}_{\alpha\beta}: p_\alpha^{-1}(\alpha) \to p_\beta^{-1}(\beta)$ is defined as the restriction of $\rho^X_{\alpha\beta}$ at $p_\alpha^{-1}(\alpha)$. Note that $SubT \simeq Hom(T, \Omega)$ in $I^\wedge$ means $h^\alpha \mapsto \alpha \in [0, 1]$

2.10 f^{-1}, $\exists f$ **and** $\forall f$ **in** $I^{\wedge}$

Let $X = (X_\alpha, \rho^X_{\alpha\beta})$, $Y = (Y_\alpha, \rho^Y_{\alpha\beta})$ be objects in $I^{\wedge}$ and $f: X \to Y$ a morphism. Then, we have

$$f^{-1}: SubY \to SubX$$

$SubY \ni Y' \mapsto f^{-1}(Y') \in SubX$, $(f^{-1}(Y'))_\alpha = f_\alpha^{-1}(Y'_\alpha)$, and for $\alpha \geqslant \beta$ $\rho^{f^{-1}(Y')}_{\alpha\beta} : f_\alpha^{-1}(Y'_\alpha) \to f_\beta^{-1}(Y'_\beta)$ is the restriction of $\rho^X_{\alpha\beta}$

$\exists f: SubX \to SubY$

$SubX \ni X' \mapsto \exists f(X') \in SubY$, $\exists f(X') = f(x')$

$(\exists f(X'))_\alpha = f_\alpha(X'_\alpha)$ and for $\alpha \geqslant \beta$ $\rho^{\exists f(x')}_{\alpha\beta}: (\exists f(X'))_\alpha \to (\exists f(X'))$ is the restriction of $\rho^Y_{\alpha\beta}$

$\forall f: SubX \to SubY$

$SubX \ni X' \mapsto \forall f(X') \in SubY$

$$(\forall f(X'))_\alpha = (Y_\alpha - f_\alpha(X_\alpha)) \cup f_\alpha\Big(\bigcap_{\alpha' \leqslant \alpha} (\rho^X_{\alpha'\alpha})^{-1}(X'_{\alpha'})\Big)$$

and for $\alpha \geqslant \beta$, $\rho^{\forall f(x')}_{\alpha\beta} : (\forall f(X'))_\alpha \to (\forall f(X'))_\beta$ is the restriction of $\rho^Y_{\alpha\beta}$

Using the bijective correspondence between subobjects and predicates in $I^{\wedge}$ we can define the mappings

$$f_*^{-1}: Hom(Y, \Omega) \to Hom(X, \Omega)$$

$$Hom(Y, \Omega) \ni p \mapsto f_*^{-1}(p) = p \cdot f$$

$$(\exists f)_*: Hom(X, \Omega) \to Hom(Y, \Omega)$$

$$Hom(X, \Omega) \ni p \mapsto (\exists f)_*(p)$$

$$(\exists f)_*(p) = (((\exists f)_*(p))_\alpha)_{\alpha \in L},\ ((\exists f)_*(p))_\alpha : Y_\alpha \to \Omega_\alpha = [0, \alpha]$$

$$((\exists f)_*(p))_\alpha(y) = \begin{cases} sup\ p_\alpha(x) & \text{if } f_\alpha^{-1}(y) \neq \emptyset \\ x \in f_\alpha^{-1}(y) & \\ 0 & \text{if } f_\alpha^{-1}(y) = \emptyset \text{ for all } y \in Y_\alpha \end{cases}$$

$$(\forall f)_*: Hom(X, \Omega) \to Hom(Y, \Omega)$$

$$Hom(X, \Omega) \ni p \mapsto (\forall f)_*(p)$$

$$(\forall f)_*(p) = (((\forall f)_*(p))_\alpha)_{\alpha \in L}, \qquad ((\forall f)_*(p))_\alpha : Y_\alpha \to \Omega_\alpha$$

$$((\forall f)_*(p))_\alpha(y) = \begin{cases} inf\ p_\alpha(x) & \text{if } f_\alpha^{-1}(y) = \emptyset \\ x \in f_\alpha^{-1}(y) & \\ 0 & \text{if } f_\alpha^{-1}(y) = \emptyset \end{cases}$$

2.11 The category of fuzzy sets, Set L

Let us consider the full subcategory of $I^{\wedge}$ denoted $Fl[0, 1]$ whose objects are $X = (X_\alpha, \rho^X_{\alpha\beta})$, where $\rho^X_{\alpha\beta}: X_\alpha \to X_\beta$, $\alpha \geqslant \beta$, are inclusions and for every $J \subseteq L$ we have $X_{SupJ} = \bigcap_{\alpha \in J} X_\alpha$.

ObSetL is the class of all pairs (X, f) where X is a set and $f: X \to L$ is a mapping, and $Hom\ ((X,f),(Y,g)) = \{h: X \to Y / g \cdot h > f$. The correspondences

$$F_{Ob}: ObSetL \to ObFl\,[0,\ 1]$$

$$(X,f) \mapsto (N^f_\alpha,\ i^{N(f)}_{\alpha\beta}),\ N_\alpha(f) = \{x \in X / f(x) \geqslant \alpha\}$$

$$F_{HOm}: Hom\,(X,f),(Y,g) \to Hom\,(F_{Ob}(X,f),\ F_{Ob}(Y,\ g))$$

$$h \mapsto h^* = (h^*_\alpha)_{\alpha \in [0,\ 1]} \text{ with } h^*_\alpha = h/N_\alpha(f)$$

define a functor $F: SetL \to Fl\,[0, 1]$. By the representation theorem [1] we have a categorical equivalence. Therefore, *SetL* is a subcategory of $I^\wedge$.

2.12 Observation If $I = \{0\}$ and $L = \{0, 1\}$, then $I^\wedge$ is Set. For this reason an object in $I^\wedge$ is called an I-set. According to (2.11) the concept of an I-set is a generalization of the concept of a fuzzy set.

3. A CATEGORICAL DESCRIPTION OF THE OPTIMIZATION PROBLEM

Let us consider I and L as in (2).

3.1 Definition Given an object (X, f) in *SetL* a minimization (maximization) problem is: find $x_0 \in X$ such that

(a) $f(x) \geqslant f(x_0)$ for all $x \in X$

(b) $f(x) \leqslant f(x_0)$ respectively.

If x_0 verifies (a) or (b) we shall write

$$f(x_0) = \min_X f, \qquad f(x_0) = \max_X f \text{ respectively}$$

The sets

$$X^{min} = \{x_0 \in X / f(x_0) = \min_X f\}$$

$$X^{max} = \{x_0 \in X / f(x_0) = \max_X f\}$$

are solutions of the optimization problem relative to the pair (X, f)

3.2 Observation The fuzzy programming problem as defined in current literature [2,3] can be reduced to definition (3.1). More precisely, if $(X, f_1, f_2, \ldots, f_n)$, $f_i: X \to L$, $i = 1, 2, \ldots, n$ are the parametra of a fuzzy programming problem, then the sets of its solutions is the set of the solutions for a maximization problem relative to the pair (X, D) where $D = f_1 \wedge f_2 \wedge \ldots \wedge f_n$

3.3 X^{min} and X^{max} in $I^\wedge$.

According to (2.11) we can see (X, f) as an object in $I^\wedge$ namely $N(f) = (N^f_\alpha,\ i^{N(f)}_{\alpha\beta})$. Therefore $\widetilde{X} = (\widetilde{X}_\alpha, \rho^{\widetilde{X}}_{\alpha\beta})$ is the object of $I^\wedge$ defined by $\widetilde{X}_\alpha = X$ and $\rho^{\widetilde{X}}_{\alpha\beta} = Id_X$ for all $\alpha, \beta \in L$ with $\alpha \geqslant \beta$. It is easy to see that $N(f) \in Sub\,\widetilde{X}$ in $I^\wedge$.

Given the bijective correspondence between subobjects and predicates, the pair $(\widetilde{X}, N(f))$ can be identified with the pair $(\widetilde{X}, \widetilde{f})$ where $\widetilde{f}: \widetilde{X} \to \Omega$

is the predicate associated to the subobject $N(f)$ of $\widetilde{X}$. Therefore, according to (2.9) we have $\widetilde{f} = (\widetilde{f}_\alpha)_{\alpha \in L}$ with $\widetilde{f}_\alpha : \widetilde{X}_\alpha \to \Omega_\alpha = [0, \alpha]$, $\widetilde{f}_\alpha(x) = sup\{\alpha' < \alpha / x \in N^f_{\alpha'}\} = sup\{\alpha' \leqslant \alpha / f(x) \geqslant \alpha'\} = f(x) \wedge \alpha$. Therefore, we have $\widetilde{f} = f_1$

If we consider in $I^\wedge$ the unique morphism $\omega_{\widetilde{X}} : \widetilde{X} \to I$, then the subobject $\widetilde{X}^{min} = \widetilde{f}^{-1}((\forall \omega_{\widetilde{X}}) * (\widetilde{f})$ of $\widetilde{X}$ is given by

$$\widetilde{X}^{min}_\alpha = \begin{cases} X^{min} & \text{if } \alpha > \lambda_{min} = \min\limits_X f \\ X & \text{if } \alpha \leqslant \lambda_{min} \end{cases}$$

and

$$\widetilde{X}^{max}_\alpha = \begin{cases} X^{max} & \text{if } \alpha \geqslant \lambda_{max} = \max\limits_X f \\ N^f_\alpha & \text{if } \alpha < \lambda_{max} \end{cases}$$

One can see that the subobjects $\widetilde{X}^{min}$ and $\widetilde{X}^{max}$ include in their structure X^{min} and X^{max}, and $N(f) \in Sub\ \widetilde{X}^{max}$.

In $I^\wedge$ the predicate corresponding to $\widetilde{X}^{min}$ is $p^{min} = (p^{min}_\alpha)_{\alpha \in L} p^{min}_\alpha : X \to \Omega_\alpha = [0, \alpha]$ wherefor $\alpha > \lambda_{min}$, $p^{min}_\alpha(x) = sup\ \{\alpha' \leqslant \alpha / x \in \widetilde{X}^{min}_{\alpha'}\}$ is α if $x \in X^{min}$ and λ_{min} if $x \notin X^{min}$ and for $\alpha < \lambda_{min}$, $p^{min}_\alpha(x) = \alpha$ for all $x \in X$.

In conclusion, considering the pair (X, f) as a pair $(\widetilde{X}, \widetilde{f})$ in $I^\wedge$, we have built, using only the topos properties of $I^\wedge$, two subobjects $\widetilde{X}^{max}$, $\widetilde{X}^{min}$ of X which define the sets X^{max} and X^{min}

$$X^{max} = \widetilde{X}^{max}_1, \qquad X^{min} = \widetilde{X}^{min}_1$$

or, equivalently,

$$X^{max} = (p^{max})^{-1}(1), \qquad X^{min} = (p^{min})^{-1}(1)$$

where $p^{max} : \widetilde{X} \to \Omega$, $p^{min} : \widetilde{X} \to \Omega$ are predicates associated with subobjects $\widetilde{X}^{max}$ and $\widetilde{X}^{min}$.

4. OPTIMIZATION PROBLEMS IN TOPOI. CONFLUENCE

Let E be a topos and Ω its object of classification.

4.1 Definition An optimization problem is a pair (X, p) where $X \in ObE$ and $p : X \to \Omega$ is a predicate.

4.2 Observation This definition is justified tacking into account the previous section which suggests the following.

4.3 Definition A minimization problem relative to (X, p) in E means to find the subobject X^{min} of X defined by the pullback

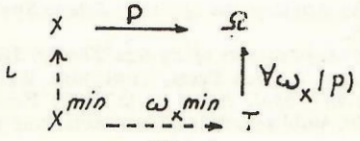


Fig. 4

where $\forall \omega_X : Hom(X, \Omega) \to Hom(T, \Omega)$ is the mapping induced by the morphism $\omega_X : X \to T$ indicated in (1.4).

A maximization problem relative to (X, p) in E means to find the subobject X^{max} of X defined by the pullback

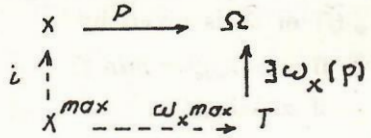


Fig. 5

where $\exists \omega_X : Hom(X, \Omega) \to Hom(T, \Omega)$ is the mapping induced by the morphism $\omega_X : X \to T$ indicated in (1.4).

In what follows, using definition (4.3), we shall reformulate in an arbitrary topos, an extension of the optimization problem. If $t : T \to \Omega$ is the morphism given in definition (1.1) (iii) we shall denote by $< t,t > : T \to \Omega \times \Omega$ the unique morphism defined by the property of Ω_X Ω.

4.4 Definition $\wedge : \Omega \times \Omega \to \Omega$ is by definition the predicate associated in E to the subobject of $\Omega \times \Omega$ defined by the monomorphism $< t, t, >$, and induces a binary operation $(p_1, p_2) \mapsto p_1 \wedge p_2$.

4.5 Definition An optimization problem in a topos E is: given $(X, p_1, p_2, \ldots, p_n)$ find the subobject X^{max} of X relative to $(X, D = p_1 \wedge p_2 \wedge \ldots \wedge p_n)$ according to definition (4.3) (ii).

In conclusion, we have defined in E for every $n \in N$ and for every $X \in ObE$ a mapping

$$O(n, X) : (Hom(X, \Omega))^n \to Sub X$$

$$O(n, X)(p_1, p_2, \ldots, p_n) = D^{-1}(\exists \omega_X(D)) \text{ where}$$

$$D = p_1 \wedge \ldots \wedge p$$

which induces through the izomorphism between subobjects and predicates a n-ary operation on $Hom(X, \Omega)$, respectively $Sub X$. $O(n, X)$ describes in a topos the correspondence between optimization problems on X and their solutions according to definition(4.5).

4.6 Observation In the topos Set the solution of a minimization problem according to definition (4.3)(i) relative to the pair (X, χ_A) where $A \subseteq X$, $\chi_A : X \to [0, 1]$ is the characteristic function of A, is $X^{min} = X - A$ and the solution of a maximization problem is $X^{max} = A$.

4.7 Observation Another way to see an optimization problem in a topos is to define $f : X \to 0$ where 0 is an ordered object of E.

REFERENCES

1. Negoita C. V., Ralescu D.A: *Applications of Fuzzy Sets to System Analysis.* Birkhäuser Verlag, Basel, 1975.
2. Negoita C. V.: *Management Applications of System Theory.* Birkhäuser Verlag, Basel, 1979.
3. Negoita C. V.: *Fuzzy Systems.* Abacus Press, Tunbridge Wells, UK, 1980.
4. Negoita C. V.: *Fuzzy Sets in Topol.* Actes de la Table Ronde sur le Flou. Université de Lyon, Juin 1980. publisched in Fuzzy sets and Systems 8(1982)

14. 03. 1982